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Large-scale vortices in rotating stratified disks

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Abstract. A very new possibility of generation of large-scale flow patterns in turbulent astrophysical bodies is discussed. When the global rotation and the fluid stratification are simultaneously involved, the Reynolds stress tensor contains contributions linear in the mean velocity. Then for sufficiently rapid rotation the system becomes unstable to large-scale motions. The mean velocity equation is very similar to the mean magnetic field equation of dynamo theory. A plane "dynamo-wave" solution of the Reynolds equation is found in a further analogy to the hydromagnetic dynamo. The numerical solution of a 1D eigenvalue problem leads to a rather low critical "dynamo number" ($\simeq 6$) implying that the new mean-flow excitation can indeed act in rotating and stratified astrophysical disks (and spheres?).

Key words: magnetohydrodynamics – turbulence – accretion disks

1. Introduction

There are several observational hints motivating the search for a mechanism capable of producing large-scale vortices in rotating astrophysical bodies. Brandt et al. (1988) reported an observation of a vortex on the solar surface. It clearly dominated the granulation and persisted for more than 1 hour. If real, such vortices may provide an important mechanism for heating of the solar corona by twisting the foot-points of coronal magnetic flux tubes (Parker 1979).

Stimulating observations have also been reported by Abramowicz et al. (1992). They suggest that hot accretion disks produce coherent vortices. Modification of X-ray spectra due to the vortices may explain the short-term variability and the low degree of polarisation. Also the global flow pattern in spiral galaxies can be mentioned here. It could be formed by azimuthally migrating large-scale horizontal vortices.

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Recently Khomenko et al. (1991) have discussed whether a large-scale flow in turbulent fluids may follow the equation,

$$\frac{\partial \tilde{\boldsymbol{\Omega}}}{\partial t} = \Gamma \text{ rot } \tilde{\boldsymbol{\Omega}} + \nu_T \Delta \tilde{\boldsymbol{\Omega}} \tag{1}$$

with

$$\tilde{\boldsymbol{\Omega}} = \text{rot } \overline{\boldsymbol{u}}, \tag{2}$$

hence

$$\text{div}\, \tilde{\boldsymbol{\Omega}} = 0, \tag{3}$$

where the coefficients ν_T and Γ are considered uniform (cf. Moffatt & Tsinober 1992). Equation (1) is identical with the well-known magnetic induction equation for the α^2-dynamo. As the quantity Γ must be a pseudo-scalar, we suppose it originating only in rotating media.

As in the dynamo theory, the presence of the Γ-term in the RHS of (1) strongly suggests the existence of a large-scale instability. The large-scale vortex, $\tilde{\boldsymbol{\Omega}}$, originates from the action of the turbulence parametrized by the coefficients Γ and ν_T.

After (1) the angular momentum conservation for the mean flow is violated. This equation can be found for a special case of *compressible* turbulent flow with prescribed helicity. Under these conditions, the momentum accumulated by the mean motion is balanced by the momentum get by the turbulence: $\overline{\rho}\, \overline{\boldsymbol{u}} = < \rho' \boldsymbol{u}' >$. For *incompressible* flows, to which case the present paper is restricted, the conservation laws are guaranteed by the symmetry properties of the stress tensor. Therefore, Eq. (1) cannot be realized.

As we shall demonstrate, however, when global rotation and the fluid stratification are simultaneously involved the Reynolds stress tensor attains a contribution linear in the mean velocity. As a consequence, the fluid is unstable to the mean flow. The analogy to the dynamo theory is not as straight as for Eq. (1) but it survives. It must be mentioned in this context that rotation and inhomogeneity are also the basic conditions for the magnetohydrodynamic α-effect (Krause & Rädler 1980). The same model of rotating and inhomogeneous turbulence is used below which has been recently applied to derive the MHD α-effect (Rüdiger & Kitchatinov 1993). Thus we expect that dynamo action and vortex generation are always corresponding and even interacting.

The estimations of the Sect. 4 demonstrate that the mean-flow instability may indeed be active under real conditions. Hence, the large-scale magnetic fields as well as the velocity fields of astrophysical bodies under diverse circumstances may have a common origin.

The mean-flow dynamics in turbulent media is governed by the Reynolds equation discussed in Sect. 2. As its new turbulence-related part the correlation tensor for a turbulence field subjected to a large-scale flow is derived in Sect. 3. Quite similar as in the dynamo theory the new non-dissipative term is able to drive a large-scale instability ("inverse cascade") if its magnitude exceeds a critical (eigen-)value (Sect. 4).

2. Macroscopy

The subject of the present paper is close to the "kinetic alpha-effect" by Frisch et al. (1987). It can be included as the RHS in the Reynolds equation governing the mean flow, $\overline{\boldsymbol{u}}$, of a turbulent fluid,

$$\frac{\partial \overline{u}_i}{\partial t} + (\overline{\boldsymbol{u}} \cdot \nabla)\,\overline{u}_i - \nu_T \Delta \overline{u}_i + \frac{1}{\overline{\rho}}\frac{\partial \overline{p}}{\partial x_i} = -\Gamma_{ijk}\overline{u}_{k,j}. \quad (4)$$

Here the tensor Γ is involved which is identical to the Λ-tensor introduced by Krause & Rüdiger (1974). It is derived in the present paper for rotating and gravitationally stratified turbulence.

We restrict ourselves to the incompressible case. The Reynolds equation then reads

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial T_{ij}}{\partial x_j} = \overline{f}_i/\overline{\rho}, \quad (5)$$

where the bars above letters signify averaging, $\overline{\boldsymbol{f}}$ denotes all the external mean forces.

The stress tensor T has the known structure

$$T_{ij} = \overline{u}_i\overline{u}_j + Q_{ij} + \frac{\overline{p}}{\overline{\rho}}\,\delta_{ij} + ... \quad (6)$$

with the correlation tensor $Q_{ij} =< u'_i u'_j >$. The standard notations for mean and fluctuating fields are used, e.g., $\boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}'$.

The tensor Q_{ij} is obviously *symmetric*: $Q_{ij} = Q_{ji}$. It is thus *not* possible to apply an ansatz such as

$$Q_{ij} \propto \Gamma\ \varepsilon_{ijk}\overline{u}_k, \quad (7)$$

(which would directly lead to (1)) because (7) violates the symmetry condition (Krause & Rüdiger 1974).

It is reasonable, however, to re-formulate the problem. To this end it is sufficient to assume the existence of a linear relation between Q and $\overline{\boldsymbol{u}}$. As a Taylor expansion we find

$$Q_{ij} = Q_{ij}^{(0)} + \ \Gamma_{ijk}\overline{u}_k \ \ - N_{ijkl}\overline{u}_{k,l} + ... \quad (8)$$

The series expansion may be truncated after the given viscosity term. For the isotropic case the latter may be formulated as usual (cf. Stix et al. 1993),

$$N_{ijkl} = \nu_T(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) \ + \ \mu_T\delta_{ij}\delta_{kl} \quad (9)$$

with ν_T and μ_T as the turbulent viscosities.

The construction of the Γ-part of Eq. (8) is more complicated. Due to its symmetry in i and j it only exists with a preferred characteristic direction, $\boldsymbol{g}$. With this anisotropy involved to its first order it becomes

$$\Gamma_{ijk} = \gamma_1\delta_{ij}g_k + \gamma_2(\delta_{ik}g_j + \delta_{jk}g_i). \quad (10)$$

If, however, the global rotation is also included, there are further contributions:

$$\Gamma_{ijk} \ = \ ... + \gamma_3(\varepsilon_{ikl}\Omega_j + \varepsilon_{jkl}\Omega_i)g_l + \gamma_4\left(\varepsilon_{ilk}g_j + \varepsilon_{jlk}g_i\right)\Omega_l + \ \gamma_5\left(\delta_{ik}\varepsilon_{jlp} + \delta_{jk}\varepsilon_{ilp}\right)\Omega_l g_p. \quad (11)$$

Only terms linear in the angular velocity, $\boldsymbol{\Omega}$, are kept. We shall find in Sect. 4 that the mean-flow instability is indeed generated by the rotationally-created part (11) of the Γ-tensor. Therefore, a basic rotation *and* an anisotropy must be simultaneously covered by the treatment.

It should be mentioned that the Γ-tensor of (10) and (11) is not another representation of the Λ-effect of the theory of stellar differential rotation (cf. Schüssler 1984). The latter has the second order in the anisotropy, $\boldsymbol{g}$, and involves the uniform rotation, $\boldsymbol{\Omega}$, rather than the mean flow, $\overline{\boldsymbol{u}}$ (Rüdiger 1989).

3. Microscopy

All our derivations belong to the quasilinear approximation in which the linearised equations are applied to derive the tensor Q_{ij}. The same method is also called the second-order correlation approximation or the first-order smoothing in literature (cf. Moffatt 1978; Krause & Rädler 1980; Rüdiger 1989).

An incompressible turbulent flow subjected to simultaneous action of global rotation and uniform mean flow obeys the (linearised) Reynolds equation,

$$\frac{\partial \boldsymbol{u}'}{\partial t} + 2\,\boldsymbol{\Omega} \times \boldsymbol{u}' + (\overline{\boldsymbol{u}} \cdot \nabla)\boldsymbol{u}' + \frac{1}{\overline{\rho}}\nabla p' - \nu_t\Delta\boldsymbol{u}' = \frac{\boldsymbol{f}'}{\overline{\rho}}, \quad (12)$$

where p' is the pressure fluctuation, ν_t is the effective viscosity produced by a small-scale background turbulence or a microscopic one, and $\boldsymbol{f}'$ is a random force driving the turbulence. It is reasonable to use the Fourier transforms in form to

$$\boldsymbol{u}'(\boldsymbol{r}, \boldsymbol{t}) = \int \hat{\boldsymbol{u}}(\boldsymbol{k}, \omega)\ \mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} d\boldsymbol{k}\ d\omega. \quad (13)$$

The flow is assumed to be incompressible, i.e.

$$\mathrm{div}\ \boldsymbol{u}' = 0. \quad (14)$$

Equation (12) now reads

$$(-i\omega + \nu_t k^2)\hat{u}_i + 2(\boldsymbol{k}^\circ \cdot \boldsymbol{\Omega})\varepsilon_{ijp}k_j^\circ\hat{u}_p + i(\overline{\boldsymbol{u}} \cdot \boldsymbol{k})\hat{u}_i = \hat{f}_i^s/\overline{\rho}, \quad (15)$$

where $\boldsymbol{k}^\circ = \boldsymbol{k}/k$ is the unit vector in the direction of the wave-vector $\boldsymbol{k}$, the pressure has been eliminated by the incompressibility condition, $\boldsymbol{k}\cdot\hat{\boldsymbol{u}} = 0$, and hats above letters signify Fourier-amplitudes.

The solution of Eq. (15) linear in $\boldsymbol{\Omega}$ and $\overline{\boldsymbol{u}}$ (but products of $\boldsymbol{\Omega}$ and $\overline{\boldsymbol{u}}$ are kept) reads

$$\hat{u}_i = \left[\delta_{ij} + \frac{2(\boldsymbol{k}^\circ\cdot\boldsymbol{\Omega})}{-i\omega+\nu_t k^2}\varepsilon_{ijp}k^\circ_p - \frac{i(\overline{\boldsymbol{u}}\cdot\boldsymbol{k})}{-i\omega+\nu_t k^2}\left(\delta_{ij} + \frac{4(\boldsymbol{k}^\circ\cdot\boldsymbol{\Omega})}{-i\omega+\nu_t k^2}\varepsilon_{ijp}k^\circ_p\right)\right]\hat{u}^{(0)}_j. \quad (16)$$

The velocity field $\hat{\boldsymbol{u}}^{(0)}$ in the above equation corresponds to the so-called "original" turbulence which the random force would drive if either the global rotation and the flow $\overline{\boldsymbol{u}}$ were absent:

$$\hat{\boldsymbol{u}}^{(0)} = \frac{\hat{\boldsymbol{f}}^s}{\overline{\rho}(-i\omega+\nu_t k^2)}. \quad (17)$$

It remains to define the original turbulence. The spatial inhomogeneity of the turbulence is important for what follows. The simplest representation for the inhomogeneous case is the quasi-isotropic model by Kitchatinov (1987). It bases on the double-Fourier method by Roberts & Soward (1975) to handle large-scale spatial inhomogeneities. It reads

$$< u^{(0)}_i(\boldsymbol{r},t)u^{(0)}_j(\boldsymbol{r}+\boldsymbol{\xi},t+\tau) >= = \int \hat{M}_{ij}(\boldsymbol{k},\boldsymbol{\kappa},\omega)e^{i\,(\boldsymbol{\kappa}\cdot\boldsymbol{r}\,+\,(\frac{\boldsymbol{\kappa}}{2}+\boldsymbol{k})\cdot\boldsymbol{\xi}-\omega\tau)}d\boldsymbol{k}d\boldsymbol{\kappa}d\omega \quad (18)$$

with

$$\hat{M}_{ij} = \frac{\hat{E}(k,\omega,\boldsymbol{\kappa})}{16\pi k^2}\left[\delta_{ij} - k^\circ_i k^\circ_j + \frac{1}{2k^2}\left(\kappa_i k_j - \kappa_j k_i\right)\right], \quad (19)$$

where $\boldsymbol{k}$ and $\boldsymbol{\kappa}$ are wave-vectors for different scales. Only first-order terms in the turbulence gradient are included. The quantity $\hat{E}$ in (19) is the Fourier-transform of the local spectrum E,

$$< u^{(0)2} >= \int_0^\infty E(k,\omega,\boldsymbol{r})\,dk\,d\omega, \quad (20)$$

with

$$E(k,\omega,\boldsymbol{r}) = \int \hat{E}(k,\omega,\kappa)\ e^{i\,\boldsymbol{\kappa}\cdot\boldsymbol{r}}\,d\boldsymbol{\kappa}. \quad (21)$$

The turbulence (18), (19) subjected to the influence of the global rotation directly leads to the MHD α-effect (Rüdiger & Kitchatinov 1993). We now show that it also drives the kinetic Γ-effect under the same conditions.

The spectral tensor of the velocity fluctuations can be found from (16) and (19) with the three principal ingredients of the present theory, i.e. rotation, uniform mean flow, and stratification involved to the first-order terms. Then, the inverse Fourier transform yields the Γ-tensor after (8):

$$\Gamma_{ijk} = \gamma_1\delta_{ij}g_k + \gamma_2\left(\delta_{ik}g_j + \delta_{jk}g_i\right) + \gamma\left(\delta_{ik}\varepsilon_{jlp}\Omega_l + \delta_{jk}\varepsilon_{ilp}\Omega_l + \varepsilon_{jkp}\Omega_i + \varepsilon_{ikp}\Omega_j\right)g_p, \quad (22)$$

where $\boldsymbol{g} = \nabla\log u_T$ is the relative gradient of the rms turbulent velocity $u_T = \sqrt{< u'^2 >}$. The coefficients in (22) are defined through

$$\gamma_1 = -\frac{1}{15}\int_0^\infty\int_0^\infty \frac{\nu_t k^2(\nu_t^2k^4+9\omega^2)}{(\nu_t^2k^4+\omega^2)^2}E(k,\omega,\boldsymbol{r})\,dk\,d\omega,$$

$$\gamma_2 = -\frac{1}{15}\int_0^\infty\int_0^\infty \frac{\nu_t k^2(\nu_t^2k^4-\omega^2)}{(\nu_t^2k^4+\omega^2)^2}E(k,\omega,\boldsymbol{r})\,dk\,d\omega,$$

$$\gamma = \frac{1}{15}\int_0^\infty\int_0^\infty \frac{(\nu_t^2k^4-3\omega^2)}{(\nu_t^2k^4+\omega^2)^2}E(k,\omega,\boldsymbol{r})\,dk\,d\omega. \quad (23)$$

Comparison with Eq. (11) shows that $\gamma_3 = \gamma_5 = \gamma$ and $\gamma_4 = 0$ for our turbulence model.

The Γ-tensor (22) is not invariant under Galilean transformations which is already known from the AKA-effect (Frisch et al. 1987, cf. Eq. (10) above). The reason is that the statistical characteristics of the turbulence driving forces are also non-invariant. The relevant non-equivalence of the reference systems can be easily found in nature. E.g., if the interstellar turbulence is indeed driven by supernova explosions then the reference frame in which the local stellar population is at rest is obviously preferred. Another reason for non-invariance are the boundaries of a body.

Note that the quasilinear approximation applied is strictly justified for small Strouhal numbers only, $S = \tau_{corr}u_T/l_{corr} \ll 1$, with τ_{corr} and l_{corr} as the correlation time and correlation length (Krause & Rädler 1980). In this limit, the typical frequency scale of the spectral function is large compared to the frequencies producing dominant contributions to the integrals (23). Then $E(k,0,\boldsymbol{r})$ can be substituted in these integrals (the white-noise approximation) to get

$$\gamma_1 = -\frac{\pi}{6}\int_0^\infty E(k,0,\boldsymbol{r})\,dk, \qquad \gamma_2 = 0,$$

$$\gamma = -\frac{\pi}{30\nu_t}\int_0^\infty \frac{E(k,0,\boldsymbol{r})}{k^2}\,dk. \quad (24)$$

Only with rotational influence involved the Γ-tensor (22) is different from a trivial contribution of an isotropic turbulent pressure in this limit. We put $\gamma_2 = 0$ in the following as suggested by (24).

4. Mean-flow instability

The averaged equation of motion in the co-rotating frame of reference reads

$$\frac{\partial\overline{u}_i}{\partial t} + (\overline{\boldsymbol{u}}\cdot\nabla)\,\overline{u}_i + 2\varepsilon_{ijk}\Omega_j\overline{u}_k + \frac{1}{\rho}\frac{\partial\overline{p}}{\partial x_i} = = \nu_T\Delta\overline{u}_i - \frac{\partial}{\partial x_j}\Gamma_{ijk}\overline{u}_k. \quad (25)$$

With (22) for the Γ-tensor the system can become unstable to a large-scale flow pattern. We consider the instability for the simple case of a plane geometry with $\boldsymbol{\Omega}\|\boldsymbol{g}$ and the system being uniform in planes normal to $\boldsymbol{\Omega}$. The mean velocity is also assumed normal to the $\boldsymbol{\Omega}$-direction and varying only along this

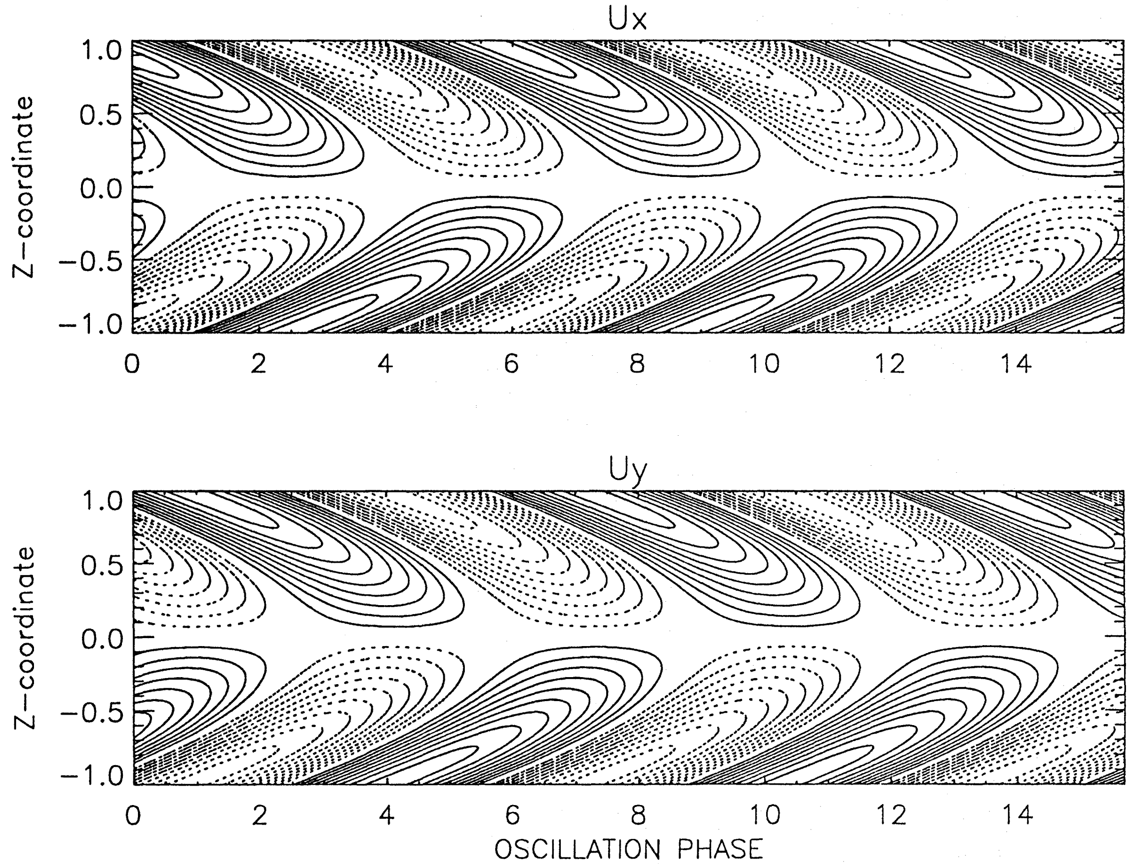

Fig. 1. Contour lines for the velocity components $\overline{u}_x$ (above) and $\overline{u}_y$ (below) of the fundamental mode (phase = σt). Positive values (solid) are distinguished from negative values (dotted), C_Γ = 5.98

direction. In a Cartesian coordinate system with its z-axis parallel to $\mathbf{\Omega}$ our assumptions are

$$\overline{\boldsymbol{u}} = \{\overline{u}_x(z), \overline{u}_y(z), 0\}, \quad \boldsymbol{g} = \{0, 0, g(z)\}. \tag{26}$$

Due to incompressibility the vertical flow, $\overline{u}_z$, identically vanishes. The components of Eq. (25) now read,

$$\begin{aligned} \frac{\partial \overline{u}_x}{\partial t} &= \nu_T \frac{\partial^2 \overline{u}_x}{\partial z^2} - \frac{\partial \Gamma \overline{u}_y}{\partial z} + 2\Omega \overline{u}_y, \\ \frac{\partial \overline{u}_y}{\partial t} &= \nu_T \frac{\partial^2 \overline{u}_y}{\partial z^2} + \frac{\partial \Gamma \overline{u}_x}{\partial z} - 2\Omega \overline{u}_x, \end{aligned} \tag{27}$$

where the coefficient

$$\Gamma = \gamma(\boldsymbol{g} \cdot \mathbf{\Omega}) \tag{28}$$

is introduced. Equations (27) are similar to the α^2-dynamo equations for plane geometry. In the simple disk-geometry they formally correspond to the vortex-equation (1).

The velocity components, $\overline{u}_x$ and $\overline{u}_y$, may be combined into the complex function,

$$W = \overline{u}_x + i\,\overline{u}_y. \tag{29}$$

The system (27) can then be written in the single complex equation,

$$\frac{\partial W}{\partial t} = \nu_T \frac{\partial^2 W}{\partial z^2} + i\frac{\partial \Gamma W}{\partial z} - 2i\Omega W. \tag{30}$$

4.1. Local analysis

If the spatial scale of the mean flow is small compared to that of the Γ-coefficient (28), Γ can be assumed to be uniform. Then a simple plane-wave solution, $W \sim \exp(ikz+\sigma t)$, can be found. Its substitution into (30) results into the dispersion relation,

$$\sigma = -\nu_T k^2 - \Gamma k - 2i\Omega. \tag{31}$$

The growth rate, Re(σ), is positive for sufficiently small k:

$$-\Gamma/\nu_T < k < 0 \ \text{for} \ \Gamma > 0, \qquad \Gamma/\nu_T > k > 0 \ \text{for} \ \Gamma < 0.$$

Hence, the system is unstable to plane-wave perturbations with sufficiently large wave-lengths. The growth rate attains its maximal value at the wave-number $k_{max} = |\Gamma|/(2\nu_T)$.

For further steps we need estimates for Γ. Simple mixing-length arguments applied to (23) suggest $\gamma \sim \tau_{corr}^2 < u'^2 > \sim l_{corr}^2$. Then from (28), $\Gamma \propto l_{corr}^2 \Omega/H$ with $H = 1/|g|$ as the stratification scale. For the scale $l_{max} = 2\pi/k_{max}$ of the disturbances corresponding to the maximum growth rate we find $l_{max}/H \sim 10/\Omega^*$, where $\Omega^* = 2\tau_{corr}\Omega$ is the Coriolis number. Therefore, the local analysis can be applied only to the very rapid rotators ($\Omega^* \gg 1$) which are seldom (if any at all) among the astrophysical disks. Hence, the local analysis is mainly limited to illustration al purposes. Next, the dimensions of the astrophysical bodies are typically not much larger compared to the inhomogeneity length and boundary conditions must be imposed on the system.

4.2. Boundary value problem

The simplest one-dimensional case with boundaries is the plane layer, a disk. The z-axis of our coordinate system is normal to the disk plane. We normalise the distances and times to the half-thickness, H, of the disk and to the diffusive time, H^2/ν_T, respectively, and rewrite Eq. (30) in dimensionless variables,

$$\frac{\partial W}{\partial t} = \frac{\partial^2 W}{\partial z^2} + iC_\Gamma \frac{\partial \hat{\Gamma} W}{\partial z} - i\ \mathrm{Ta}^{1/2}\ W, \tag{32}$$

where $\hat{\Gamma}$ is a dimensionless function of order unity, $\Gamma = \Gamma_0 \hat{\Gamma}(z)$, scaling the kinetic Γ-effect, $\mathrm{Ta} = 4\Omega^2 H^4/\nu_T^2$ is the Taylor number, and $C_\Gamma = \Gamma_0 H/\nu_T$. The boundaries of the disk are at $z = \pm 1$. Stress-free boundary conditions are imposed, $T_{zx} = T_{zy} = 0$, which in our notation read

$$\frac{\partial W}{\partial z} + i\hat{\Gamma} C_\Gamma W = 0. \tag{33}$$

As usual, the linear stability analysis reduces to an eigenvalue problem with $W = \exp(\sigma t)\hat{W}(z)$. The Taylor number dependence of (32) can be "included" in the eigenvalue σ,

$$\left(\sigma + i\ \mathrm{Ta}^{1/2}\right) \hat{W}(z) = \frac{\partial^2 \hat{W}(z)}{\partial z^2} + iC_\Gamma \frac{\partial \hat{\Gamma}(z)\hat{W}(z)}{\partial z}, \tag{34}$$

so that only one parameter, C_Γ, remains.

The disk stratification is symmetric about the mid-plane, $z = 0$, but the Γ must be an anti-symmetric function. The simple ansatz, $\hat{\Gamma}(z) = -\sin(\pi z/2)$, is used in our model. An infinite series of eigenvalues and corresponding eigenfunctions exists. For $C_\Gamma = 0$, all the eigenvalues, $\sigma' = \sigma + i\ \mathrm{Ta}^{1/2}$, are real and negative. We derived the C_Γ-dependences of the five smallest in absolute magnitude eigenvalues numerically. The sign of C_Γ does not influence the eigenvalues. While C_Γ increases, the eigenvalues become complex showing oscillatory solutions. The real parts of the eigenvalues are negative until C_Γ reaches C_{cr} at which the first marginally stable flow pattern appears.

Our result is $C_{cr} = 5.98$ with the normalised frequency $\Im(\sigma') = 3.77$. The corresponding eigenfunction - the "fundamental mode" - is displayed in Fig.1. The flow is antisymmetric about the mid-plane of the disk (dipole-parity solution). At sufficiently large C_Γ all considered eigenfunctions become excited. It may be expected from analogy with dynamo theory, however, that only the fundamental mode is stable in the weakly nonlinear regime (Krause & Meinel 1988).

The C_Γ-parameter is very similar to the corresponding C_α of dynamo models. $C_{cr} = 5.98$ has the same order of magnitude as the C_α's expected for various astrophysical objects, e.g., galaxies. The considered mean-flow generation is thus anticipated to work there. Of course, the above theory is only a first view to the possibilities. More elaborated 2D or even 3D models are needed. Only with such models the horizontal structure of a generated flow pattern can be resolved.

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