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Slow magnetosonic oscillations with $m \gg 1$ in a dipole magnetosphere with rotating plasma

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[1] A mechanism of excitation of azimuthal small-scale ($m \gg 1$) slow magnetosonic (SMS) oscillations by the external currents in the near-equatorial region of the terrestrial magnetosphere is proposed. A theoretical study of the structure and spectrum of such oscillations is performed in the model of the magnetosphere with the dipole magnetic field and rotating plasma. In the direction along field lines of the geomagnetic field the oscillations are waves standing between the magnetically conjugated points. The amplitude of the magnetospheric SMS oscillations with $m \gg 1$ decreases from the equatorial plane to the ionosphere at scales much lower then the field line length. So one is not able to observe the SMS-oscillation field in the vicinity of the ionosphere and on the surface of the Earth. In the direction across the magnetic shells, the standing SMS waves have a structure typical for resonant oscillations. Latitudinal distributions of eigenfrequencies of several first harmonics of SMS oscillations with the polarization similar to the polarization of poloidal and toroidal Alfvén waves in the plane perpendicular to the field lines are drawn. *INDEX TERMS:* 2740 Magnetospheric Physics: Magnetospheric configuration and dynamics; 2736 Magnetospheric Physics: Magnetosphere/ionosphere interactions; 2799 Magnetospheric Physics: General or miscellaneous; *KEYWORDS:* Slow magnetosonic oscillations; Dipole-like magnetosphere; Structure of resonant SMS oscillations.

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1. Introduction

[2] The magnetosphere of the Earth is an equilibrium plasma configuration where various types of MHD oscillations can be excited. In the spectrum of electromagnetic oscillations of the magnetosphere they belong to its lowest part with frequencies from a few Hz to a fractions of mHz [*Guliel'mi*, 1979].

[3] There are three branches of MHD oscillations in the homogeneous plasma: the Alfvén waves, fast magnetic sound (FMS) and slow magnetic sound (SMS). The Alfvén waves propagate almost along the field lines of the magnetic field. Under some conditions, the SMS waves also are able to canalize in the direction of magnetic field lines. The group velocity of a FMS wave is directed along the wave vector. Due to all this, the Alfvén and SMS waves are often called directed modes, whereas the FMS oscillations are called an isotropic mode. Unlike the Alfvén and FMS oscillations, the SMS oscillations exist only in the plasma with a finite (different from the zero) gas pressure.

[4] In the inhomogeneous plasma, all three branches of the MHD oscillations are related. This relation is realized at resonant surfaces, where an interaction of various oscillation branches occurs. The mechanism of the field line resonance [*Chen and Hasegawa*, 1974; *Southwood*, 1974; *Tamao*, 1965] is the most known mechanism. In the process of monochromatic FMS wave propagation in the plasma inhomogeneous along one of the transverse coordinates, there occurs an amplification of the Alfvén oscillations at the resonant magnetic shell. The later is determined by the equality between the FMS-wave frequency and local frequency of the Alfvén oscillations. Similar process is possible at interaction of FMS and SMS oscillations. For the case of one-dimensionally inhomogeneous plasma, the theory of this phenomenon was developed by *Yumoto* [1985]. For the dipole model of the magnetosphere, the theory of such resonant interaction was presented by *Leonovich et al.* [2006] where the interaction was called magnetosonic resonance.

[5] It should be noted that such interaction is effective only for rather large-scale in the azimuthal direction oscillations. In axisymmetrical models of the magnetosphere (including the model with the dipole magnetic field), an arbitrary dis-

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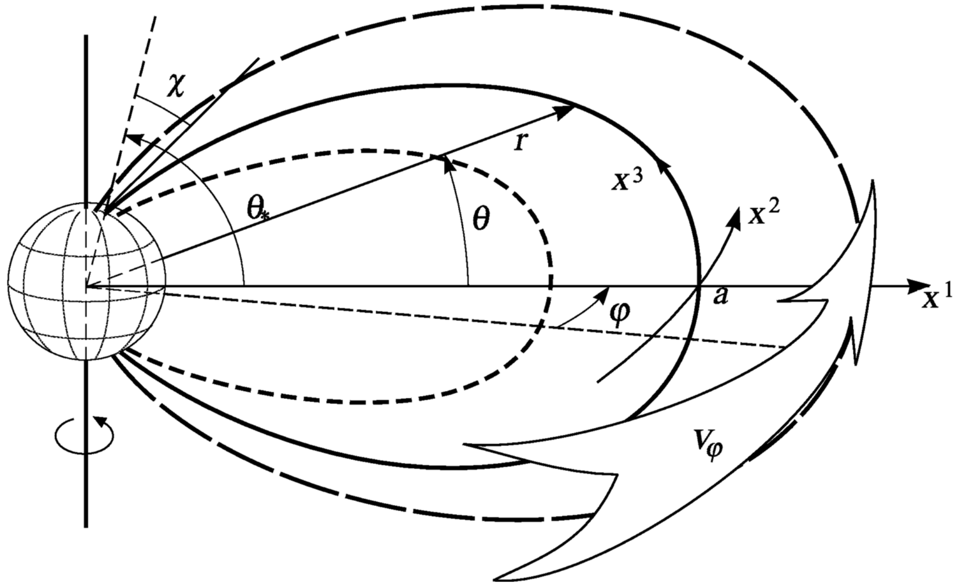

Figure 1. Curved orthogonal coordinate system (x^1, x^2, x^3) connected to the magnetic field lines and nonorthogonal coordinate system (a, ϕ, θ) used in the numerical calculations.

turbance may be presented as the sum of azimuthal harmonics of the form $\exp(im\varphi)$ where m is the azimuthal wave number and φ is the azimuthal angle. The FMS oscillations with $m \sim 1$ are able to penetrate into the magnetosphere with the amplitude large enough for a resonant amplification of the directed model of MHD oscillations. The waves with $m \gg 1$ almost do not penetrate into the inner magnetosphere. So for a resonant amplification of the Alfvén and SMS waves, one should consider sources localized at resonant magnetic shells. For the Alfvén waves with $m \gg 1$, *Leonovich and Mazur* [1993] proposed as such a source the external currents in the ionosphere. Unfortunately, one can not use such source for the SMS oscillations with $m \gg 1$. *Leonovich et al.* [2006] showed that the amplitude of standing SMS waves decreases sharply from the equatorial plane to the ionosphere, so the ionosphere can play a role neither in the process of excitation, nor in the process of dissipation of these oscillations. *Southwood* [1977] suggested as a source of the SMS oscillations with $m \gg 1$ their resonant interaction to the Alfvén waves. The theory of this process was developed for the dipole model of the magnetosphere with finite plasma pressure [*Taylor and Walker*, 1987; *Walker*, 1987; *Walker and Pekrides*, 1996]. However *Leonovich et al.* [2006] showed that the eigenfrequencies of the main harmonics of the Alfvén and SMS oscillations at the closed field lines differ by 2–3 orders of magnitude. Therefore, the interaction of these branches of MHD oscillations should be very inefficient.

[6] As a mechanism of excitation of localized SMS oscillations with a large azimuthal wave number, we consider in this paper the currents in the near-equatorial region of the magnetosphere. Out of these sources, the most evident is the ring current developing during strong geomagnetic disturbances in the near-equatorial region of the magnetosphere. Other components of the magnetospheric current system are also able to serve as a source of SMS oscillations. Moreover, the influence of the magnetospheric plasma motion on the structure and spectrum of the SMS eigenoscillations is studied for the first time. *Kozlov et al.* [2006] performed a similar study for the Alfvén waves.

[7] The paper has the following structure. In Section 2 the environment model is presented and the main equations describing the structure and spectrum of SMS oscillations with large azimuthal wave numbers in axial-symmetric magnetosphere with rotating plasma are derived. The field-aligned structure and spectrum of the standing SMS waves with the poloidal and toroidal polarization are studied in Sections 3 and 4. An exact expression for the polarization splitting of the spectrum is also obtained. In Section 5 the model equation determining the structure of the SMS oscillations with $m \gg 1$ across magnetic shells is obtained and its solution is found. Analytical expressions describing field components of the resonant SMS waves in the vicinity of the toroidal resonant magnetic shell are obtained in Section 6. The results of the numerical solution of the equations describing the structure and spectrum of the studied oscillations are presented in Section 7. The main results of the work are summarized in the Conclusions.

2. The Environment Model and Main Equations

[8] For solving of the formulated problem we use the self-consistent model of the magnetosphere with the dipole field and plasma rotating in the azimuthal direction (Fig-

ure 1). The detailed description of the model was presented by *Leonovich et al.* [2004]. The dynamical equilibrium of the plasma configuration is maintained in this model by the centrifugal forces and gradient of the gas kinetic pressure.

[9] We introduce the curved orthogonal coordinate system (x^1, x^2, x^3) connected to the field lines of the magnetic field. The coordinates x^3 and x^1 are directed along and across the field lines, respectively, whereas x^2 completes the coordinate system up to the right-hand side one. The length element squared is determined in this coordinate system as

$$ds^2 = g_1(dx^1)^2 + g_2(dx^2)^2 + g_3(dx^3)^2$$

We will assume that the plasma and magnetic field are homogeneous along the x^2 coordinate. For description of the MHD oscillations we use the equation system of an ideal MHD:

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla P + \frac{1}{4\pi}[\mathrm{curl}\,\mathbf{B}\times\mathbf{B}] - \frac{1}{c}[\tilde{\mathbf{J}}\times\mathbf{B}] \tag{1}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \mathrm{curl}[\mathbf{v}\times\mathbf{B}] \tag{2}$$

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\mathbf{v}) = 0 \tag{3}$$

$$\frac{d}{dt}\frac{P}{\rho^\gamma} = 0 \tag{4}$$

where $\mathbf{B}$ and $\mathbf{v}$ are vectors of the magnetic field strength and the plasma motion velocity, respectively, ρ and P are the plasma density and pressure, respectively, γ is the adiabatic index, and $\tilde{\mathbf{J}}$ is the density of the external magnetospheric currents.

[10] The stationary $(\partial/\partial t = 0)$ system of equations (1)–(4) describes the distribution of the parameters of the undisturbed magnetosphere: $\mathbf{B}_0, \mathbf{v}_0, \rho_0$ and P_0. We linearize system (1)–(4) relative to small disturbances related to MHD oscillations. We designate the disturbed components of the field as $\tilde{\mathbf{B}}, \tilde{\mathbf{v}}, \tilde{\rho}$ and $\tilde{P}$. We will consider monochromatic oscillations $\propto \exp(ik_2x^2 - i\omega t)$ where ω is the oscillation frequency, k_2 is the azimuthal wave number (if x^2 is the azimuthal angle, then $k_2 = m = 1, 2, 3, \ldots$). After linearization of the component of the vector equation, (1) takes the form

$$-i\bar{\omega}\rho_0 v_1 - \frac{1}{2}\tilde{\rho}\Omega^2(\nabla_1 g_2) - \rho_0 v_2\Omega(\nabla_1 \ln g_2) =$$

$$-\nabla_1\tilde{P} + \frac{1}{4\pi}\frac{1}{\sqrt{g_3}}B_0(\nabla_3 B_1 - \nabla_1 B_3) - \frac{1}{4\pi\sqrt{g_2}}J_2B_0 \tag{5}$$

$$-i\bar{\omega}\rho_0 v_2 + \rho_0 v_1(\nabla_1\Omega)\frac{g_2}{g_1} +$$

$$\rho_0 v_1\Omega\frac{1}{g_1}(\nabla_1 g_2) + \rho_0 v_3\Omega\frac{1}{g_3}(\nabla_3 g_2) =$$

$$-ik_2\tilde{P} + \frac{1}{4\pi}\frac{1}{\sqrt{g_3}}B_0(\nabla_3 B_2 - ik_2B_3) + \frac{1}{4\pi\sqrt{g_1}}J_1B_0 \tag{6}$$

$$i\bar{\omega}\rho_0 v_3 + \frac{1}{2}\tilde{\rho}\Omega^2(\nabla_3 g_2) + \rho_0 v_2\Omega(\nabla_3 \ln g_2) = \nabla_3\tilde{P} \tag{7}$$

where v_i and B_i $(i = 1, 2, 3)$ are the covariant components of the disturbed velocity $\tilde{\mathbf{v}}$ and magnetic field $\tilde{\mathbf{B}}$ respectively,

$$J_i(\omega, k_2) = \frac{1}{2\pi}\int\int_{-\infty}^{\infty}\tilde{J}_i\exp(i\omega t - ik_2x^2)dtdx^2$$

are the Fourier components of the density of the external currents in the magnetosphere, $\bar{\omega} = \omega - k_2\Omega$ is the wave frequency modified by the Doppler effect, Ω is the angular velocity of the rotation of the magnetospheric plasma, and $\nabla_i \equiv \partial/\partial x^i$.

[11] For the description of the MHD oscillation field it is convenient to use presentation of the oscillation fields $\tilde{\mathbf{B}}$, $\tilde{\mathbf{v}}$, $\tilde{\rho}$, $\tilde{\mathbf{P}}$ via potentials [*Leonovich and Mazur*, 1989]. According to the Helmholtz expansion theorem [*Korn and Korn*, 1968], an arbitrary vector field in every point of which its first derivative is determined may be presented in the form of a sum of potential and eddy fields. For the vector field $\tilde{\mathbf{E}}$ this expansion has the form

$$\tilde{\mathbf{E}} = -\nabla_\perp\varphi + \nabla_\perp\times\mathbf{\Psi}$$

where $\nabla_\perp \equiv (\nabla_1, \nabla_2)$, and φ and $\mathbf{\Psi} = (0, \zeta, \psi)$ are the scalar and vector potentials, respectively.

[12] Using linearized equations (1)–(4) and linearized equation of magnetic field line freezing-in

$$\tilde{\mathbf{E}} = -\frac{1}{c}(\mathbf{v}_0\times\tilde{\mathbf{B}} + \tilde{\mathbf{v}}\times\mathbf{B}_0)$$

one can express the disturbed field components via the potentials φ, ζ and ψ. The φ and $\psi = \psi_F + \psi_S$ potentials describe the Alfvén and magnetosonic oscillations of the plasma, respectively (ψ_F and ψ_S correspond to FMS and SMS oscillations, respectively). For the resonant SMS oscillations considered here, the input of the potential ζ is negligible and below we omit it in our calculations. Since the interaction of the Alfvén and SMS waves is negligibly small [*Leonovich et al.*, 2006] and the FMS waves with large azimuthal numbers $m \gg 1$ almost do not penetrate into the magnetosphere, we will take in the further calculations $\varphi = 0$ and $\psi_F = 0$.

[13] Substituting the expressions for the components of SMS oscillations into (6), we obtain after some transformations the equation for SMS waves:

$$\widehat{L}_S^T\frac{1}{g_1}\nabla_1^2\psi - \widehat{L}_S^P\frac{k_2^2}{g_2}\psi =$$

$$-\frac{4\pi\omega}{k_2c^2}\left(1 + \frac{S^2}{\bar{\omega}^2}\bar{D}_3\right)A^2\sqrt{\frac{g_3}{g_1}}J_1 \tag{8}$$

where

$$\widehat{L}_S^P(\bar{\omega}) = \frac{\omega}{\bar{\omega}}A^2 + S^2 + \frac{\omega}{\bar{\omega}}\frac{S^2}{\bar{\omega}^2}\bar{D}_3A^2$$

$$\widehat{L}_S^T(\bar{\omega}) = \widehat{L}_S^P + \frac{\Omega}{k_2\bar{\omega}} A^2 \tilde{D}_3 g_2 + \frac{\Omega}{k_2\bar{\omega}} S^2 \bar{D}_3 g_2$$

are the poloidal and toroidal field-aligned operators, respectively,

$$\widetilde{D}_3 = \nabla_3 \frac{g_2}{\sqrt{g}} \nabla_3 \frac{g_1}{\sqrt{g}}$$

$$\bar{D}_3 = \frac{B_0\sqrt{g_3}}{P_0^\sigma\sqrt{g}} \nabla_3 \frac{P_0^\sigma\sqrt{g}}{\rho_0 g_3} \nabla_3 \frac{\rho_0}{B_0\sqrt{g_3}}$$

$S = \sqrt{\gamma P_0/\rho_0}$ is the speed of sound, and $A = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity.

[14] We assume here that the structure of the considered azimuthally small-scale SMS waves resembles the structure of similar Alfvén waves. There is, however, one significant difference. The field of standing SMS waves is localized in the vicinity of the equatorial plane [*Leonovich et al.*, 2006]. This means, in particular, that the ionosphere neither can be a source of SMS waves nor can absorb effectively their energy. Therefore, both the generation and dissipation of SMS oscillations should be of a magnetospheric origin. The SMS oscillations can be effectively absorbed by ions of the background plasma, because the typical velocity of their propagation in the plasma is of the order of the thermal velocity of ions. The transverse external currents in the magnetosphere generated in the periods of geomagnetic field disturbances are able to serve as a source.

[15] The theory of the standing azimuthally small-scale Alfvén waves [*Leonovich and Mazur*, 1993] states that such oscillations are generated by a monochromatic source at the poloidal resonant magnetic shell where their azimuthal wavelength is much shorter than the radial one. Then these oscillations drift across the magnetic shells to the toroidal resonant magnetic shell where the radial wavelength becomes much less than the poloidal one. In the process of this shift, the polarization of the Alfvén wave changes from the poloidal to the toroidal one. In the vicinity of the toroidal resonant surface, there occurs a complete absorption of the Alfvén wave energy due to the dissipation in the ionosphere. We will assume that the azimuthally small-scaled SMS waves have the similar structure, except the differences mentioned above and related to the generation and dissipation mechanisms.

3. Structure in the Vicinity of Poloidal Resonant Shell

[16] Now we consider the structure of SMS waves with $m \gg 1$ in the vicinity of the poloidal resonant surface. The typical wavelength of the main harmonics of standing SMS waves in the field-aligned direction is of the order of the length of the field line itself. We will see in the calculations below that the typical wavelength of such oscillations across magnetic shells is much less than the field-aligned wavelength. In the vicinity of the poloidal resonant surface, the following relation is fulfilled: $|k_2| \gg |\nabla_1\psi/\psi| \gg |\nabla_3\psi/\psi|$. This fact makes it possible to look for solution of (8) by the different scale method, writing the potential ψ in the form

$$\psi = U(x^1)(P(x^1, x^3) + h(x^1, x^3)) \exp(ik_2 x^2 - i\omega t) \qquad (9)$$

where function $U(x^1)$ and $P(x^1, x^3)$ describe in the main order the small-scale structure of oscillations along the x^1 coordinate and the structure of the oscillations along field lines of the magnetic field, respectively. The typical scale of $P(x^1, x^3)$ variations along x^1 is much larger than the scale of $U(x^1)$ variations. The small correction $h(x^1, x^3)$ describes the oscillations in higher orders of the disturbance theory.

[17] As far as the ionosphere almost does not influence the structure of standing SMS waves, we will use the homogeneous boundary conditions

$$\psi|_{x^3_\pm} = 0 \qquad (10)$$

where the signs "±" refer to the points of the field line crossings with the ionospheres of the Northern and Southern hemispheres. One can obtain the equation for the field-aligned structure if only the terms of the main order ($\sim k_2^2\psi$) of the disturbance theory are left in (8):

$$\widehat{L}_S^P P = 0 \qquad (11)$$

[18] The poloidal eigenfunctions $P_N(x^1, x^3)$, ($N = 1, 2, 3, \ldots$ is the field-aligned wave number) and the corresponding values of the poloidal eigenfrequencies $\bar{\omega} = \Omega_{PN}$ are the solution of equation (11). We will assume that the eigenfunctions P_N are normalized by the following condition

$$\int_{l_-}^{l_+} \frac{pP_0^\sigma}{g_2 g_3} \frac{A^2}{S^2} P_N^2 dl = 1 \qquad (12)$$

where $p = \sqrt{g_1/g_2}$.

[19] For harmonics with $N \gg 1$ one can find the solution of equation (11) in the Wentzel–Kramers–Brillouin (WKB) approximation. Using the standard method (see, e.g. [*Leonovich and Mazur*, 1993], we write the solution normalized by condition (12) within the first two orders of the WKB approximation in the form

$$P_N(x^1, x^3) = \sqrt{\frac{2}{t_S} \frac{g_3 g_2^{3/2}}{P_0^\sigma g_1^{1/2}} \frac{S}{A^2}} \sin\left(\Omega_{PN} \int_{l_-}^{l} \frac{dl}{S}\right) \qquad (13)$$

where

$$\Omega_{PN} = \frac{\pi N}{t_S} \qquad t_S = \int_{l_-}^{l_+} \frac{dl}{S}$$

is the time of the run between the magnetically conjugated ionospheres with the local speed of sound. One can find the solution of equation (11) for the main harmonics ($N \sim 1$) only numerically.

[20] In the next order of the disturbance theory we have from equation (8)

$$\widehat{L}_{ST}(\bar{\omega})\frac{1}{g_1}P_N\nabla_1^2U_N - \widehat{L}_{SP}(\bar{\omega})\frac{1}{g_2}P_Nk_2^2U_N =$$

$$-\frac{4\pi\omega}{k_2c^2}\left(1+\frac{S^2}{\bar{\omega}^2}\bar{D}_3\right)A^2\frac{\sqrt{g_3}}{\sqrt{g_1}}J_1$$

Multiplying this expression by $P_0^\sigma g_1 P_N/S^2\sqrt{g}$ and integrating along the field line, we obtain the equation determining the transverse structure of SMS waves in the vicinity of the poloidal resonant shell:

$$-\left(\frac{k_2\Omega}{\Omega_{PN}}\left(\frac{\Omega_{PN}-(\bar{\omega}+i\gamma_N)}{\bar{\omega}}-\right.\right.$$

$$\left.\frac{\Omega_{PN}^3-(\bar{\omega}+i\gamma_N)^3}{\bar{\omega}^3}\right)-$$

$$\left.\frac{\Omega_{PN}^2-(\bar{\omega}+i\gamma_N)^2}{\bar{\omega}^2}\right)k_2^2U_N+$$

$$(\alpha_{T1}-\frac{k_2\Omega}{\Omega_{PN}}\alpha_{T2})\nabla_1^2U_N = I \tag{14}$$

where

$$\alpha_{T1} = \int\frac{P_0^\sigma}{\sqrt{g}}\frac{(\Omega_{PN}+k_2\Omega)A^2+\Omega_{PN}S^2}{\Omega_{PN}S^2}P_N^2dx^3$$

$$\alpha_{T2} = \int\frac{P_0^\sigma g_2}{\sqrt{g}}\frac{\Omega_{PN}^2}{k_2^2}\times$$

$$\frac{(\Omega_{PN}+k_2\Omega)A^2+\Omega_{PN}S^2}{\Omega_{PN}S^2}\frac{A^2}{S^2}P_N^2dx^3$$

$$I = -\int\frac{P_0^\sigma g_1}{S^2\sqrt{g}}P_N\frac{4\pi\omega}{k_2c^2}\left(1+\frac{S^2}{\bar{\omega}^2}\bar{D}_3\right)A^2\frac{\sqrt{g_3}}{\sqrt{g_1}}J_1$$

The attenuation decrement γ_N caused by the dissipation of SMS oscillations at their interaction with ions of the background plasma is introduced here phenomenologically. One can obtain its exact expression only in the scope of the kinetic approach. It is known that in a plasma with inhomogeneous distribution of the ion and electron temperature ($T_e \gg T_i$), the value of γ_N is small ($\mathrm{Im}\,\omega/\mathrm{Re}\,\omega \sim \sqrt{m_e/m_i} \ll 1$) [*Krall and Trivelpiece*, 1973]. In this case a mode with almost no attenuation can exist. In the magnetosphere of the Earth, the condition $T_e \gg T_i$ can be fulfilled in the regions of the maximal concentration of high-energy electrons, in the regions with intense field-aligned currents, or in the dawn sector of the magnetosphere during substorms. In the rest of the magnetosphere, most probably, the inverse relation between the electron and ion temperatures $T_e \lesssim T_i$ is fulfilled. In this case the decrement of the studied SMS oscillations is no more a small value. Estimations show that $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega \sim 0.1$ in the homogeneous plasma. In the inhomogeneous plasma, the attenuation decrement can decrease slightly due to braking of conditions of the resonant interaction of waves with ions. In this case SMS waves exist as forced oscillations.

4. Structure in the Vicinity of the Toroidal Resonant Shell

[21] It the same way we study the structure of standing SMS waves in the vicinity of the toroidal resonant surface. In this region the relation $|\nabla_1\psi/\psi| \gg |k_2| \gg |\nabla_3\psi/\psi|$ is fulfilled. We will look for the potential ψ in the form

$$\psi = V(x^1)(T(x^1,x^3)+t(x^1,x^3))\exp(ik_2x^2-i\omega t) \tag{15}$$

Here the functions $V(x^1)$ and $T(x^1,x^3)$ describe the structure of the oscillations along the x^1 coordinate in the main order and the field-aligned structure, respectively, and $t_N(x^1,x^3)$ is a small correction.

[22] Leaving in (8) the terms $\sim \nabla_1^2\psi$ we obtain the equation determining the field-aligned structure of the toroidally polarized standing SMS waves:

$$\widehat{L}_S^T T = 0 \tag{16}$$

Together with the boundary conditions in the ionosphere:

$$T(x^1,x_\pm^3) = 0$$

(16) defines the toroidal eigenfunctions $T_N(x^1,x^3)$, ($N = 1,2,3,\ldots$) and toroidal eigenfrequencies Ω_{TN}. We chose the normalization for the T_N function in the form

$$\int_{l_-}^{l_+}\frac{p^{-1}P_0^\sigma}{g_1g_3}\frac{A^2}{S^2}T_N^2dl = 1 \tag{17}$$

[23] For the harmonics with $N \gg 1$ in the first two orders of the WBK approximation, the solution of (16) normalized by condition (17) will be written in the form

$$T_N(x^1,x^3) = \sqrt{\frac{2}{t_S}\frac{g_3g_1^{3/2}}{P_0^\sigma g_2^{1/2}}\frac{S}{A^2}}\sin\left(\Omega_{TN}\int_{l_-}^{l}\frac{dl}{S}\right) \tag{18}$$

where $\Omega_{TN} = \pi N/t_S$. The results of numerical solution of equations (11) and (16) for the first two harmonics are shown in Figures 2 and 3.

[24] The polarization splitting of the spectrum (the residual of the toroidal and poloidal eigenfrequencies $\Delta\Omega_N = \Omega_{TN} - \Omega_{PN}$ is one of the main characteristics of the azimuthally small-scaled SMS oscillations. This splitting determines the distance between the poloidal and toroidal resonant shells. Between these shells the field of standing SMS

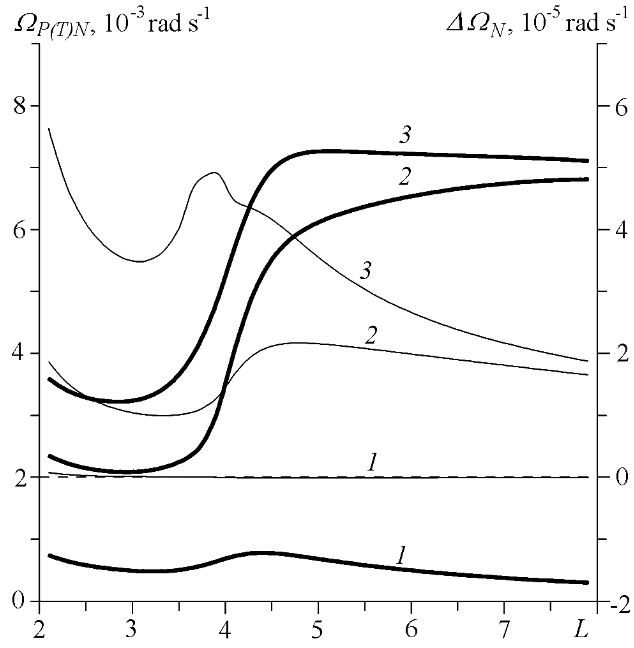

Figure 2. Distribution across the magnetic shells of the eigenfrequencies of standing SMS waves $\Omega_{P(T)N}$ (the left-hand axis) and of the polarization splitting of the spectrum $\Delta\Omega_N = \Omega_{PN} - \Omega_{TN}$ (the right-hand axis) for the first three harmonics.

waves is localized. The exact value for this parameter can be obtained if one multiply equations (11) and (16) to T_N and P_N, respectively, integrates along the field line, and subtract one from the other:

$$\Delta\Omega_N = \frac{\Omega}{k_2}\left(\Omega_{TN}^2 \int \frac{P_0^\sigma g_2}{\sqrt{g}} \frac{A^2}{S^4} P_N T_N dx^3 - \right.$$

$$\left. \int \frac{P_0^\sigma g_2}{\sqrt{g} g_3} \frac{A^2}{S^2} (\nabla_3^2 \ln P_0^{\sigma-1}) \right) \Big/$$

$$2 \int \frac{P_0^\sigma}{\sqrt{g}} \frac{A^2}{S^2} \frac{\Omega_{TN} + k_2\Omega}{\Omega_{TN}} P_N T_N dx^3 \tag{19}$$

Acting in the same way as in the previous Section, we obtain the equation describing the transverse structure of standing SMS waves in the vicinity of the toroidal resonant surface:

$$\left(\frac{k_2\Omega}{\Omega_{TN}}\left(\frac{\Omega_{TN} - (\bar{\omega} + i\gamma_N)}{\bar{\omega}} - \frac{\Omega_{TN}^3 - (\bar{\omega} + i\gamma_N)^3}{\bar{\omega}^3}\right) - \right.$$

$$\left. \frac{\Omega_{TN}^2 - (\bar{\omega} + i\gamma_N)^2}{\bar{\omega}^2}\right) \nabla_1^2 V_N -$$

$$(\alpha_{P1} + \frac{k_2\Omega}{\Omega_{TN}} \alpha_{P2}) k_2^2 V_N = 0 \tag{20}$$

where

$$\alpha_{P1} = \int \frac{P_0^\sigma}{\sqrt{g}} \frac{(\Omega_{TN} + k_2\Omega)A^2 + \Omega_{TN} S^2}{\Omega_{TN} S^2} T_N^2 dx^3$$

$$\alpha_{P2} = \int \frac{P_0^\sigma g_2}{\sqrt{g}} \frac{\Omega_{TN}^2}{k_2^2} \times$$

$$\frac{(\Omega_{TN} + k_2\Omega)A^2 + \Omega_{TN} S^2}{\Omega_{TN} S^2} \frac{A^2}{S^2} T_N^2 dx^3$$

5. Structure Across Magnetic Shells

[25] In order to obtain the complete structure of standing SMS waves across the magnetic shells, one should complete the solutions obtained in the vicinity of the poloidal and toroidal resonant surfaces by the solution in the intermediate region and perform their joining. It is a rather cumbersome procedure (see *Leonovich and Mazur* [1993]). To avoid it, we use a different method based on creation and solution of a

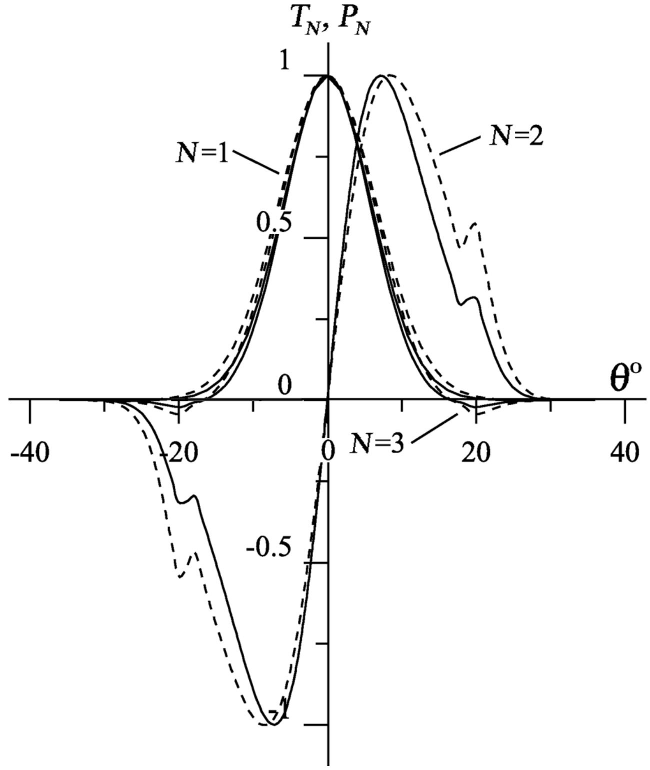

Figure 3. The structure of standing SMS waves with the toroidal (solid curves) and poloidal (dashed curves) polarizations. The graphs of the poloidal P_N and toroidal T_N eigenfunctions with a unite amplitude for the first three harmonics are presented. The amplitude of standing SMS waves is rapidly decreasing from the equatorial plane to the ionosphere. So SMS oscillations can not be observed on the Earth's surface and in its vicinity.

model equation making it possible to describe the transverse structure of the oscillations within the entire region of their existence [*Leonovich and Mazur*, 1997].

[26] In the vicinity of the corresponding resonant surfaces, one can present the poloidal and toroidal eigenfrequencies in the form

$$\Omega_{P(T)N} = \bar{\omega}\left(1 - \frac{x^1 - x^1_{P(T)N}}{L}\right) \qquad (21)$$

where $x^1_{P(T)N}$ are coordinates of the resonant surfaces and $L^{-1} = \nabla_1 \Omega_{PN}/\Omega_{PN}$. Combining equations (14) and (20) and taking into account (21), we obtain the model equation:

$$\sqrt{|\alpha_T|}\left(\frac{x^1 - x^1_{TN}}{L} + i\frac{\gamma_N}{\bar{\omega}}\right)\nabla_1^2 W_N -$$

$$\sqrt{|\alpha_P|}\left(\frac{x^1 - x^1_{PN}}{L} + i\frac{\gamma_N}{\bar{\omega}}\right)k_2^2 W_N = \frac{I}{\sqrt{|\alpha_T|}} \qquad (22)$$

where

$$\alpha_T = \alpha_{T1} + \frac{k_2\Omega}{\Omega_{TN}}\alpha_{T2}, \qquad \alpha_P = \alpha_{P1} + \frac{k_2\Omega}{\Omega_{TN}}\alpha_{P2}$$

Introducing a dimensionless transverse coordinate

$$\xi = (x^1 - x^1_{TN})/(x^1_{TN} - x^1_{PN})$$

we rewrite equation (22) in the form

$$(\xi + i\varepsilon)\frac{\partial^2}{\partial\xi^2}W_N - \kappa^2(\xi + 1 + i\varepsilon)W_N = b_N \qquad (23)$$

One can consider the coefficients of this equation

$$\varepsilon = \gamma_N L/\Omega_N \Delta_N$$

$$\kappa^2 = \sqrt{|\alpha_P/\alpha_T|}k_2^2\Delta_N^2$$

$$b_N = IL/\alpha_T \Delta_N$$

$$\Delta_N = x^1_{TN} - x^1_{PN}$$

almost constant, because they vary only slightly at the scale of the localization of the solution $W_N(\xi)$ we are looking for.

[27] One can find a solution of (23) presenting the looked-for function $W_N(\xi)$ in the form of a Fourier integral

$$W_N(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{W}_N(k)e^{ik\xi}dk$$

Substituting this expression into (23) we obtain the differential equation of the first order for the $\tilde{W}_N(k)$ function which is easy to solve (see *Leonovich and Mazur* [1997]). Performing the inverse Fourier transformation we obtain the solution of the initial equation in the form

$$W_N(\xi) = ib_N\int_0^{\infty}\frac{\exp[ik(\xi + i\varepsilon) + i\kappa\,\mathrm{arctg}(k/\kappa)]}{k^2 + \kappa^2}dk \qquad (24)$$

[28] Now we consider the behavior of $W_N(\xi)$ in the vicinity of the toroidal resonant surface ($\xi \to 0$) and at the asymptotics ($|\xi| \to \infty$). At $\xi \to 0$ the main part of the integral (24) is provided at $k \gg 1$, so in the power index of the exponent of the integrand one can take $\mathrm{arctg}(k/\kappa) \approx \pi/2$ and leave only k^2 in the denominator. In this case one can easily obtain expression for the second derivative

$$W_N''(\xi) \overset{\xi\to 0}{\approx} \frac{b_N e^{i\kappa\pi/2}}{(\xi + i\varepsilon)}$$

Integrating it we obtain

$$W_N(\xi) \overset{\xi\to 0}{\approx} b_N e^{i\kappa\pi/2}(\xi + i\varepsilon)\ln(\xi + i\varepsilon) \qquad (25)$$

At $|\xi| \to \infty$ the main part of the integral is provided at $k \ll 1$, so in $\mathrm{arctg}(k/\kappa)$ and in the denominator of the integrand one can take $k = 0$. After that the integral is easily calculated

$$W_N(\xi) \overset{|\xi|\to\infty}{\approx} -\frac{b_N}{\kappa^2}\,\frac{1}{\xi + i\varepsilon} \qquad (26)$$

Thus, the amplitude of SMS oscillations at distancing from the resonant surfaces decreases at asymptotic.

6. Structure of the SMS-Oscillations Field in the Vicinity of the Toroidal Resonant Surface

[29] Now we consider the behavior of the components of the SMS-oscillation field in the vicinity of the toroidal resonant surface ($\xi = 0$). To do this, we express the disturbed components of the magnetic and electric fields, plasma motion velocity, and its pressure via the ψ potential. Using in the vicinity of the toroidal resonant shell expression (25) for the function $U_N(\xi)$, we obtain

$$B_{1N} \approx -i\bar{B}_N\left(\frac{g_1}{\sqrt{g}}\nabla_3\frac{g_2}{\sqrt{g}}T_N\right)\frac{1}{\Delta_N}\ln(\xi + i\varepsilon)$$

$$B_{2N} \approx \bar{B}_N\left(\frac{g_2}{\sqrt{g}}\nabla_3\frac{g_2}{\sqrt{g}}T_N\right)\times$$

$$\left(\frac{\Omega}{\omega}\frac{1}{\Delta_N^2}\frac{1}{(\xi + i\varepsilon)} + \frac{(\nabla_1\Omega)}{\bar{\omega}}\frac{1}{\Delta_N}\ln(\xi + i\varepsilon)\right) +$$

$$\bar{B}_N\left(\frac{g_2}{\sqrt{g}}\nabla_3\frac{g_1}{\sqrt{g}}T_N\right)k_2\frac{\bar{\omega}}{\omega}(\xi + i\varepsilon)\ln(\xi + i\varepsilon)$$

$$B_{3N} \approx i\bar{B}_N\frac{1}{g_1}T_N\frac{1}{\Delta_N^2}\frac{1}{(\xi + i\varepsilon)} -$$

$$i\bar{B}_N\frac{1}{g_2}T_N k_2^2(\xi + i\varepsilon)\ln(\xi + i\varepsilon)$$

where the designation $\bar{B}_N = (c/\bar{\omega})b_N e^{i\kappa\pi/2}$ is used. At $\varepsilon \to 0$ the azimuthal B_{2N} and field-aligned B_{3N} components of the disturbed magnetic field have a singularity $\sim \xi^{-1}$ and the radial component B_{1N} has a logarithmic singularity. The main component of the disturbed magnetic field B_{3N} almost does not depend on the plasma motion velocity. In the vicinity of the toroidal resonant surface $|B_{2N}| \gg |B_{1N}|$. The existence of the singularity of the B_{2N} component depends on the terms proportional to the angular velocity of the plasma rotation Ω and to its gradient. In the case of the immobile plasma when $\Omega(x^1) \equiv 0$, the azimuthal component B_{2N} has no singularities in the amplitude distribution, so in the vicinity of the resonant surface $|B_{2N}| \ll |B_{1N}|$.

[30] We obtain for the electric field components

$$E_{1N} \approx i\bar{E}_N \frac{g_1}{\sqrt{g}} T_N \frac{\omega^3 - (k_2\Omega)^3}{\omega(\omega^2 - (k_2\Omega)^2)} (\xi + i\varepsilon)\ln(\xi + i\varepsilon) -$$

$$i\bar{E}_N \frac{g_2}{\sqrt{g}} T_N \frac{k_2\Omega^2}{\omega\bar{\omega}} \frac{1}{\Delta_N^2} \frac{1}{(\xi + i\varepsilon)}$$

$$E_{2N} \approx -\bar{E}_N \frac{g_2}{\sqrt{g}} T_N \frac{1}{\Delta_N} \ln(\xi + i\varepsilon)$$

$$E_{3N} \approx -i\bar{E}_N \left(\nabla_3 \frac{g_2}{\sqrt{g}} T_N\right) \frac{\Omega}{\bar{\omega}} \frac{1}{\Delta_N} \ln(\xi + i\varepsilon)$$

where $\bar{E}_N = b_N e^{i\kappa\pi/2}$. The azimuthal E_{2N} and field-aligned E_{3N} components have logarithmic singularities, and the radial component E_{1N} has the strongest singularity $\sim \xi^{-1}$, so $|E_{2N}| \ll |E_{1N}|$ in the vicinity of $\xi = 0$. The relation between the transverse components of the disturbed electric field changes to the opposite one ($|E_{2N}| \gg |E_{1N}|$) in the model without plasma rotation.

[31] We have for the components of the velocity field in the vicinity of the resonant surface

$$v_{1N} \approx -\bar{v}_N \frac{1}{\sqrt{g_3}} T_N \frac{1}{\Delta_N} \ln(\xi + i\varepsilon)$$

$$v_{2N} \approx -i\bar{v}_N \left(\frac{1}{\sqrt{g_3}} T_N \frac{\bar{\omega}}{\omega} k_2 (\xi + i\varepsilon)\ln(\xi + i\varepsilon) + \right.$$

$$\left. \frac{g_2}{g_1\sqrt{g_3}} T_N \frac{\Omega}{\omega} \frac{1}{\Delta_N^2} \frac{1}{(\xi + i\varepsilon)} \right)$$

$$v_{3N} \approx i\bar{v}_N \left(\frac{1}{\rho_0} \nabla_3 \frac{A^2\rho_0}{\sqrt{g_3}} T_N \right) \frac{1}{\bar{\omega}^2} k_2^2 (\xi + i\varepsilon)\ln(\xi + i\varepsilon) -$$

$$i\bar{v}_N \left(\frac{1}{\rho_0} \nabla_3 \frac{A^2\rho_0}{g_1\sqrt{g_3}} T_N \right) \frac{1}{\bar{\omega}^2} \frac{1}{\Delta_N^2} \frac{1}{(\xi + i\varepsilon)}$$

where $\bar{v}_N = (c/B_0)b_N e^{i\kappa\pi/2}$. The behavior of the disturbed plasma velocity in the transverse direction is similar to the behavior of the magnetic field. If we take the plasma to be immobile, then $|v_{2N}| \ll |v_{1N}|$ We have an inverse relation $|v_{2N}| \gg |v_{1N}|$ in the model with moving magnetospheric plasma. Thus, the SMS oscillations in the plane transverse to the field lines have a toroidal and poloidal polarizations in the models without plasma motion and with it, respectively. The plasma motion provides no significant influence on the main component of the velocity field v_{3N}.

[32] The expression for the disturbed pressure in the vicinity of the toroidal resonant surface has the form

$$\tilde{P}_N \approx i\frac{B_0\bar{B}_N}{4\pi} k_2^2 \frac{1}{\sqrt{g_3}} T_N (\xi + i\varepsilon)\ln(\xi + i\varepsilon) -$$

$$i\frac{B_0\bar{B}_N}{4\pi} \frac{1}{g_1\sqrt{g_3}} T_N \frac{1}{\Delta_N^2} \frac{1}{(\xi + i\varepsilon)}$$

and does not depend on the plasma rotation velocity. In a homogeneous plasma, the localized SMS waves have a typical singularity: the gas-kinetic pressure and pressure of the magnetic field of such waves oscillate in antiphase with amplitudes close by magnitude, so the total disturbed pressure of these oscillations is almost zero. One can easily check that for the resonant SMS oscillations considered here in the zero order of the disturbance theory, the condition $\tilde{P} + B_0 B_{||}/4\pi \approx 0$ (where $B_{||} = B_3/\sqrt{g_3}$ is the physical field-aligned component of the disturbed magnetic field) is also fulfilled.

7. Results of Numerical Solution

[33] We will find numerically the solution of equations (11) and (16) (describing the structure of the resonant SMS oscillations) for the main harmonics of standing SMS waves. We take the geomagnetic field to be a dipole one such as it takes place in the inner part of the magnetosphere. We use the coordinate system (a, ϕ, θ) connected to field lines of the magnetic field (see Figure 1). Here a is the equatorial radius of the field line, ϕ is the azimuthal angle, and θ is the latitude counted from the equator. In this coordinate system, the length of the radius vector of a point at the field line is written as

$$r = a^2 \cos^2\theta$$

and the element of the length along the field line has the form

$$dl = a\cos\theta\sqrt{1 - 3\sin^2\theta}\, d\theta$$

The strength of the dipole magnetic field is determined by the equation

$$B_0(a,\theta) = B_0 \left(\frac{a_0}{a}\right)^3 \frac{\sqrt{1 - 3\sin^2\theta}}{\cos^6\theta}$$

and the components of the metric tensor have the form

$$g_1 = \frac{\cos^6\theta}{1 - 3\sin^2\theta} \qquad g_2 = a^2\cos^6\theta \qquad (27)$$

We did not succeed in obtaining a simple expression of the same kind for the g_3 component. One can determine it via the ratio of the segments of field lines with the equatorial radii a_0 and a crossed by two close coordinate surfaces $x^3 = \text{const}$ at latitudes θ_0 and θ respectively:

$$\frac{g_3(a,\theta)}{g_3(a_0,\theta_0)} = \left(\frac{a}{a_0}\right)^6 \left(\frac{\cos\theta}{\cos\theta_0}\right)^{12} \frac{1-3\sin^2\theta_0}{1-3\sin^2\theta}$$

We will take the plasma distribution in the plane of the magnetic meridian using the model of the magnetosphere by *Leonovich et al.* [2004].

[34] Figure 2 shows the distributions of the poloidal and toroidal eigenfrequencies of the first three harmonics of standing SMS waves. They were obtained as a result of numerical solution of equations (11) and (16) with homogeneous boundary conditions on the ionosphere. Figure 2 shows also the distributions of the polarization splitting of the spectrum across the magnetic shells for the first three harmonics of SMS waves. Unlike for the Alfvén waves, the frequencies of the main harmonics of SMS waves differ considerably from their values in the WBK approximation. For example, for the first harmonic of SMS waves, the exact value of the frequency differs from its WBK value by a factor of 5.

[35] Figure 3 shows the field-aligned structures of the first three harmonics of standing SMS waves. It should be noted that the spectrum and structure of the SMA oscillations with $m \gg 1$ almost coincide with the case $m \sim 1$. A rapid decrease of their amplitude is also observed at the approaching the ionosphere. So SMS oscillations can not be observed on the surface of the Earth and in its vicinity. The most probable place of their detection is the near-equatorial magnetosphere in the region of closed field lines.

[36] Figure 4 shows the dependence of the ratio of the polarization splitting of the spectrum to its eigenfrequency ($\Delta\Omega_1/\Omega_1$) on the azimuthal wave number $k_2 \equiv m$ and the transverse coordinate $L = a/R_E$ (R_E is the Earth radius) for the first harmonic of SMS oscillations. The value of $\Delta\Omega_1/\Omega_1$ is very small ($\sim 10^{-3} - 10^{-4}$) almost for all values of the m and L parameters. Moreover, a spectral splitting sign reversal is observed for the first harmonics (it is its feature). This means that there exists a shell $x^1 = \bar{x}^1$ at which the toroidal and poloidal frequencies coincide $\Omega_{T1}(\bar{x}^1) = \Omega_{P1}(\bar{x}^1)$. Dissipation of the energy of such oscillations occurs at the same shell they where excited at, in the similar way as it occurs at the magnetosonic resonance [*Leonovich et al.*, 2006].

[37] Figure 5a shows the transverse structure of the oscillations with $\kappa = 0.1$ for the cases of a moderate ($\varepsilon = 0.1$) and strong ($\varepsilon = 1$) attenuation. The structure of these oscillations is of a typical resonant type. The maximal oscillation amplitude is observed at a toroidal resonant surface. The characteristic scale of localization of oscillations across the magnetic shells in this case is determined by the dissipation level. Figure 5b shows the structure of the oscillations for $\kappa = 20$. At moderate attenuation, the structure of a "running wave" type is distinctly seen. The amplitude of the oscillations in the vicinity of the poloidal resonant shell where their generation occurs is much larger then in the vicinity of the toroidal shell where they are absorbed. So on the whole, one may consider their polarization as a poloidal one. It should be noted that such a structure of SMS waves

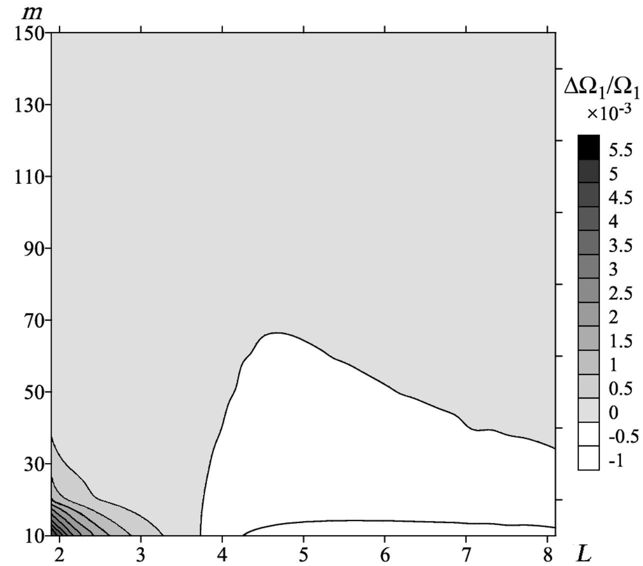

Figure 4. The dependence of the ratio of the spectrum polarization splitting to the eigenfrequency ($\Delta\Omega_1/\Omega_1$) on the azimuthal wave number m and the magnetic shell parameter $L = a/R_E$ for the first harmonics of SMS oscillations. The value of $\Delta\Omega_1/\Omega_1$ is very small ($\sim 10^{-3} - 10^{-4}$) and changes its sign in the outer magnetosphere for some values of m.

is rather hypothetical. In the real magnetosphere, one can not obtain the relation between parameters required for the structure realization.

[38] Figure 6 shows the distribution of the parameter κ value in the plane formed by the values of the azimuthal wave numbers m typical for the observed in the magnetosphere geomagnetic pulsations and the magnetic shell parameter L. One can see that in the entire magnetosphere $\kappa < 1$. This means that all the SMS oscillations observed in the magnetosphere of the Earth should have a resonant structure. Observation in the daytime magnetosphere of SMS oscillations of the "running wave" type is impossible.

8. Conclusions

[39] At the end we summarize the main results of our investigation.

[40] (1) We have developed the theory of the slow magnetosonic oscillations with high azimuthal wave numbers in a model of the magnetosphere with rotating plasma. The structure and the spectrum of such oscillations are investigated both analytically and numerically. These oscillations have some peculiarities. It is shown that these oscillations similar to the Alfvén waves propagate along the magnetic field lines. The SMS waves like Alfvén waves due to high conductivity of the Earth's ionosphere have the form of standing waves along the magnetic field lines. Nevertheless as it is confirmed by numerical simulations the amplitude of the standing SMS waves drastically decreases from equatorial

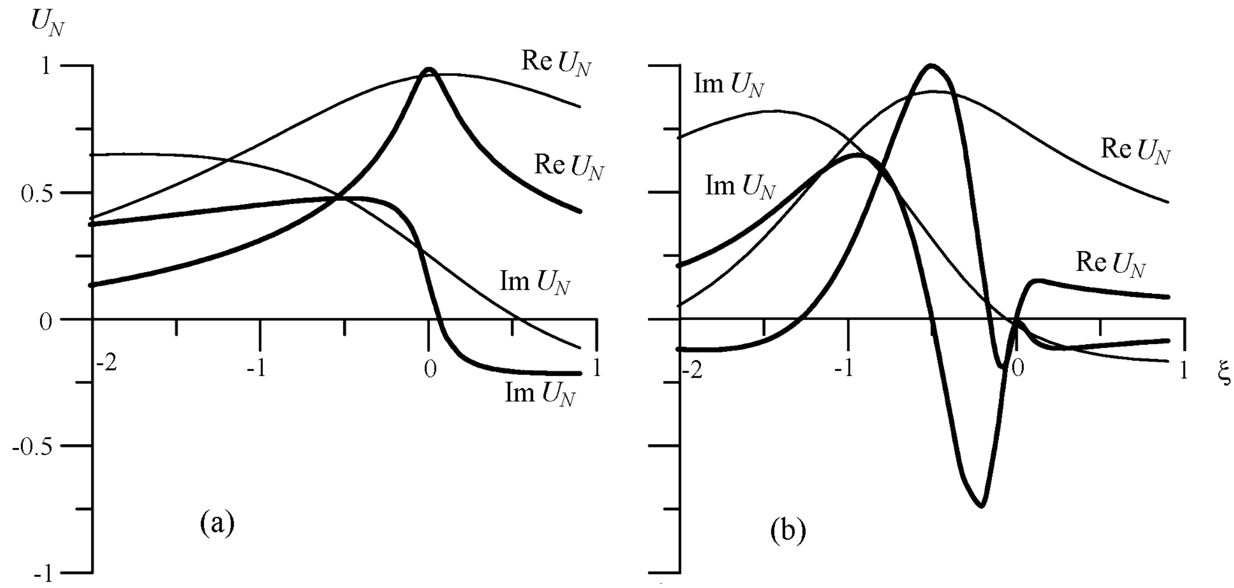

Figure 5. The structure of SMS waves across the magnetic shells in two limiting cases. Panel (a) shows the dependencies of the real and imaginary parts of the U_N function describing the resonant type of oscillations for which $\kappa_N = 0.1$. Panel (b) shows the real and imaginary parts of the U_N function describing the oscillation structure of the "running wave" type for which $\kappa_N = 20$. Thick and thin curves correspond to the cases of moderate $\varepsilon = 0.1$ and strong ($\varepsilon = 1$) attenuation, respectively. The resonant structure (panel (a)) is more likely to be observed in the real magnetosphere.

plane towards the ionosphere. As the intensities of generation and dissipation are proportional to the amplitude, the influence of the ionosphere on these oscillations can be neglected. Possible source of the SMS oscillations can be situated on the same magnetic shells in the near equatorial region of the magnetosphere. In our paper the external magnetospheric currents are considered as such source. The dissipation of such oscillations is determined by local properties of the background plasma. This dissipation is strong enough because the phase speed of the SMS oscillations is close to the averaged thermal speed of ions in the background plasma ($\omega/k_\parallel \sim S$). The attenuation can be so strong that the presence of the SMS waves in the form of eigenmodes becomes practically impossible. It should be mentioned that SMS oscillations can serve as one of the channels of dissipation of the ring current energy. Being excited in the equatorial plane by time dependant component of ring current these oscillations can transmit their energy to the particles of the background plasma.

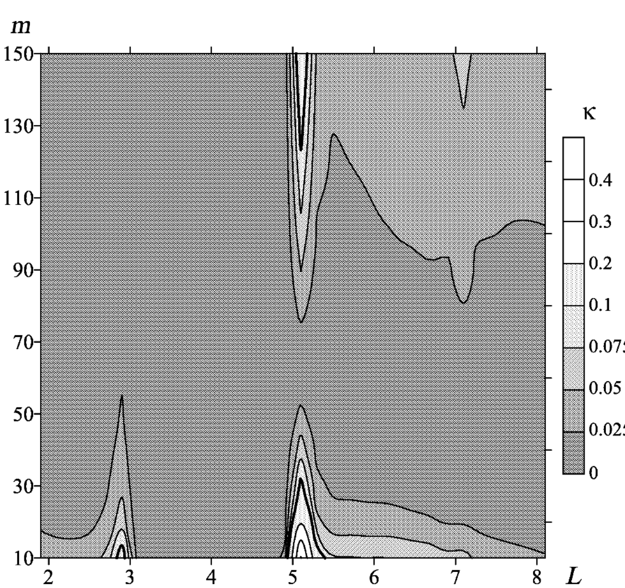

Figure 6. The dependence of the κ parameter on the azimuthal wave number m and magnetic shell parameter $L = a/R_E$. For $\kappa > 1$ the transverse structure of the of the SMS wave has the form of the propagating wave. At the same time for $\kappa < 1$ it has a typical resonance structure. As it is seen from the figure, for the whole magnetosphere $\kappa < 1$. Observation in the daytime magnetosphere of the SMS oscillations of the propagating wave is impossible.

[41] (2) Analytical investigation of the structure of the SMS oscillations across the magnetic shells shows that these oscillations similar to Alfvén waves can have two types of the transverse structures. The shape of the transverse structure is determined by the value of $\kappa \sim k_2\Delta_N$. If $\kappa < 1$ the oscillations across the magnetic shells take the form of the resonance peak while for $\kappa > 1$ they have the structure of the propagating wave. While investigating Alfvén oscillations it was shown that the conditions under which these

oscillations have the form of a propagating wave across the magnetic shells are fulfilled within the considered model only for the first harmonic of the standing Alfvén waves. This is connected with the anomalously large equatorial distance between the resonance shells for this harmonic. The equatorial splitting of the shells for the SMS oscillations is several orders of magnitude less than for the main harmonics of the Alfvén waves. The values of the parameter κ for small-scale SMS oscillations are less than unity. As a result the SMS oscillations with high azimuthal wave numbers always have resonance structure across the magnetic shells.

[42] The most probable region for registration of the localized SMS oscillations with high azimuthal numbers is the near equatorial region of the magnetosphere where the amplitude of these oscillations achieves maximal values [*Takahashi et al.*, 1987]. As an indicator of the SMS oscillations can be used the peculiarity that the total disturbed pressure for these oscillations is approximately equal to zero (in the presence of the longitudinal component of the disturbed magnetic field [*Glassmeier et al.*, 1999; *Woch et al.*, 1990]).

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