HIERARCHY OF CYCLIC SOLAR ACTIVITY CHANGES

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Abstract. A hierarchy of cyclic changes in solar activity is studied based on energetic and phase wavelet analyses. The phase wavelet spectrum of yearly number of sunspot groups shows fork-like bifurcations. Separate 11-yr cycles are combined in pairs and wave trains to make up complete magnetic cycles with complex synergies. The phase spectrum also indicates the existence of different regimes in solar activity cyclic changes. This variability seems to be controlled by changes in the solar dynamo regime.

The number of sunspot groups is characterized by the sunspot number index, which has quantified solar activity since 1610 (Hoyt and Schatten, 1998). Changes in sunspot activity are characterized by a multiscale and hierarchical structure. Figure 1 shows the modulus of the continuous wavelet transform of the yearly number of sunspot groups using the Morlet wavelet, following Lawrence, Cadavid, and Ruzmaikin (1995) and Frick *et al.* (1997). The cyclic mode of about 11 yr scale is clearly delineated in the spectrum. Long-term changes (the Gleissberg cycle) are seen in the spectrum as an energy enhancement on a 50–100 yr time scale. There are edge distortions near the ends of the wavelet spectrum if the analyzing wavelets go partly outside the available data. The confidence range is shown by dots on the wavelet spectrum.

The phase wavelet spectrum clearly shows a cyclic structure, even if the amplitudes of temporal changes are small. The phase of the wavelet transform for the time series of the sunspot index is shown in Figure 2. It is remarkable that this phase spectrum resembles the phase spectrum of a process generated by the period doubling mechanism (Feigenbaum, 1978). A numerical experiment was carried out to verify our conclusions about the period doubling. A period-doubling process in its pure form is generated by the logistic map $x_{n+1} = rx_n(1 - x_n)$ at appropriate r. It turns out that phase wavelet spectrum of sunspot index and that for the logistic sequence (e.g., at $r = 3.8$) show a striking qualitative similarity. The bifurcations are clearly seen as branching black or white patterns in contrast with the background in Figure 1. These illustrate a locus of the phase if it is less than fifteen degrees, which we call the zero-phase lines. The time coordinate of the zero-phase line corresponds to a maximum of the variations at a given time scale.

The horizontal cross-section of the phase wavelet spectrum describes phase changes for a given time scale. For example, in a cross-section at a 10–11 yr

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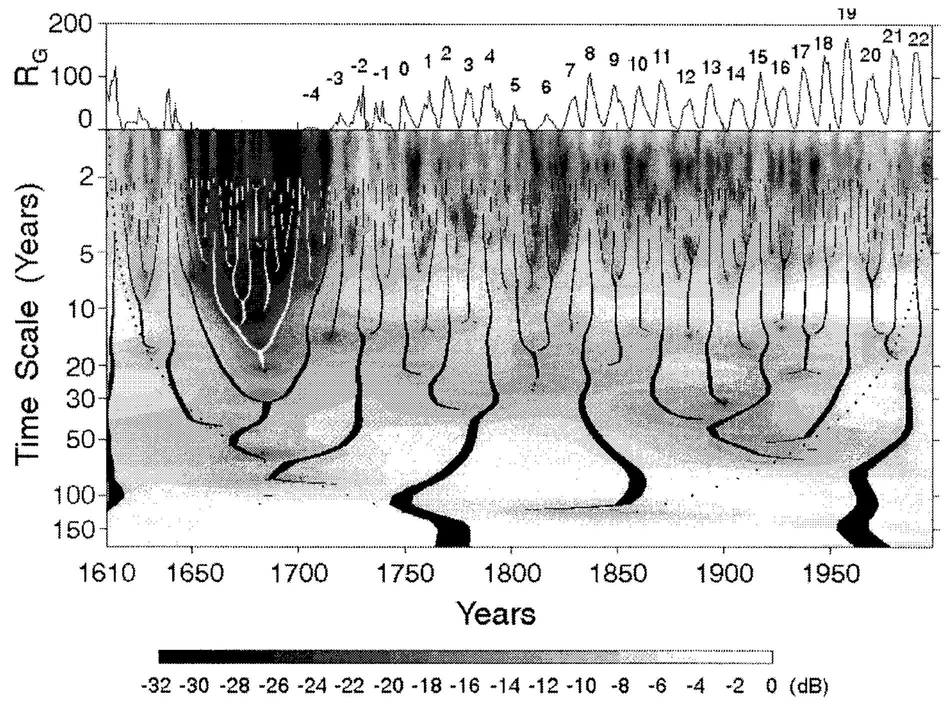

Figure 1. Yearly number of sunspot groups (*above*), and the modulus of wavelet transform normalized to its maximum value (*below*). Numbers of activity cycles are indicated at the top.

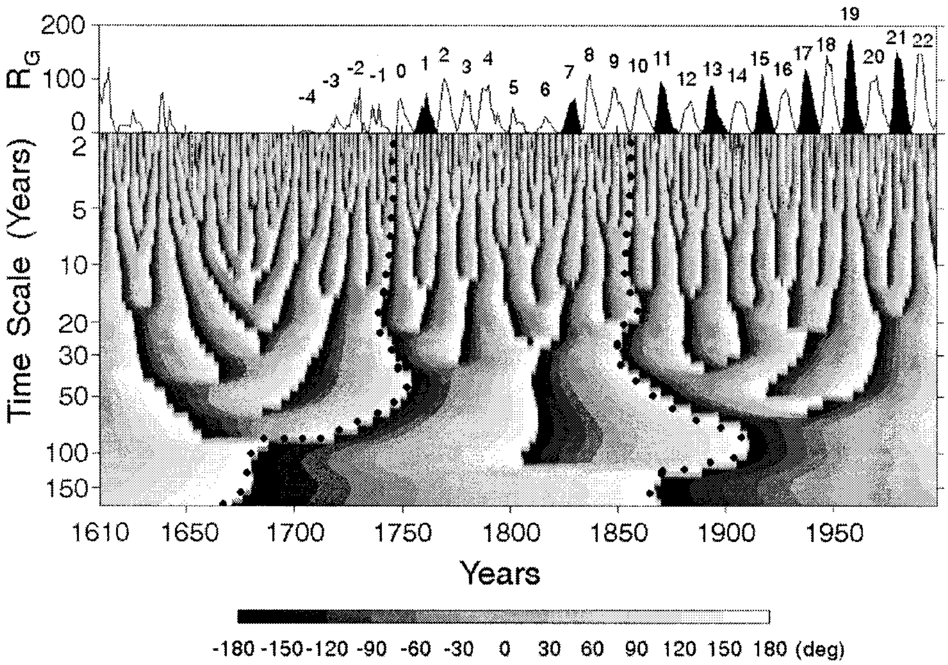

Figure 2. Number of sunspot groups, and phase wavelet spectrum.

time scale, alternating light and dark bands describe corresponding activity cycles. Zero-phase lines alternate more or less regularly at this time scale. As a rule, these lines represent separate branches of greater structures, twice as large in time scale. Bifurcations between the 10–20 yr time scale show relations of adjacent 11-yr cycles, which are coupled in pairs forming separate magnetic cycles, according to Hale's law. A causal relation between adjacent 11-yr cycles also appears as an amplitude alternation from an even to odd cycle according to their Zürich numbers (Gnevyshev and Ohl, 1948). One can see directly from the sunspot index plot that every odd cycle exceeds the preceding even cycle in amplitude, beginning with the cycle 10 to the present day. But only two pairs of cycles (0, 1 and 6, 7) have the same relation before cycle 10.

The phase wavelet spectrum and the skeleton of zero-phase lines describe complicated changes at greater time scales. Bifurcations in the 'time-scale' space located near points with coordinates about (1690, 70), (1770, 120), and (1960, 70) are also seen, where all the values are measured in years. The outer branches of the first bifurcation enclose the Maunder minimum when a low activity regime existed. It is usually dated from 1645 to 1715 (Eddy, 1976), when low and less regular variations of solar activity occurred. The internal structure of these variations is shown as white branching patterns against the background of general activity decrease. Two branches of the second bifurcation enclose a suppression of solar activity for cycles 5–7, known as the Dalton minimum (Frick *et al.*, 1997).

We can also trace lines of phase changes from cycle to cycle running between black and white large features which are marked by black spots over the white edges of the phase structures. The lines which start from the greatest time scale will be referred to as phase separating lines or separatrixes. Spots in Figure 2 mark two of these lines over the white edges. The separatrixes reach the smallest time scale, thus indicating some special time points. These time points indicate the beginning and end of large structures in the phase space of basic parameters describing the evolution of the dynamic system that generates the magnetic field of the Sun. There are independent tree-like structures branching on different time scales between the separating lines. Wavelet analysis of non-stationary time series shows that such regions are often characterized by their own intrinsic regimes.

The phase separating lines are physically meaningful, and their significance could be understood by taking into account all the peculiarities of the signal, including the character of its non-stationarity. The separatrixes continuing to smaller time scales indicate the time interval from 1745 to 1855. Altogether there are ten 11-yr cycles occurring in this interval, but the alternation according to the rule by Gnevyshev and Ohl takes place only in cycle pairs 0–1 and 6–7. An odd cycle is colored black if it has a higher amplitude than that of the preceding even cycle. Therefore, we conclude that there is no regular amplitude alternation from even cycle to odd cycle during the time interval marked by the phase separatrixes 1745 –1855. It is remarkable that the regular alternation according to (Gnevyshev and Ohl, 1948) is strictly satisfied for all cycle pairs since 1855 after the phase separatrix. This alternation characterizes the cyclic mode which exists to the present time. Thus, the phase separating lines indicate special time points when the solar dynamo changes its regime. The dynamo regime of low and less regular variations is terminated by 1745, and more or less regular cyclic variations were established between 1745 and 1855; these heliocycles are characterized by irregular amplitude alternation from the even cycle to the odd one. Since 1855 to the present, the dynamo regime has operated with a regular alternation between even and odd cycles: namely, every odd cycle exceeded the previous even cycle.

Thus, the separatrixes indicate large patterns of the phase space of basic parameters which describe the evolution of solar activity as a dynamic system. An alternative explanation of the phase separatrixes raises a question as to the quality of data reconstruction. Does the last separatrix approximately indicate the beginning of accurate data, when Wolf started a regular sunspot count? Does the first separatrix indicate the initial time from which a more or less reliable reconstruction is possible? If we suppose that all the reconstruction is reliable, the height of coming cycle 23 is of critical importance in the scheme outlined above. If the maximum height of the coming cycle turn out to be lower than that of the preceding cycle, this might imply that a new regime of the solar dynamo is beginning to operate.

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References

Eddy, J. A.: 1976, *Science* **192**, 1189.
Feigenbaum, M. J.: 1978, *J. Stat. Phys.* **19**, 25.
Frick, P., Galyagin, D., Hoyt, D. V., Nesme-Ribes, E., Schatten, K. H., Sokoloff, D., and Zakharov, V.: 1997, *Astron. Astrophys.* **328**, 670.
Gnevyshev, M. N. and Ohl, A. I.: 1948, *Astron. Zh.* **25**, 18.
Hoyt, D. V. and Schatten, K. H.: 1998, *Solar Phys.* **179**, 189.
Lawrence, J. K., Cadavid, A. C., and Ruzmaikin, A. A.: 1995, *Astrophys. J.* 455, 366.