Generation of Alfvén Wave Parallel Electric Field in Curved Magnetic Field due to the Coupling With the Compressional Mode

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Abstract

A new generation mechanism of the parallel electric field of Alfvén waves in the magnetosphere is suggested. In a homogeneous plasma, Alfvén wave has neither parallel electric nor parallel magnetic field. The parallel electric field is usually supposed to be generated by small kinetic effects like electron inertia, which is usually described as a coupling of MHD Alfvén wave with electrostatic wave. Another possibility is considered in the current paper. Due to the coupling with the compressional mode in a curved magnetic field, Alfvén mode gains field-aligned component of the wave magnetic field, which is especially considerable in a finite pressure plasma. Then, the parallel magnetic field of the wave is coupled with the parallel electric field according to the kinetic quasi-neutrality condition (akin to the coupling of these two fields in the slow magnetosonic mode). As a result, Alfvén mode acquires parallel electric field, which is shown to be considerably larger than that due to the coupling with the electrostatic mode.

1 Introduction

It has been suspected for long time that Alfvén waves can be responsible for the auroral arcs intensification [1]. Recently, this idea has received experimental support through observations by the POLAR spacecraft of Alfvén waves carrying significant electromagnetic power to the auroral ionosphere [2]. The acceleration of the electrons is usually thought to take place due to the parallel electric field of Alfvén waves ($E_{\|}$). But in usual MHD treatment, this field is zero, $E_{\|} = 0$. It means that kinetic effects must be taken into account. One such effect is electron inertia which provides coupling of Alfvén wave with electrostatic mode (which is characterized by finite $E_{\|}$) resulting the formation of the so-called inertial Alfvén wave [1]. The parallel to transverse electric field ratio is proportional to wave frequency to ion gyrofrequency ratio, so this mechanism is not very effective for ultra-low-frequency modes. In this paper, another approach is suggested which takes into account the coupling between Alfvén and compressional (large parallel magnetic field, $B_{\|}$) modes due to plasma inhomogeneity.

2 The parallel electric field of Alfvén wave in a uniform plasma

Let us consider the ordinary scenario of the parallel electric field generation. In a homogeneous plasma, Alfvén wave has no parallel magnetic field, but transverse and parallel components of electric

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field are coupled due to the electron inertia. When the wave frequency ω is much less than the gyrofrequency ω_c, the quasi-neutrality condition holds

$$\sum_{i,e} \frac{q^2}{m} \left\langle \frac{\partial F}{\partial \varepsilon}\phi_\parallel + (1 - J_0^2)\frac{\partial F}{\partial \varepsilon}\psi + \frac{\omega}{k_\parallel v_\parallel - \omega}\frac{\partial F}{\partial \varepsilon} J_0^2 \phi_\parallel \right\rangle = 0. \tag{1}$$

Here the sum over the particle species is assumed, F is the particle distribution function, $J_n = J_n(k_\perp v_\perp/\omega_c)$ is the n^{th} order Bessel function, ε is the particle energy, $v_\perp$ and $v_\parallel$ are the transverse and the longitudinal components of the particle velocity, $k_\perp$ and $k_\parallel$ are the transverse and the longitudinal components of the wave vector, $\langle ... \rangle$ denotes the averaging over the velocity space, and the functions $\phi_\parallel$ and ψ determine parallel and perpendicular components of the wave electric field through the equations

$$E_\parallel = -ik_\parallel \phi_\parallel, \ E_\perp = -ik_\parallel \psi.$$

The second equation relating these variables is the vorticity equation:

$$\frac{c^2}{4\pi\omega^2}k_\parallel^2 k_\perp^2 \psi = -\sum_{i,e}\frac{q^2}{m}\left\langle (1 - J_0^2)\frac{\partial F}{\partial \varepsilon}\psi + (1 - J_0^2)\frac{\partial F}{\partial \varepsilon}\phi_\parallel \right\rangle \tag{2}$$

(see, e.g., [3]). Solution of the system (1,2) for the case of hot electrons ($\omega \ll k_\parallel v_\parallel$) is

$$E_\parallel = -\frac{k_\perp^2 v_e^2}{\omega^2}\frac{m_e}{m_i}\frac{\omega^2}{\omega_{ci}^2}E_\perp. \tag{3}$$

Due to the small ratio ω/ω_{ci} the inequality $E_\parallel \ll E_\perp$ holds in a uniform plasma.

3 The scenario

According to the present scenario, Alfvén mode acquires longitudinal component of magnetic field $B_\parallel$ due to plasma nonuniformity, and $B_\parallel$ is coupled with wave parallel electric field due to finite electron mass m_e.

3.1 Coupling of Alfvén and compressional modes in nonuniform plasma

There are several ways how plasma nonuniformity can provide coupling of the Alfvén and compressional modes. The first is field line curvature in finite pressure plasma (e.g., [4]):

$$B_\parallel = \frac{ck_y}{\omega}\left(\frac{4\pi P'}{B_0^2} + \frac{2}{R}\frac{6\pi P}{B_0^2}\right)\psi \sim \frac{ck_y}{\omega}\frac{\beta}{L_\perp}\psi. \tag{4}$$

Here k_y is wave vector azimuthal component, P is plasma pressure, R is the field line curvature radius, β is plasma to magnetic pressure ratio, and $L_\perp$ is the characteristic scale of transverse nonuniformity.

Even better known is coupling of Alfvén and compressional modes due to Alfvén speed variation which causes the Alfvén resonance; see, e.g. [5]. This is effective only for transverse large scale wave. Yet another kind of coupling is caused by the field line convergence, as first studied by Leonovich and Mazur [6].

3.2 Coupling of the parallel electric field and parallel magnetic field

Let us neglect the coupling between the Alfvén (finite $E_\perp$) and the electrostatic (finite $E_\parallel$) modes, but take into account the coupling between the electrostatic and the compressional (finite $B_\parallel$) modes. Then the quasi-neutrality condition becomes:

$$\sum_{i,e}\frac{q^2}{m}\left\langle \frac{\partial F}{\partial \varepsilon}\phi_\parallel + \frac{\omega}{k_\parallel v_\parallel - \omega}\frac{\partial F}{\partial \varepsilon}(J_0^2\phi_\parallel + \frac{v_\perp J_0 J_1}{k_\perp c}B_\parallel)\right\rangle = 0. \tag{5}$$

This equation couples $E_\parallel$ and $B_\parallel$ fields. After integrating, we get

$$\left[\frac{q^2}{m_e v_e^2} - \frac{q^2 k_\parallel^2}{m_i \omega^2}\right]\phi_\parallel = \frac{q}{B}B_\parallel \tag{6}$$

(the imaginary part of the frequency is assumed small, which is not important for our analysis).

In order to show how this equation works let us consider the slow magnetosonic mode. For this purpose, consider the perpendicular Ampere low:

$$\frac{4\pi k_\perp}{c}\phi_\parallel \sum \frac{q^2}{m}\left\langle v_\perp J_1 \frac{\omega}{k_\parallel v_\parallel - \omega}\frac{\partial F}{\partial \varepsilon} J_0\right\rangle + B_\parallel\left[k_\perp^2 + \frac{4\pi k_\perp}{c}\sum \frac{q^2}{m}\left\langle \frac{\omega}{k_\parallel v_\parallel - \omega}\frac{\partial F}{\partial \varepsilon}\frac{v_\perp^2 J_1^2}{k_\perp c}\right\rangle\right] = 0.$$

After integrating we get:

$$\frac{q}{B}4\pi n\phi_\parallel + B_\parallel = 0. \tag{7}$$

The solution of the system (6, 7) is the the slow magnetosonic dispersion relation:

$$\frac{k_\parallel^2}{\omega^2} = \frac{1}{v_s^2} + \frac{1}{A^2}$$

(here $v_s = v_e\sqrt{m_e/m_i}$ is ion-sound speed, and $A = B/\sqrt{4\pi n m_i}$ is Alfvén speed).

For the frequencies higher than $v_s k_\parallel$, Eq. (6) reduces to

$$\frac{q^2}{m_e v_e^2}\phi_\parallel = \frac{q}{B}B_\parallel. \tag{8}$$

Now we can express $\phi_\parallel$ from (8) and $B_\parallel$ from (4). Finally we get

$$E_\parallel = \frac{m_e v_e^2}{qB_0}\frac{ck_y}{\omega}\left(\frac{4\pi P'}{B_0^2} + \frac{2}{R}\frac{6\pi P}{B_0^2}\right)E_\perp. \tag{9}$$

4 Conclusion

Let us compare the estimate (9) with the one from the ordinary scenario (3):

$$\frac{E_\parallel}{E_\parallel^{ord}} \sim \frac{\beta}{k_\perp L_\perp}\frac{\omega_{ci}}{\omega}.$$

It can be seen than the present scenario is more effective due to large ω_{ci}/ω ratio for Alfvén waves, especially for not so large $k_\perp$ values.

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References

[1] Hasegawa, A., Particle acceleration by MHD surface wave and formation of aurora, *J. Geophys. Res.*, **81**, 5083–5090, 1976

[2] Keiling A., J.R. Wygant, C. Cattell, W. Peria, G. Parks, M. Temerin, F.S. Mozer, C.T. Russell, and C.A. Kletzing, Correlation of Alfvén wave Poynting flux in the plasma sheet at 47 RE with ionospheric electron energy flux, *J. Geophys. Res.*, **107** (A7), doi:10.1029/2001JA900140, 2002

[3] Chen, L. and A. Hasegawa, Kinetic theory of geomagnetic pulsations, 1, Internal excitations by energetic particles, *J. Geophys. Res.*, **96**, 1503–1512, 1991

[4] Mager, P. and D. Yu. Klimushkin, Theory of azimuthally small-scale Alfvén waves in an axisymmetric magnetosphere with small but finite plasma pressure, *J.Geophys.Res.*, **107** (A11), 1356, doi:10.1029/2001JA009137, 2002

[5] Glassmeier, K.H., C. Othmer, R. Cramm, M. Stellmacher, and M. Engebretson, Magnetospheric field line resonance: a comparative approach, *Surveys Geophys.*, **20**, 61–109, 1999

[6] Leonovich, A.S. and V.A. Mazur, A theory of transverse small-scale standing Alfvén waves in an axially symmetric magnetosphere, *Planet. Space Sci.*, **41**, 697–717, 1993