# ON PHYSICAL CONDITIONS IN THE CHROMOSPHERE ABOVE SUNSPOT UMBRAE

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**Abstract.** By means of an inversion of H and K Ca II line profiles the temperature and electron density in the chromosphere above the umbrae of two sunspots have been estimated. The temperature gradient 5 K km$^{-1}$ exceeds the corresponding values in both quiet regions and plages. At a height of about 1500 km the umbra becomes hotter than the quiet region. At a temperature of about 10 000 K the temperature gradient increases sharply. The electron density at 1500 km is approximately the same as that in the quiet chromosphere at the same height.

## 1. Introduction

There is a paucity of information on the chromosphere above sunspot umbrae. Only a few attempts have been made to investigate the temperature and pressure distributions (e.g. Livshits *et al.*, 1966; and Baranovsky, 1974a, b).

This paper estimates the temperature $T_e$ and electron density $N_e$ distribution above the umbrae of two large sunspots. Our method is based on a solution of the rate equations for a 5 level model of the Ca II ion. In these equations, $T_e$ and $N_e$ enter via the collisional rate coefficients. Values of the other unknowns in the equations – the population densities and radiative rates – are estimated on the basis of previous studies: Inversion of the observed H and K line profiles (Teplitskaja and Efendieva, 1976) provided the distribution of the source functions $S_K(\tau_K)$, $S_H(\tau_K)$ and the mean radiation intensities $\bar{J}_K(\tau_K)$, $\bar{J}_H(\tau_K)$ (Teplitskaja and Efendieva, 1975; Efendieva and Turova, 1975). From these data some of the population ratios and radiative rate coefficients may be inferred. The metastable state $3^2D$ plays an important role in the rate equations, and we estimate its influence on the basis of calculations by Shine and Linsky (1974). Finally, the optical depth scale $\tau_K$ in the K line centre had been transformed into a geometrical height $h$ by Teplitskaja and Efendieva (1974).

## 2. Basic Equations

The statistical equilibrium equation for each of the five levels in question (cf. Figure 1) has a standard form, and there is no need to write it in detail. Note only that transitions to the continuum and stimulated emissions will be neglected.

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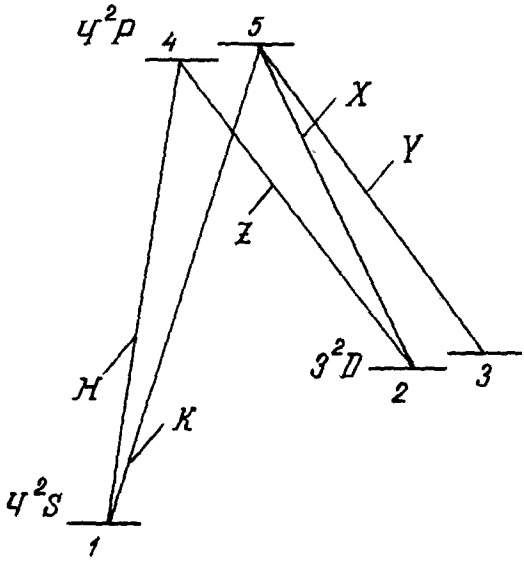


Fig. 1. The adopted model of the Ca II atom.

Let $i$ refer to the lower of the two levels of a transition $i \to k$. The rate coefficients for collisional de-excitation may be represented with good accuracy by

$$c_{ki} = (a_{ki} + b_{ki} T_e^{-1/2}) N_e \,, \tag{1}$$

where $a_{ki}$ and $b_{ki}$ are constants depending on the particular transition. They are chosen to fit the atomic data given by Shine and Linsky (1974). The rates of collisional excitations $c_{ik}$ are expressed through $c_{ki}$ as

$$c_{ik} = \frac{g_k}{g_i} e^{-h\nu_{ik}/kT_e} c_{ki} \,; \tag{2}$$

$g_i$, $g_k$ are the statistical weights.

If the source functions $S_{ik}$ and the mean intensities $\bar{J}_{ik}$ of the lines formed by the permitted transitions between the five levels are known, then any two of five equations of statistical equilibrium would be sufficient to determine the electron temperature $T_e$ and the electron density $N_e$ for the following reasons: Assuming complete redistribution, which seems to be justified for the central parts of the line (Shine *et al.*, 1975), the relative population may be derived from the known values of the source function by using

$$S_{ik} = \frac{2h\nu_{ik}^3}{c^2} \left( \frac{g_k}{g_i} \frac{n_i}{n_k} - 1 \right)^{-1} . \tag{3}$$

The values of $\bar{J}_{ik}$ yield the rates of radiative processes. Then $T_e$ and $N_e$ enter the rate equations by means of (1) and (2). (In Equation (3) we have retained the stimulated emission for the sake of completeness.)

However, it is very difficult in practice to specify the source functions of all five lines from observations. Apart from the merely technical problem of simultaneous registration of two widely separated spectral regions with a sufficiently high spectral resolution there are more serious difficulties.

Firstly, the inversion of a line profile yields the source function on a line optical depth scale. To consider the statistical equilibrium equations in a specific layer of the atmosphere one must know the relation between the scales of optical depths of lines. For example,

$$\tau_X \sim \int_0^{\tau_K} \frac{\phi_X(0, \tau_K)}{\phi_K(0, \tau_K)} \frac{S_K(\tau_K)}{S_X(\tau_K)} \mathrm{d}\tau_K ,$$

where $\phi(0, \tau_K)$ is a normalized absorption function at the line center at the depth $\tau_K$. To bring $\tau_X$ and $\tau_K$ – optical depths in the X and K line centres – into agreement it is necessary to have a relation between $S_X$ and $\tau_K$. This is not deducible from direct observations.

Secondly, the levels of UV and IR line formation (i.e. the levels where $\tau_{UV}$ and $\tau_{IR}$ are close to unity) are separated in space. Any information based on measurements of the IR lines must be extrapolated to be useful at the height of formation of the H and K line cores.

Thus, the five rate equations (of which only four are independent) contain more than four unknown quantities. For the solution of the problem additional conditions are required. Before introducing these additional conditions it is convenient to combine the equations for levels 2 and 3, 4 and 5. If they are added in pairs the IR lines may be regarded as a whole, without knowledge of relations between the triplet components. We express the total radiative rate from the state $3^2D$ (later on – the state II) in terms of the total radiative rate from the state 1 and characterize it by parameter

$$\gamma = \frac{0.4B_{25}\bar{J}_X + 0.6B_{35}\bar{J}_Y + 0.4B_{24}\bar{J}_Z}{(\mathrm{UV})}, \tag{4}$$

where

$$(\mathrm{UV}) = B_{15}\bar{J}_K + B_{14}\bar{J}_H . \tag{5}$$

$B_{ik}$ are the Einstein coefficients of absorption. Expression (4) includes the numerical factors $g_1 = 2$, $g_2 = 4$, $g_3 = 6$, $g_4 = 2$, $g_5 = 4$.

Three parameters appearing in the rate equations – (UV), $n_{41} = n_4/n_1$, and $n_{51} = n_5/n_1$ are known from our previous work on the H and K lines. The remaining unknowns are $T_e$, $N_e$, $\gamma$, and $n_{II1} = (n_2 + n_3)/n_1$. The values of $n_{II1}$ and $n_{51}$ are connected by relationship

$$\frac{n_{II1}}{n_{51}} = \frac{g_2}{g_5}\left(1 + \frac{2h\nu_{25}^3}{c^2}\frac{1}{S_X}\right) + \frac{g_3}{g_5}\left(1 + \frac{2h\nu_{35}^3}{c^2}\frac{1}{S_Y}\right).$$

Shine and Linsky (1974) discussed in detail the problem of H and K line formation in an idealized atmosphere close in its properties to the HSRA (Gingerich *et al.*, 1971). In particular they showed that

$$S_X(h) \approx S_Y(h);$$

therefore, with sufficient accuracy,

$$\frac{n_{II\,1}}{n_{51}} \approx \frac{2.5 S_X + 1.6025 \times 10^{-3}}{S_X}.$$

If we write

$$\frac{S_X}{B_X(T_e)} = \beta \frac{S_K}{B_K(T_e)},$$

where $B_X(T_e)$ and $B_K(T_e)$ are the corresponding Planck functions, then the ratio $S_X/S_K$ is expressed through the parameter $\beta$ and the known function of temperature,

$$f(T_e) = \frac{B_X(T_e)}{B_K(T_e)}. \tag{6}$$

Consequently,

$$\frac{n_{II\,1}}{n_{51}} \approx \frac{2.5\beta f(T_e) S_K + 1.6025 \times 10^{-3}}{\beta f(T_e) S_K} \equiv \varphi(T_e, \beta). \tag{7}$$

By introducing the two unknown parameters $\gamma$ and $\beta$, we obtain the statistical equilibrium equations for the states $4^2S$, $3^2D$, and $4^2P$ in the following working form:

$$c_{21}\varphi(T_e, \beta) n_{51} + c_{41}(n_{51} + n_{41}) - c_1 = (\mathrm{UV}) - A_{41}(n_{51} + n_{41}), \tag{8}$$

$$\begin{aligned} &c_{II}\varphi(T_e, \beta) n_{51} - c_{4II} n_{41} - c_{5II} n_{51} - c_{1II} = \\ &\quad = A_{42} n_{41} + (A_{52} + A_{53}) n_{51} - \varphi(T_e, \beta)(\mathrm{UV})\gamma n_{51}, \end{aligned} \tag{9}$$

$$\begin{aligned} &-c_{II45}\varphi(T_e, \beta) n_{51} + c_4 n_{41} + c_5 n_{51} - c_{145} = \\ &\quad = (\mathrm{UV}) - (A_{41} + A_{42}) n_{41} - (A_{51} + A_{52} + A_{53}) n_{51} + \\ &\quad + \varphi(T_e, \beta)(\mathrm{UV})\gamma n_{51}. \end{aligned} \tag{10}$$

The various symbols are defined by:

$$\begin{aligned} c_1 &= c_{12} + c_{13} + c_{14} + c_{15}, \\ c_{II} &= 0.4(c_{21} + c_{24} + c_{25}) + 0.6(c_{31} + c_{34} + c_{35}), \\ c_{4II} &= c_{42} + c_{43}, \\ c_{5II} &= c_{52} + c_{53}, \\ c_{1II} &= c_{12} + c_{13}, \end{aligned}$$

$$c_{\mathrm{II}45} = 0.4(c_{24}+c_{25})+0.6(c_{34}+c_{35}),$$

$$c_4 = c_{41}+c_{42}+c_{43},$$

$$c_5 = c_{51}+c_{52}+c_{53},$$

$$c_{145} = c_{14}+c_{15}.$$

In deducing Equations (8) – (10) we assumed that the relative populations of the levels 2 and 3 follow a Boltzmann distribution with negligible energy separation of the levels. That is, we assume that $n_2/n_3 = g_2/g_3$. The $A_{ki}$'s are the Einstein coefficients of spontaneous emission.

From the expression for the collisional rate coefficients (Equation (1)) we see that the left hand sides of Equations (8) – (10) are proportional to the electron density $N_e$.

Any two of the independent equations of the set (8) – (10) may be solved for $T_e$ and $N_e$ when the parameters $\beta$ and $\gamma$ are known. To estimate $\gamma$, we computed the values of $\bar{J}_{ik}$ in an idealized atmosphere using the values of $S_K/B_K$, $S_H/B_H$, $S_X/B_X$, $S_Y/B_Y$, and $S_Z/B_Z$ given by Shine and Linsky (1974). From this we found $\gamma$ as shown in Figure 2. At depth, where $\tau_K \geqslant 50$, $\gamma$ varies almost parallel to its LTE value, so the ratio $\gamma/\gamma_{LTE}$ slowly decreases from $\approx 1.0$ in deeper layers to a minimum of $\approx 0.9$ at the top of this portion of the atmosphere. As the opacity in the IR lines diminishes, $\gamma$ increases again and a local maximum appears at $\tau_K \approx 5$. Thus, the value of $\gamma$ at large depths depends on the local temperature; closer to the surface it is controlled by the relation between the

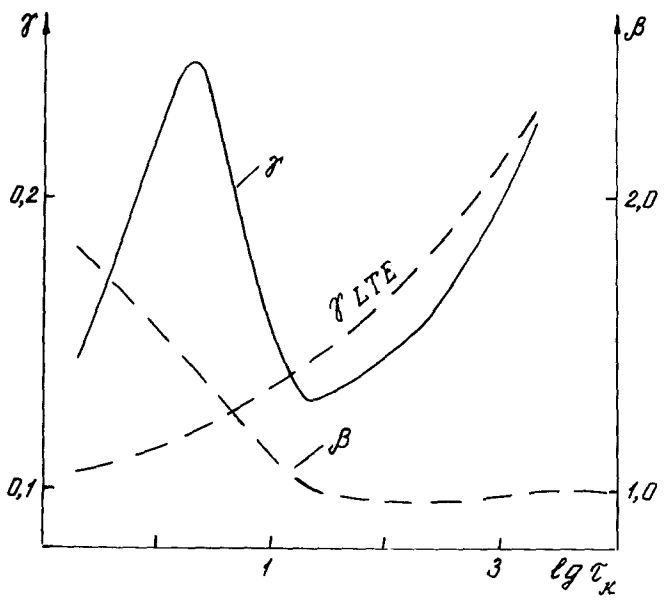


Fig. 2. Variations of the parameters $\beta$ and $\gamma$ with optical depth in the idealized chromospheric model.

optical depth scales of the two multiplets. To exclude the temperature influence on $\gamma$ it is convenient to use the ratio $\gamma/\gamma_{\mathrm{LTE}}$ instead of $\gamma$.

Shine and Linsky (1974) also noted that in the region of IR triplet formation the parameter $\beta$ is equal to unity with an accuracy of 10%. They pointed out that this behaviour of $\beta$ has a straightforward explanation: $\beta$ is determined not by the atmospheric model but by the relative values of collisional rates. In the region of resonance doublet formation $\beta$ increases and reaches about 1.6. The depth at which a rise in $\beta$ starts is close to that at which the increase of $\gamma$ begins; probably, it too depends on the relations between the scales of optical depths in both multiplets (Figure 2).

On the basis of the above behaviour of the parameters $\beta$ and $\gamma$ we adopt the limits

$$0.95 \leqslant \beta \leqslant 1.6\,, \tag{12a}$$

$$0.85 \leqslant \gamma/\gamma_{\mathrm{LTE}} \leqslant 1.0 \quad \text{for} \quad \tau_{\mathrm{K}} \geqslant \tau'_{\mathrm{K}}\,, \tag{12b}$$

$$1.0 < \gamma/\gamma_{\mathrm{LTE}} \quad \text{for} \quad \tau_{\mathrm{K}} < \tau'_{\mathrm{K}}\,. \tag{12c}$$

Here $\tau'_{\mathrm{K}}$ is some critical value of $\tau_{\mathrm{K}}$. Since the relations between optical depth scales in the sunspot umbra are unknown, the value of $\tau'_{\mathrm{K}}$ remains to be determined.

## 3. Distributions of Temperature and Electron Density

Equations (8) – (10) under the constraints (12) have no unique solutions for $T_e$ and $N_e$; one may only determine the limits within which the solutions are confined. The search for these limits is carried out as follows. At the deepest point ($\tau_{\mathrm{K}} = 10^4$) LTE is assumed and $T_e$ is determined by the Planck formula. At the next point (e.g. $\tau_{\mathrm{K}} = 10^3$), the values of $T_e$ and $N_e$ corresponding to $0.95 \leqslant \beta \leqslant 1.0$ are obtained, and those satisfying system (8) – (10) under condition (12b) are selected. As we move further upwards, the small values of $\beta \approx 1$ are gradually eliminated because there are no positive roots $N_e$ of the system (8) – (10), for reasonable values of $T_e$. Above a certain height one succeeds in obtaining a positive gradient $\mathrm{d}T_e/\mathrm{d}h$ only by using (12c) instead of (12b). This condition determines the critical value of $\tau'_{\mathrm{K}}$. For $\tau_{\mathrm{K}} < \tau'_{\mathrm{K}}$ there is a wide range of roots of the above system, corresponding to the numerous values of $\beta$ and $\gamma$ that are consistent with the requirements of (12a) and (12c). The largest and smallest values of $T_e$ delimit the boundaries of the band of possible solutions. Thus, a complete objective set of results is obtained with minimal restrictions on the numbers $\beta$ and $\gamma$.

There is an additional criterion of reliability of the results: the gradient of $N_e$ in the chromospheric layers must not be too large. Competition between two processes – increase in the degree of ionization and decrease in total density – leads to an almost constant electron density throughout an extensive portion of the atmosphere. This is clearly seen in the HSRA, the model by Vernazza *et al.* (1973), and the plage models of Shine and Linsky (1974).

Table I lists the data used in our calculations ($S_K$, $S_H$, and (UV)). The source functions were deduced from the H and K line profiles measured in five areas near the centres of the umbrae of two large sunspots (Teplitskaja and Efendieva, 1975). Since the source functions vary but little within each umbra we use only the average from each of the two spots, which we denote by I and II. The columns for the case denoted by II were found by using data closer to the centre of the line profiles, and so they correspond to a larger range of optical depths than the case denoted by I.

TABLE I

The initial data for model computing

| | | Umbra II | | | Umbra I | | |
|---|---|---|---|---|---|---|---|
| $\tau_K$ | $h$, km | $S_K \times 10^6$ | $S_H \times 10^6$ | (UV)$\times 10^{-5}$ | $S_K \times 10^6$ | $S_H \times 10^6$ | (UV)$\times 10^{-5}$ |
| 5 | 1770 | 2.291 | 2.101 | 1.367 | | | |
| 10 | 1720 | 2.061 | 1.994 | 1.298 | 2.559 | 2.030 | 1.510 |
| 50 | 1560 | 1.507 | 1.499 | 0.9782 | 1.739 | 1.701 | 1.217 |
| 100 | 1480 | 1.280 | 1.248 | 0.8287 | 1.481 | 1.490 | 0.9690 |
| 500 | 1290 | 0.8218 | 0.7305 | 0.5168 | 0.9029 | 0.8567 | 0.5800 |
| $10^3$ | 1200 | 0.6635 | 0.5477 | 0.4080 | 0.6502 | 0.5972 | 0.4134 |
| $10^4$ | 890 | 0.3444 | 0.3600 | 0.2286 | 0.2195 | 0.1805 | 0.1349 |

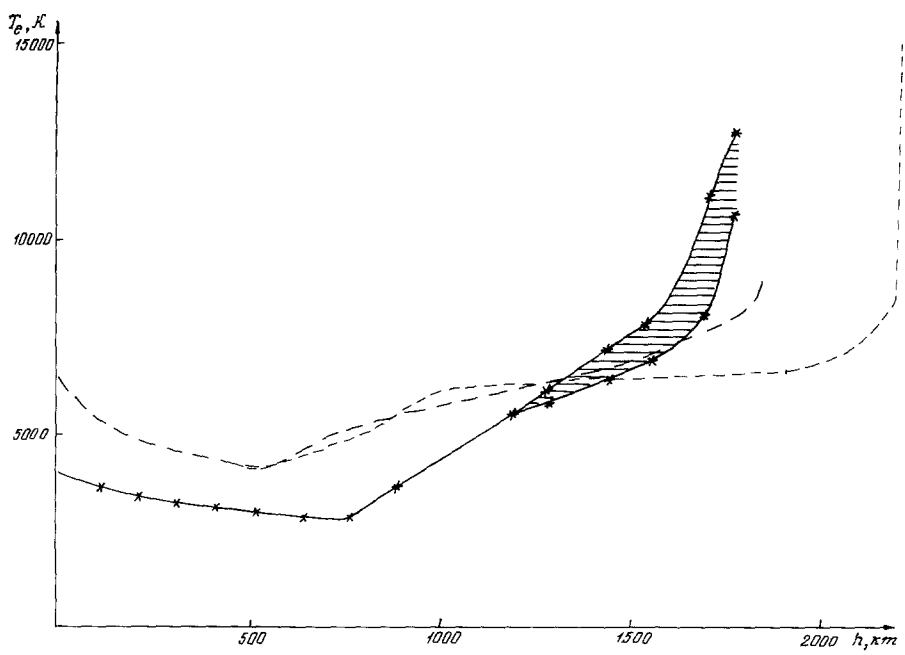


Fig. 3. Temperature distributions. Quiet region models: long dashes, Gingerich *et al.* (1971); short dashes, Vernazza *et al.* (1973). Sunspot umbral models: crosses, umbral photospheric layers from Kneer (1972); asterisks, chromospheric layers of umbra II.

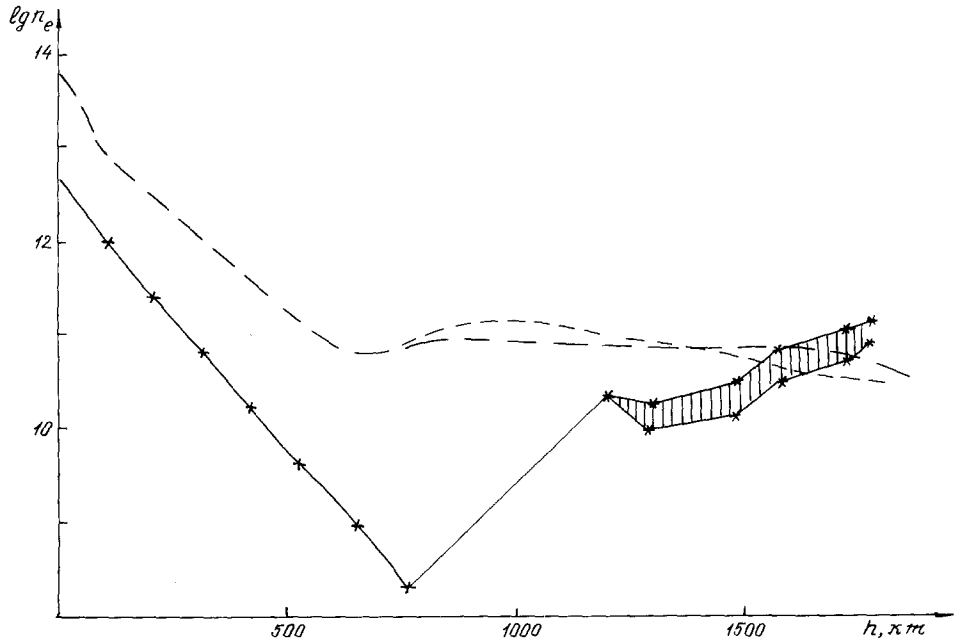


Fig. 4. Electron density distributions. Symbols similar to those in Figure 3.

Figures 3 and 4 show the results of calculations for group II of the line profiles. The hatched strips show the range of possible models above the sunspot umbra. In the photospheric layers of the umbra the distributions of $T_e$ and $N_e$ are taken in accordance with Kneer (1972). For comparison, the same figures give corresponding distributions in the models of the quiet chromosphere (Gingerich *et al.*, 1971; Vernazza *et al.*, 1973). Figure 5 presents the results for group I of the line profiles. For the sake of comparison the medians of the strips $T_e$ and $N_e$ in Figures 3 and 4 are traced by heavy lines.

## 4. Discussion

Comparison of the distribution curves $T_e(h)$ in Figure 3, as well as $T_e(m)$ given in Figure 3 of the paper by Shine and Linsky (1974) reveals some common features in the thermal structure of undisturbed and active regions of the chromosphere. The main one is the presence of a jump at approximately the same value of temperature – 8000 – 9000 K. The geometrical height of the level at which the jump occurs is different for different regions. In the umbra this height seems to be closer to the height of the jump in plages, rather than the height in quiet regions.

Up to the level where the jump occurs, the temperature in the umbra varies more rapidly than that in the undisturbed chromosphere. This fact was previously inferred by Efendieva (1973) from indirect data on the gradient $dT_e/dh$. At a height of about 1500 km the temperatures of the umbra and quiet region become equal and

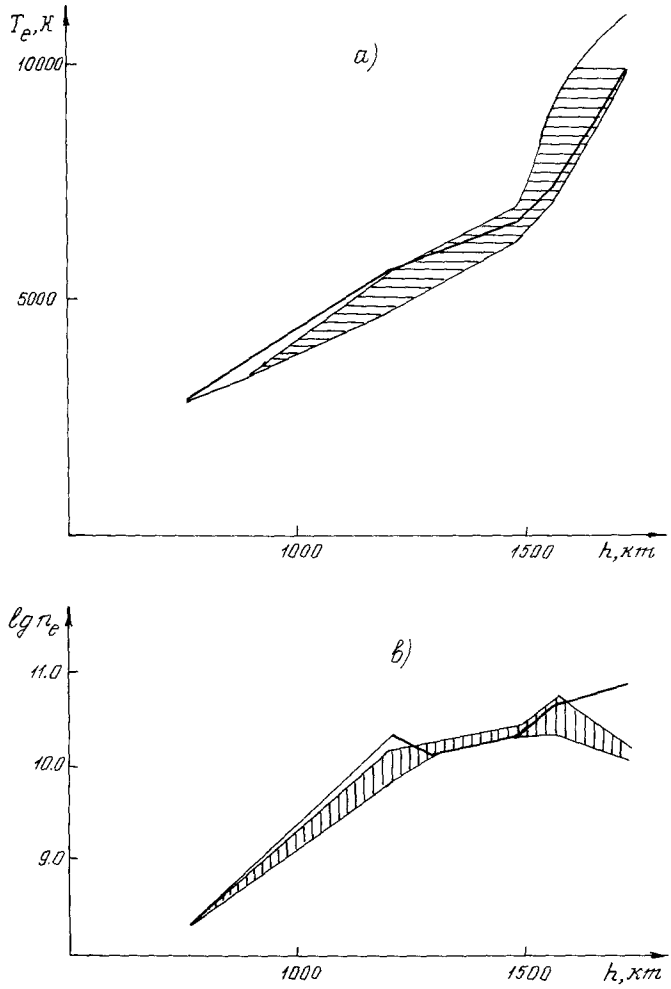


Fig. 5. Temperature and electron density in umbra I. Solid lines represent the medians of hatched strips in Figures 3 and 4.

above this height the umbra becomes hotter. Above the level of the jump, however, the gradient $dT_e/dh$ above the spot is evidently not as large as in the quiet chromosphere, and it is not precluded that $T_e$ in the transition region above the umbra is less than that in the surrounding material. This would be consistent with the axis-symmetric distribution of $T_e$ in the sunspot-related loops observed in the EUV (Foukal, 1975): the internal parts of the tubes are cooler than their periphery.

The electron density can be determined by using Equations (8) – (10) only where the radiation field deviates significantly from equilibrium. For that reason there is a gap between the last value of $N_e$ given by Kneer (1972) for a height about 800 km and our value of $N_e$ for a height 1200 km. The interval 800 – 1200 km must contain

the minimum of $N_e$. The quiet chromosphere models (HSRA and Vernazza *et al.*) also have shallow minima in the run of $N_e$ while much more pronounced minima occur in the plage models of Shine and Linsky (1974). A rapid increase in $N_e$ above the temperature minimum level results from a steeper rise of temperature in the umbra, which increases the degree of ionization.

In summary, we see that there is a sequence of plane-parallel models for different components of the Sun's chromosphere – the quiet region with a slow increase in temperature and almost constant electron density, then the plages of various brightness, and finally the sunspot umbra, where the temperature and ionization degree grow most rapidly. Table II combines data from Shine and Linsky (1974) and from our study of the umbra, to illustrate the existence of a sequence of temperature gradients.

TABLE II

The sequence of the chromospheric models; $dT_e/dh$ (K km$^{-1}$)

| Quiet region | Strong plages | Umbra of a large sunspot |
|---|---|---|
| 2.5 | 3.7 | 5.2 |

The distributions of $T_e(h)$ and $N_e(h)$ in the umbrae of different sunspots are not necessarily equal (cf. Figure 5 where curves are close but not identical). A similar behaviour has been found in plages (Shine and Linsky, 1974). The details of the distributions depend on individual properties of sunspots. In particular the phase of their development may be important (Baranovsky and Stepanyan, 1976).

This study involves two sunspots. It seems important to us to apply the algorithm for line profile inversion to investigate other sunspots of different sizes and age as well as plages of different brightness. This should reveal more completely the sequence of solar chromospheric regimes. This is also of interest for the problem of the structure of stellar chromospheres.

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