CENTER-TO-LIMB VARIATIONS OF THE Ca II H AND K LINES IN SUNSPOT UMBRAE

S. A. GRIGORYEVA and I. P. TUROVA
Institute of Solar-Terrestrial Physics, Russian Academy of Sciences, Siberian Branch, P.0. Box 4026, Irkutsk, 664033, Russia

(Received 18 February 1997; accepted 2 October 1997)

Abstract. A comparison of theoretical and observed Ca II H and K line profiles in sunspot umbrae has been made for different sunspot positions on the solar disk. Four semi-empirical static umbral models were used in calculations: the SUNSPOT model of Avrett (1981), and the models of Staude (1982), Maltby *et al.* (1986), and Severino, Gomez, and Caccin (1994). The models suggested by Avrett, Maltby *et al.*, and Severino, Gomez, and Caccin reproduce the center-to-limb evolution of the shape of observed profiles. The best agreement with profile parameters obtained from observations is given by the Severino, Gomez, and Caccin model.

1. Introduction

Amongst data on spectra of solar features, of particular interest is information about the center-to-limb intensity behaviour of the continuum and Fraunhofer line profiles. Our investigation is devoted to the chromosphere above sunspots. At present it is well known how the sunspot umbral intensity varies from the center to the limb at the photospheric level, and even the center-to-limb behaviour of the umbra depending on the solar cycle phase is well understood (see, e.g., Albregtsen, Jorås, and Maltby, 1984; Maltby, 1994). For chromospheric layers, however, observations of the center-to-limb behaviour of the sunspot umbra are virtually unavailable.

The best diagnostic tools for studying the solar chromosphere are strong lines, such as Lα, Mg II h and k, Ca II H and K; however, only the lines last-named are accessible to the ground-based observer. We now consider them in greater detail.

It is known that in a quiet region the cores of the Ca II H and K lines reveal the limb darkening. It looks the same in both the absorption and emission parts of profiles (Zirker, 1967). The presence of the limb darkening in the Ca II H and K lines has yielded a unique opportunity to verify the radiative transfer theory while calculating chromospheric line profiles. So, computations based on the complete frequency redistribution (CRD) assumption provide the limb brightening in the K_1 and K_2 parts (e.g., Milkey and Mihalas, 1973). Accounting only for partial frequency redistribution (PRD) effects made it possible to reproduce properly the observed limb darkening in the H and K lines for spatially averaged profiles in the quiet-Sun chromosphere.

As far as the sunspot umbra is concerned, the H and K lines show a peculiar kind of behaviour. In the 1960s a number of publications addressing these lines in

Solar Physics **179:** 17–30, 1998.

active regions appeared (see, e.g., Mustel and Tsap, 1960; Bumba, 1960; Paciorek, 1965; Engvold, 1966, 1967a, b). At that time the question of the H and K profile shape in the umbra was extensively discussed. All the above-cited authors pointed out that the profiles in the umbra are narrow and asymmetric, unlike those in quiet regions. Mustel and Tsap (1960), and also Paciorek (1965) reasoned that H and K emission cores are one-peaked in the umbra. Bumba (1960) and Engvold (1966, 1967a, b) considered that the central absorption $H_3(K_3)$ is always present in the umbral spectrum, and Engvold (1967a, b) found that the separation of emission peaks increases from the disk center to the limb. Teplitskaja and Effendieva (1971) showed that in the central part of the sunspot umbra the H and K line profiles can have both a one-peaked and two-peaked structure and that their shape depends on the sunspot position on the disk; at the disk center they represent single peaks, and at a value of the heliocentric angle $\theta > 45^\circ$ they become two-peaked. Teplitskaja and Firstova (1976) made measurements of H and K profiles at different points on the solar disk using a reasonably extensive set of statistical data. It should be noted that the above results refer to the features $H_3(K_3)$ and $H_2(K_2)$ and do not give an idea of the center-to-limb behaviour of the features $H_1(K_1)$.

Based on the references cited we may list the main peculiarities of the center-to-limb behaviour of the features $H_{232}(K_{232})$ in the sunspot umbra: (i) As the sunspot moves from the center to the limb, the emission core changes in its shape: a single asymmetric peak at the center becomes two-peaked as it approaches the limb. When the sunspot moves to the disk limb, the central absorption appears. (ii) The top of a single emission peak is blueshifted, although the line center can also be redshifted. If a red shift is present, it is more clearly pronounced for the K line than for the H line. (iii) In two-peaked self reversals, the blue peak is usually brighter than the red peak for both lines. For the H line near the limb, however, the intensity of the red and blue peaks is nearly the same, and sometimes the red peak becomes even brighter than the blue peak. (iv) The limb darkening is seen at $H_3(K_3)$. (v) The emission core broadens as a whole to the limb.

We did not find in the literature any theoretical reconstruction of the observed center-to-limb behaviour of H and K profiles, though at the present time there are some well-founded models of sunspot umbrae.

The aim of the present work is to verify if the observed peculiarities of Ca II H and K profiles for different points on the solar disk can be reconstructed within the framework of the known semi-empirical models. While on the subject of the center-to-limb variations of the profile, we keep in mind not only the intensity variations in some particular parts of the profile but also the behaviour of such characteristics as the shape of the profile, the wavelength position at minimum intensity $H_1(K_1)$, and the half width. Besides, the role of the PRD effects on the theoretical reconstruction of the center-to-limb behaviour of the profiles is of interest.

Four semi-empirical static umbral models were used in the calculations: model SUNSPOT of Avrett (1981), Staude's (1982) model, Maltby *et al.* (1986) model, and Severino, Gomez, and Caccin's (1994) model.

2. Behaviour of the Ca II H and K Lines at Different Sunspot Positions on the Solar Disk

The shape of Ca II H and K line profiles is an important source of information about physical conditions in layers where lines are formed. It is representative of both the temperature run and the dynamic regime in the chromosphere. Carlsson and Stein (1995), when treating spatially and temporally resolved profiles, concluded that in magnetic field-free internetwork regions there is no emission most of the time and, hence, no general chromospheric temperature rise up to $\approx$ 1000 km. Enhanced chromospheric emission, when present, can be produced by shock waves which generate short intervals of high temperatures without any outward increase in the mean gas temperature. The fact that emission is present everywhere on the disk in the spatially resolved Mg II spectra, is associated by Carlsson and Stein with the magnetic field effects in layers where the h and k lines are formed.

Unlike the quiet chromosphere, in the sunspot umbra the Ca II H and K lines are always present in emission. Possibly this is due to the presence of a magnetic field. No case of observations with high spatial and temporal resolution which could reveal at once how the magnetic field variations influence the H and K lines within the umbra, has come to our notice. On the other hand, we know that the shape of Ca II H and K emission peaks is very changeable over the same umbra. After Beckers and Tallant (1969) had found umbral flashes (UF), it became apparent that the diversity of H and K profiles also depends on the dynamic processes in the chromospheric layers of the umbra. Mattig and Kneer (1978), Kneer, Mattig, and Uexküll (1981), Turova, Teplitskaja, and Kuklin (1983), Lites (1986) have also investigated the spatial and temporal behaviour of Ca II H and K emission peaks in the presence of oscillations. Lites (1992), in a recent extensive review, summarizes the present status of the problem of investigating oscillations in sunspots, in both the observational and theoretical aspects (see also a review by Staude, 1994).

At the particular position in the umbra and instant of time, the shape of the H and K lines is determined by the phase of the oscillation. The central intensity may change 3–4 times in the minimum and maximum of UF. For studying the behaviour of Ca II H and K line profiles in umbrae at different positions on the solar disk, it is necessary to refer not only to the same point in the sunspot umbra but also to the same phase of the oscillations which is not a simple problem. An investigation of sunspot umbrae is also complicated by the effects of scattered light which not only depends upon seeing conditions but also variously distorts line profiles at different sunspot positions on the disk. As this takes place, $H_2(K_2)$ regions suffer less from the effects of the scattered light than $H_1(K_1)$ as they are bright emission features in the umbra. Hence it is possible to treat the absolute and relative line intensities in the umbra only together with a careful correction for the scattered light; moreover, it is desirable to have a large number of observations at different positions on the disk.

Table I
Reference lines in the neighbourhood of the H and K lines

K line				H line			
Elem.	λ, Å	EP, eV	W, mÅ	Elem.	λ, Å	EP, eV	W, mÅ
Fe I	3932.64	3.72	52.9	Fe I	3967.43	3.29	54.0
Fe I	3935.31	2.83	25.2	Fe I	3969.27	1.48	103.0
Fe I	3935.81	2.82	66.4	Cr I	3969.75	2.53	30.7
Co I	3935.98	0.93	59.4	Fe I	3970.40	3.06	42.9

Although the sunspot umbra is not uniform in its magnetic structure and there are oscillations, the presence of emission in the Ca II H and K lines, we believe, indicates that the chromospheric rise of temperature in sunspot umbrae is more possible than in a quiet region. Because of this we discuss the center-to-limb behaviour of the Ca II H and K lines in the context of the semi-empirical static chromospheric umbral models.

2.1. Observational data

We used spectrograms of one sunspot – the leader of group No. 751 according to *Solar-Geophysical Data* taken in the H and K Ca II lines. The pictures were obtained at the Sayan Solar Observatory in July 1981. A detailed description of the data and reduction methods may be found in Turova, Teplitskaja, and Kuklin (1983) and Turova (1994). Originally these data were meant for a different problem which did not require taking into account the scattered light; therefore, scattered light parameters were not recorded.

In this paper, observational data obtained on 19 July 1981 and 24 July 1981, corresponding to the sunspot position $\mu = \cos\theta = 0.45$ and $\mu = 0.97$, were used.

The umbral spectra which we used involved a bright feature; despite this, however, a quiet region of the umbra was clearly identifiable. Intensity variations in it had a very small amplitude, approximately 6 times as small as in the bright feature. Several profiles with the least intensity were chosen from the profiles measured in the course of the time series in this quiet area of the umbra (including the wing up to ≈ 2 Å), and they were then averaged. Thus, we obtained an averaged 'quiet' profile for each of the indicated values of μ.

Since our observational data were not corrected for the scattered light, to solve the problem formulated in this paper we have chosen such profile characteristics which were most stable to distortions by the scattered light (the shape, the positions of minimum intensity $H_1(K_1)$, and the half-width).

In addition to the observations described above, we used results reported by Teplitskaja and Firstova (1976) because they are based on a study of several tens of sunspots at different points of the disk. The measurements were made in minimum

Table II
Characteristics of umbral models

Model	$T_{\min}$, K	$H_{\min}$, km	dT	H_r, km	$T_{\max}$, K	$H_{\max}$, km
Avrett	3500	450	3.42	1590	199 500	4276
Maltby *et al.*	3400	404 (mean)	3.05	1816	12 300	2126
Staude	3000	511	5.08	1496	42 000	1820
Severino *et al.*	2910	548	4.31	1590	63 100	2308

intensity regions of the continuum in the umbra where no local perturbations, such as a UF maximum or moustaches, were visible. For most of these measurements, no corrections for the scattered light were introduced; therefore, we are using them for a qualitative discussion only.

The wavelength scale in the neighbourhood of the H and K lines was established using reference lines of a quiet region of photospheric origin; they are listed in Table I, where EP is lower excitation potential, and W is equivalent width of the line. Areas of the quiet region were chosen on both sides of the sunspot projected onto the slit. The positions of the H and K line centers were determined separately on each side, and subsequently they were found for the sunspot umbra by interpolation. Corrections for rotational and orbital motion of the Earth and for the gravitational red shift of these lines and of the H and K lines are the same. It may be suggested that corrections for solar rotation at corresponding atmospheric levels are also close to each other.

However, Samain (1991) found one more systematic line shift in a quiet region, the red shift of about 1 km s^{-1} inherent in all lines which he measured, which, except for one C I line, form in the upper photosphere. The source of this shift remains unknown; it seems likely that it has a complicated combined nature. The reference lines listed in Table I form in different layers of the quiet photosphere, and also in the low photosphere where no red shift is detectable (see, e.g., Samain, 1991; Dravins, Lindegren, and Nordlund, 1981). Such a 'heterogeneous' character of the reference lines used leads to the fact that the H and K line centers in umbral spectra were measured as accurate as 1 km s^{-1}.

2.2. Calculated Ca II H and K line profiles for the sunspot umbra

The theoretical profiles were computed using code MULTI (Carlsson, 1986) for four models. The temperature structure of these models is shown in Figure 1 where we limited ourselves to the value of $T = 15\,000$ K in order to illustrate more dramatically their difference in the photosphere and the low chromosphere. Table II compares some model parameters which illustrate their individual peculiarities. Here $T_{\min}$ is the temperature in the region of temperature minimum, $H_{\min}$ is the height corresponding to $T_{\min}$, dT is the average temperature gradient in the chromosphere, H_r is the height, at which the temperature plateau terminates and

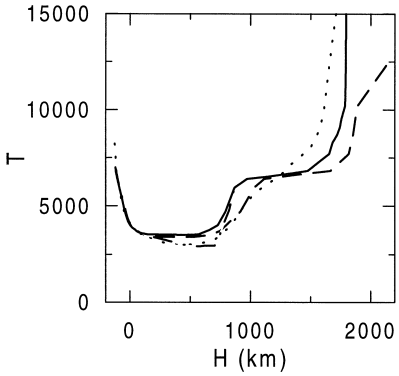

Figure 1. Temperature vs height for four umbral models; model SUNSPOT (solid curve); Staude's model (dotted curve); Maltby *et al.* model (dashed curve); Severino, Gomez, and Caccin's model (dash-dotted curve). Height is measured outward from the level at which $\tau_{5000} = 1$, where τ_{5000} is the continuum optical depth at 5000 Å.

an abrupt temperature rise in the chromosphere begins, $T_{\max}$ is the maximum temperature in the chromosphere, and $H_{\max}$ is the height corresponding to $T_{\max}$.

Staude's model does not include the temperature plateau; the height at which the temperature gradient in the chromosphere increases abruptly, was taken to be H_r for this model.

PRD effects were taken into account using Uitenbroek's code which is complementary to MULTI; we were kindly provided with it by the author. The Ca II ion is represented as five levels plus continuum. Atomic parameters of Ca II, as in Carlsson (1986), are taken from Shine and Linsky (1974). All bound-free transitions are calculated approximately by specifying the emission temperature T_r that is fixed in the photosphere. For the transitions from levels $4^2S_{1/2}$, $3^2D_{3/2}$, $3^2D_{5/2}$, values of T_r are taken from Lites and Skumanich (1982). For the transitions from levels $4^2P_{1/2}$ and $4^2P_{3/2}$ we accept a somewhat smaller value of T_r, namely 4600 K, which was obtained through comparison with an exactly calculated transition (Grigoryeva, Turova, and Teplitskaya, 1989). The eight-point quadrature in μ is applied.

To reproduce the blue shift of the emission peak from the center of the unperturbed line, the asymmetry of the Ca II H and K line profiles and the center-to-limb variation in asymmetry, we included, as was done by Lites and Skumanich (1982), in our computation the systematic mass downflow in the chromospheric layers of the umbra, the velocity of which decreases linearly from the upper point of the model to a height slightly above the temperature minimum. Lites and Skumanich (1982), in similar calculations for their umbral model, derived the downflow velocity $V_{\rm sys}$ from the relationship $\rho V_{\rm sys} = C$, where ρ is the mass density. At the upper point of their model $\rho = 8.198 \times 10^{-15}$ g cm^{-3} and $V_{\rm sys} = 4 \times 10^6$ cm s^{-1}, such that $C = 3.28 \times 10^{-13}$ g^{-1} s^{-1} cm^{-2}. For the models of Staude (1982) and Maltby *et al.* (1986), we took the value of $V_{\rm sys}$ at upper points of the models to be

Table III
Adopted downflow velocity in the chromosphere

Reference	V_{sys}, km s^{-1}
Avrett (SUNSPOT), 1981	6.2
Staude, 1982	3.8
Maltby *et al.*, 1986	4.0
Severino, Gomez, and Caccin, 1994	3.4

close to that given by this relationship. For the SUNSPOT models of Avrett (1981) and for the model of Severino, Gomez, and Caccin (1994), extremely high values of V_{sys} in the upper chromosphere are obtained, with which we were unable to find a solution. Therefore, for these models, we had to depart from the continuity condition, and we took such a behaviour of the systematic velocity which makes it possible to obtain a stable solution. Table III lists the values of V_{sys} taken for the upper point (H_{max} from Table II) of each of the four models.

It should be noted that it is a rather artificial expedient to include a flow for reproducing the asymmetry of the H and K line profiles. However, keeping within the framework of static models, we have only this tool for making the shape of synthesized profiles fit observations. The presence of oscillations in the umbral chromosphere requires taking into account dynamic effects both when constructing a model and when interpreting chromospheric line profiles. As has been pointed out above, the shape of H and K line profiles is determined largely by the oscillation phase. In the future we intend to perform calculations with dynamic processes taken into account. At the same time we believe that calculations in terms of static models are also very useful because they describe the 'instantaneous' state of the chromosphere and can be used as the basis for calculations including the dynamics.

The synthesized Ca II H and K line profiles were corrected by invoking macroturbulence in the calculations. For all models, except for Staude's model, it was assumed that $V_{macro} = 5$ km s^{-1}, and for Staude's model $V_{macro} = 3$ km s^{-1} was taken (V_{macro} is the most probable velocity of a Gaussian distribution). The V_{macro} values were selected in such a way that the computed profiles could reasonably reproduce the shape of the observed ones.

Figures 2(a–d) show synthesized PRD profiles of the H line for the models considered at three points on the disk. Figure 2(e) gives time-averaged observed H line profiles at two points on the disk (sunspot of AR 751).

From the comparison of Figures 2(a–d) and 2(e) it follows that the shape of the profiles calculated from the SUNSPOT model and the models of Severino, Gomez, and Caccin and Maltby *et al.* and the shape of the observed profiles are alike. The calculated profiles reproduce qualitatively the evolution of the shape of the observed profiles as the sunspot moves from the center to the limb. A similar conclusion follows also from the comparison of the calculated profiles with Figure 1 from

Table IV
Shift $\Delta\lambda_0$ of the H line bisector (all shifts are in km s^{-1})

Intensity	Avrett	Staude	Maltby *et al.*	Severino *et al.*	Observ.
$\mu = 0.98$					$\mu = 0.97$
100%	−3.93	0	−4.91	−3.40	−3.10
70%	−2.42	0.11	−4.46	−2.76	−1.66
50%	−0.94	0.15	−2.27	−0.38	−0.57
20%	−0.15	0.19	0.15	0.57	−0.04
$\mu = 0.41$					$\mu = 0.45$
70%	−1.21	0.15	0.11	0.45	−0.83
50%	−0.49	0.11	0.26	0.57	−0.64
20%	−0.04	0.11	0.34	0.68	−0.98

Teplitskaja and Firstova (1976), which shows a variation in the shape of H and K line profiles for five points on the disk ($\mu = 0.94$, 0.72, 0.51, 0.41, and 0.24). The synthesized profiles show the same direction of the asymmetry, the transition to a two-peaked shape, the blue shift of the top of a single peak, and even demonstrate such a fine detail as a change of asymmetry sign of the H line profile near the limb. For Staude's model we were unable to achieve a likelihood of the shape of the calculated and observed profiles.

The behaviour of the K line profile in all four models is completely identical to that of the H line to the point of asymmetry of red and blue peaks at $\mu = 0.10$; therefore, we do not plot it. It should be noted, however, that, unlike the H profile, we did not find any observational confirmation of the asymmetry inversion of the K profile near the limb. Thus, computations do not reconstruct this difference between the H and K lines. The reason for this discrepancy can be due to the fact that the cross redistribution was not taken into account in our computations. Besides, it is possible that the adopted $V_{\rm sys}$ run might be not exactly correct in upper umbral layers.

For a qualitative evaluation of the asymmetry, we determined the shift $\Delta\lambda_0$ of the H line bisector in several parts of the profiles. The values of $\Delta\lambda_0$ for four models and observations are given in Table IV.

It follows from Table IV that the bisectors of the observed profiles are consistently blueshifted for both values of μ. The random error of determination of $\Delta\lambda_0$ is $\approx \pm 0.5$ km s^{-1}. For $\mu = 0.97$, the position of the line bisector is monotonically redshifted from the top of the peak to the wings, but it does not change sign, that is, throughout the height of the profile it remains on the blue side. Such a behaviour of the value of $\Delta\lambda_0$ somewhat differs from that summed by Teplitskaja and Firstova (1976) from spatially averaged profiles. According to their observations, the H line center in the wing region has either a red or zero shift.

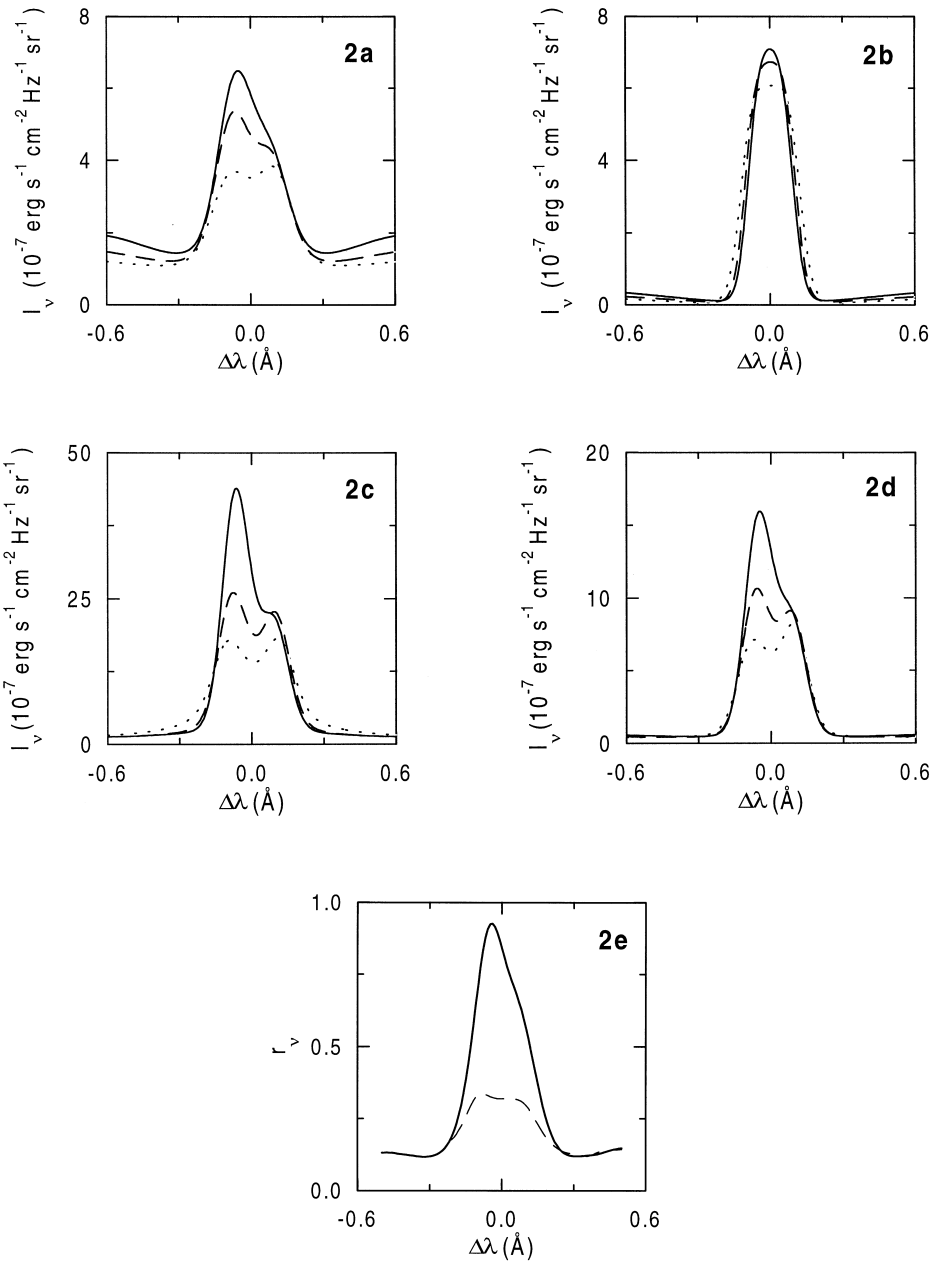

Figure 2. Synthesized PRD profiles of the H line for four umbral models and observations. (a) model SUNSPOT: $\mu = 0.10$ (dotted curve); $\mu = 0.41$ (dashed curve), $\mu = 0.98$ (solid curve). (b) Staude's model. (c) Maltby *et al.* model. (d) Severino, Gomez, and Caccin's model. (e) Time-averaged observed H line profiles: $\mu = 0.45$ (dashed curve); $\mu = 0.97$ (solid curve); the intensities are given in quasi-continuum units ($\lambda = 3954.2$ Å).

For $\mu = 0.45$, the behaviour of the value of $\Delta\lambda_0$ is more complicated. Because the profiles at this point on the disk have now become two-peaked, we limited ourselves to a maximum intensity level of 70%, lest the bisector pass through the peaks. The bisector is arch-shaped, with a redward convexity, although the shifts themselves are directed blueward. Figure 1 from Teplitskaja and Firstova (1976) gives bisectors of a similar shape for the values of $\mu = 0.41$ and $\mu = 0.51$. The difference lies in the direction of the shifts; for the AR 751 sunspot all shifts are blueward.

The uncertainty in the determination of observed values of $\Delta\lambda_0$, which is associated with the accuracy of determining the position of the H and K line centers (1 km s^{-1}), does not permit us to interpret them in terms of movements in layers, in which these lines originate.

The calculated profiles for the disk center from all models show a tendency similar to observed profiles: the greatest blue shift corresponds to the top of the peak; in the direction toward the wings the blue shift decreases, and for the models of Maltby *et al.* and Severino, Gomez, and Caccin it becomes blueward. The exception is Staude's model where the top of the peak is not shifted, and in the direction toward the wing the shifts increase redwards.

For $\mu = 0.41$, the behaviour of the calculated and observed profiles is different: three models show a monotonic red shift of the line bisector in the direction from the top to the wing, and the exception is Staude's model which gives the opposite picture.

On the whole, it can be said that by a more successful choice of the systematic flow velocity (and by referencing to the absolute wavelength scale by, for example, imprinting the spectrum of a laboratory source) it is quite probable that a fuller quantitative coincidence of the bisector shifts of calculated and observed profiles can be achieved. But the chief thing is that such an approach makes it possible to reproduce on a qualitative level the complicated character of the evolution of profiles during the disk passage of the sunspot. Of course, incorporating the downflow in calculations does not exclude alternative explanations which must be based on constructing dynamic models of the chromosphere. However, the result which we obtained for the disk center, is perhaps real evidence of the presence of vertical downward movements in the umbral chromosphere.

Let us now consider the intensities of the calculated profiles. As has already been pointed out, we do not compare absolute values of intensities with observations chiefly because the observations were not corrected for scattered light. Furthermore, the observed intensities of H_2 (K_2) in umbral spectra are extremely varied, changing from one umbral portion to another as well as with time.

The intercomparison of the calculated profiles shows that their central intensities are also quite diverse; they depend strongly on the model and on the sunspot position on the disk. The brightest profiles are obtained from the models of Maltby *et al.* and Severino, Gomez, and Caccin, and the least bright profiles are those calculated by the SUNSPOT model. The models of Maltby *et al.* and Severino, Gomez,

and Caccin give higher values of relative level populations that participate in the formation of the H and K Ca II lines, than the SUNSPOT model, which leads to increased emission in the cores of these lines. The difference of these three models lies not only in the temperature structure. In the models of Maltby *et al.* and Severino, Gomez, and Caccin, values of N_e exceed substantially corresponding values of the SUNSPOT model in chromospheric layers.

The profiles calculated from Staude's model occupy an intermediate position with respect to the brightest and the least bright for all values of μ.

It is interesting to trace the influence of partial redistribution effects upon the center-to-limb variation of H and K line intensities in the umbra. Is it similar to the influence that was found earlier for a quiet region? Let us compare Figures 2(a) and 3. Figure 3 shows the result of CRD-calculations of the H line for the SUNSPOT model. One can see that the computations made, accounting for PRD, predict the limb-darkening in all frequencies of the emission reversal. The CRD computations give a somewhat different picture. Whereas the behaviour of PRD and CRD profiles is the same at $H_2(H_3)$, CRD profiles predict the limb brightening at H_1. The distinction observed between the center-to-limb behaviour of the synthesized PRD and CRD profiles is similar to that known for a quiet region and plages (e.g., Heasley and Kneer, 1976, their Figure 2; Shine, Milkey, and Mihalas, 1975, their Figure 11).

Figure 4 gives the intensity of the feature H_1 at seven points on the solar disk for the four models with PRD effects taken into account. It is evident that the H_1 intensities differ greatly for different models, as also do the central intensities. For the disk center, for instance, the largest value (from the SUNSPOT model) and the smallest value (from Staude's model) differ nearly by a factor of 15. The degree of limb darkening is also different. It is most pronounced for the SUNSPOT model. For the other three models, the limb darkening of H_1, while present, is very small. In the model of Severino, Gomez, and Caccin, at the point $\mu = 0.10$, the H_1 intensity even slightly exceeds the intensity at the point $\mu = 0.24$.

Let us consider some characteristics of the observed profiles which are virtually independent of the scattered light influence, and compare them with the calculations. Since the behaviour of the H and K profiles is identical, we give them for the H line only.

The center-to-limb variations of the minimum intensity position $\Delta\lambda(H_1)$ are shown in Figure 5(a). The four models all provide monotonic increase of this parameter from the center to the limb. The best agreement with the observations is shown by the SUNSPOT model and the model of Severino, Gomez, and Caccin.

Figure 5(b) illustrates the variations in the H and K profile half width along the disk. All the models considered show its gradual increase from the center to the limb. The model of Severino, Gomez, and Caccin agrees best with the observations.

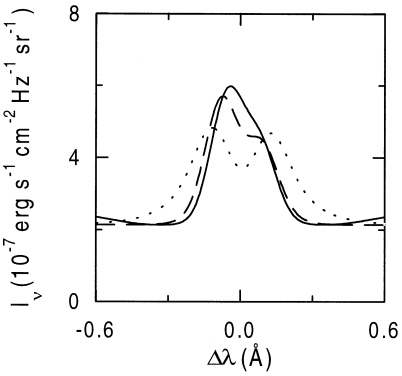

Figure 3. CRD profiles of the H line, SUNSPOT model. The meaning of line types is the same as in Figure 2.

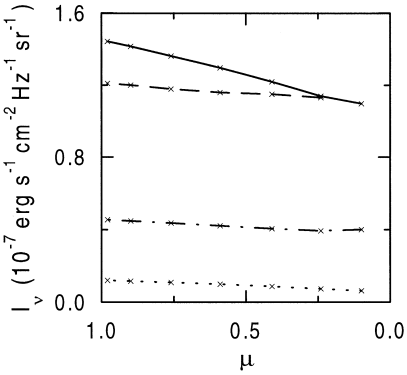

Figure 4. Intensity at H_1 vs the location of sunspot on the solar disk for four umbral models. The meaning of the line types is the same as in Figure 1.

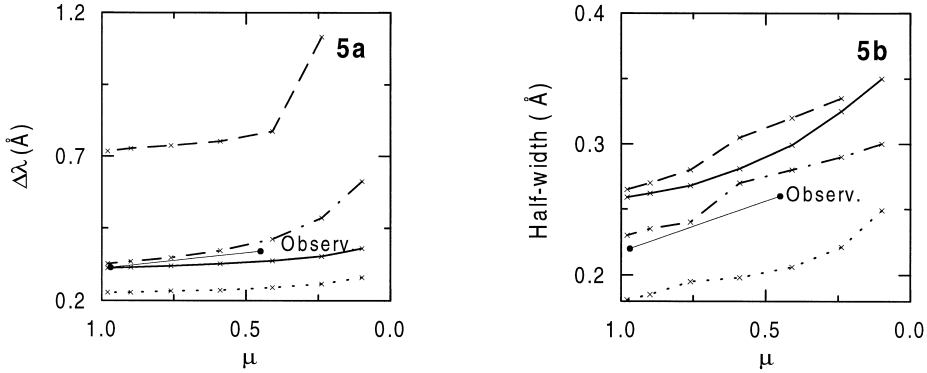

Figure 5. (a) Wavelength separation $\Delta\lambda(H_1)$ from the line center in Å at different points on the disk. Observational values (thin solid curve); the rest of the curves are as in Figure 1. (b) The same as in (a), but for half width.

3. Conclusion

The study presented here suggests the following conclusions:

(1) The semi-empirical models of Avrett, of Severino, Gomez, and Caccin, and of Maltby *et al.* reproduce the center-to-limb behaviour of the observed H line profiles in the umbra: the asymmetry direction, the transition to a two-peaked shape, the blue shift of the top of a single peak, and even a systematic red shift of the bisector line for the disk center. Staude's model does not reproduce the observed evolution of the shape of the H line profiles on the disk.

(2) Central intensities of the profiles calculated from the four umbral models, and also the intensities of the features H_1 and K_1 differ substantially from each other both at the disk center and on the limb, which is caused by individual properties of each of the models.

(3) Synthesized profiles for all models, with the PRD effect taken into account, show a certain degree of limb darkening throughout the emission inversion.

(4) The neglect of PRD effects when synthesizing H and K profiles in the umbra leads to the situation known for a quiet region and plages: it results in a limb brightening in the $H_1(K_1)$ region.

(5) The model of Severino, Gomez, and Caccin predicts best (as compared to the other three models) the observed values of peak separation $\Delta\lambda(H_1)$ and the H line half width. The model of Avrett also gives values of $\Delta\lambda(H_1)$ similar to the observed values. The model of Maltby *et al.* gives broader profiles, and Staude's model gives narrower profiles, both in the H_1 region and at 50% of maximum intensity.

Acknowledgements

We are indebted to Dr G. Severino for kindly making the data on the sunspot umbral model available to us. We are deeply grateful to Dr R. B. Teplitskaya for valuable suggestions and discussions on the preparation of the manuscript. We wish to thank Dr H. Uitenbroek for providing us with PRD code. We are also indebted to V.G. Skochilov for taking part in some computer calculations, and to V. G. Mikhalkovsky for his assistance in preparing the English version of the manuscript. This work was supported by the Russian Foundation for Fundamental Research, project number 95–02–04208a. We also thank the referee, Dr P. Maltby, for useful comments and constructive suggestions.

References

Albregtsen, F., Jorås, P. B., and Maltby, P.: 1984, *Solar Phys.* **90**, 17.

Avrett, E. H.: 1981, in L. Cram and J. H. Thomas (eds.), *The Physics of Sunspots*, Sacramento Peak Observatory, p. 359.

Beckers, J. M. and Tallant, P. E.: 1969, *Solar Phys.* **7**, 351.
Bumba, V.: 1960, *Izv. Krymsk. Astrofiz. Obs.* **23**, 212.
Carlsson, M.: 1986, *Uppsala Astron. Report*, No. 33, p. 1.
Carlsson, M. and Stein, R. F.: 1995, *Astrophys. J.* **440**, L29.
Dravins, D., Lindegren, L., and Nordlund, Å.: 1981, *Astron. Astrophys.* **96**, 345.
Engvold, O.: 1966, *Astrophysica Norvegica* **10** (6), 101.
Engvold, O.: 1967a, *Solar Phys.* **2**, 234.
Engvold, O.: 1967b, *Astrophysica Norvegica* **10** (9), 173.
Grigoryeva, S. A., Turova, I. P., and Teplitskaya, R. B.: 1989, *Issledovaniya po geomagnetizmu, aeronomii i fisike Solntsa*, Irkutsk, No. 87, p. 131.
Heasley, J. N. and Kneer, F.: 1976, *Astrophys. J.* **203**, 660.
Kneer, F., Mattig, W., and Uexküll, M. v.: 1981, *Astron. Astrophys.* **102**, 147.
Lites, B. W.: 1986, *Astrophys. J.* **301**, 1005.
Lites, B. W.: 1992, in J. H. Thomas and N. O. Weiss (eds.), *Sunspots: Theory and Observations*, Kluwer Academic Publishers, Dordrecht, Holland, p. 261.
Lites, B. W. and Skumanich, A.: 1982, *Astrophys. J. Suppl.* **49**, 293.
Maltby, P.: 1994, in R. J. Rutten and C. J. Schrijver (eds.), *Solar Surface Magnetism*, NATO ASI Series C 433, Kluwer Academic Publishers, Dordrecht, Holland, p. 179.
Maltby, P., Avrett, E. H., Carlsson, M., Kjeldseth-Moe, O., Kurucz, R. L., and Loeser, R.: 1986, *Astrophys. J.* **306**, 284.
Mattig, W. and Kneer, F.: 1978, *Astron. Astrophys.* **65**, 11.
Milkey, R. W. and Mihalas, D.: 1973, *Astrophys. J.* **185**, 709.
Mustel, E. R. and Tsap, T. T.: 1960, *Izv. Krymsk. Astrofiz. Obs.* **22**, 75.
Paciorek, J.: 1965, *Czechoslovak Academy of Sciences Astronomical Inst. Publ.* No. 51, p. 69.
Samain, D.: 1991, *Astron. Astrophys.* **244**, 217.
Severino, G., Gomez, M. T., and Caccin, B.: 1994, in R. J. Rutten and C. J. Schrijver (eds.), *Solar Surface Magnetism*, NATO ASI Series C 433, Kluwer Academic Publishers, Dordrecht, Holland, p. 169.
Shine, R. A. and Linsky, J. L.: 1974, *Solar Phys.* **39**, 49.
Shine, R. A., Milkey, R. W., and Mihalas, D.: 1975, *Astrophys. J.* **199**, 724.
Solar-Geophysical Data: 1981, No. 444, Part 1, p. 22.
Staude, J.: 1982, *HHI-STR-Report* No. 14, Category S, p. 24.
Staude, J.: 1994, in R. J. Rutten and C. J. Schrijver (eds.), *Solar Surface Magnetism*, NATO ASI Series C 433, Kluwer Academic Publishers, Dordrecht, Holland, p. 189.
Teplitskaja, R. B. and Effendieva, S. A.: 1971, *Soln. Dann.* No. 3, 80.
Teplitskaja, R. B. and Firstova, N. M.: 1976, *Solar Phys.* **48**, 103.
Turova, I. P.: 1994, *Solar Phys.* **150**, 71.
Turova, I. P., Teplitskaja, R. B., and Kuklin, G. V.: 1983, *Solar Phys.* **87**, 7.
Zirker, J. B.: 1967, *Solar Phys.* **3**, 164.