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Cosmic-ray kinetics in a random anisotropic reflective non-invariant magnetic field

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Abstract. The problem of high-energy charged particle propagation in a turbulent medium with a "frozen-in" statistically anisotropic reflective non-invariant magnetic field attracts much attention at the present time. We show that the existence of a large-scale electric field, of which the value is determined by a magnetic helicity, leads to the appearance of an effective mechanism of particle acceleration. We derive the kinetic equation for the particle distribution function and calculate its diffusion approximation. An analysis of the effectiveness of acceleration mechanisms in the observed spectrum of cosmic rays shows that the newly considered mechanism dominates over the second-order Fermi particle acceleration in the energy range of up to 1 GeV.

Key words: cosmic rays: acceleration – kinetic theory: diffusion approximation

1. Introduction

The problem of the propagation of high-energy charged particles in stochastic magnetic fields (MF) in a turbulent conductive medium has important meaning for the solution of various astrophysical problems, as well as in plasma physics (Berezinskij et al. 1984; Krommes 1984). The process of the multiple scattering of particles in regularly transferred random MF inhomogeneities was first rigorously considered by Dolginov & Toptygin (1966). Following this, a detailed investigation of the processes of diffusion, convective transport and acceleration of charged particles in the cosmic plasma were conducted by many authors (e.g. Jokipii & Parker 1970; Kulsrud & Pearce 1969; Jokipii 1971; Galperin et al. 1971; Klimas & Sandri 1971; Skilling 1971, 1975; Jones et al. 1973; Volk 1975; Goldstein 1976; Webb & Gleeson 1979; Vainstein & Kichatinov 1981; Gurevich et al. 1983; Ptuskin 1984; Dorman & Shogenov 1985).

The concrete character of turbulence within media has essential meaning in describing the processes of cosmic ray (CR) particle interaction with stochastic MF (Smith et al. 1990). The kinetic coefficients, which describe the particle propagation, are obviously derived under the supposition that the CR particles propagate in the isotropic MF turbulence, as well as in the isotropic hydrodynamic (HD) velocity field (VF) (Toptygin 1973, 1985). The particle acceleration by random magnetohydrodynamic (MHD) waves, which are transversally or longitudinally polarized with respect to the direction of the large-scale MF, has been investigated by Tverskoy (1967a, b).

Past experimental data suggested that cosmic MF fluctuations are statistically anisotropic and display the property of reflective non-invariance (cf. Belcher & Davis 1971; Matthaeus and co-workers 1981, 1982, 1984, 1990). Along with this fact, the anisotropy of MHD turbulence has a tendency to change itself (Carbone & Veltri 1990). It is well-known through the theory of the turbulent dynamo (cf. Vainstein et al. 1980; Moffat 1978; Krause & Radler 1980) that the large-scale electric field (EF) is generated when gyrotropic turbulence is present in the medium. This EF essentially influences the process of the charged particle propagation. The process of CR particle transport in the statistically anisotropic reflective non-invariant MF has been examined by Kichatinov & Matyukhin (1981), Bieber et al. (1987), Dorman et al. (1988), Burger & Bieber (1990). These studies have mainly analyzed the pitch angle and/or spatial diffusion of particles. Kichatinov (1983) showed that the large-scale EF, which is generated in the non-mirrorsymmetric turbulence, creates the possibility of appearance of an effective mechanism of CR particle acceleration. This acceleration mechanism generates the observed power-law spectrum of CR in spatially homogeneous case. Because of having a sufficiently high effectiveness for energy transfer into particles, this mechanism (see also Kichatinov 1983a; Dorman et al. 1984) may compete with the classical Fermi acceleration mechanism (Fermi 1949; Tverskoy 1967a, b). Recently, Earl et al. (1988) have rederived the equation of transport for CR, including the energy changes of particles owing to CR viscosity. The rederivation of the transport equation was carried out by Webb (1985, 1989) for the regime of relativistic flows. Dung & Schlickeiser (1990) discussed the influence of the cross and magnetic helicities of the Alfvénic slab plasma turbulence on the energy changes of CR. They show that the transport coefficients, and thus the diffusion approximation of the kinetic equation, are very sensitive to the turbulence model chosen.

In this paper, we develop the rigorous theory of high-energy particle propagation in a randomly anisotropic reflective non-invariant MF in turbulent conductive media. In Sect. 2, we derive a kinetic equation which describes both the multiple scattering and the acceleration of particles, applying the functional method

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of averaging in the stochastic ensemble of random fields. The diffusion approximation is calculated in Sect. 3, in which are given expressions for the particle flux concentration, as well as for particle flux density in the momentum modulus space. Also in Sect. 3, the kinetic coefficients driving the transport processes of CR particles are calculated. Then, in Sects. 4 and 5, we distinguish between the propagation of particles in a strong regular MF (Sect. 4) and the propagation when the stochastic component of MF essentially exceeds the regular one (Sect. 5). Finally, the analysis of the effectiveness of particle acceleration by the large-scale EF is performed in Sect. 6, where it is shown that for a given value of the MF, the mechanism considered is the most effective for "magnetizing" particles into the MF, i.e. if the particle kinetic energy is below 1 GeV.

2. Kinetic equation

Let us consider a collisionless plasma consisting of charged particles moving in the magnetic field $\boldsymbol{H}(\boldsymbol{r}, t)$, which has a regular component $\boldsymbol{H}_0$ and a random one $\boldsymbol{H}_1(\boldsymbol{r}, t)$:

$$\boldsymbol{H}=\boldsymbol{H}_0+\boldsymbol{H}_1, \tag{1}$$

$$\langle \boldsymbol{H}\rangle=\boldsymbol{H}_0. \tag{2}$$

The angle brackets denote averaging over the statistical ensemble of fields. The random components vary irregularly in space and time, and the scales of their changes are shorter than a correlation length L_c or a correlation time τ_c of the random fields. We will suppose that the MF is "frozen-in" to the plasma, moving with HD velocity $\boldsymbol{u}(\boldsymbol{r}, t)$, which has also both regular $\boldsymbol{u}_0$ and random $\boldsymbol{u}_1(\boldsymbol{r}, t)$ components:

$$\boldsymbol{u}=\boldsymbol{u}_0+\boldsymbol{u}_1, \tag{3}$$

$$\langle \boldsymbol{u}\rangle=\boldsymbol{u}_0. \tag{4}$$

Because of the frozen-in MF, there exists in the plasma an induced EF

$$\boldsymbol{E}(\boldsymbol{r}, t)=[\boldsymbol{u}\boldsymbol{H}]/c, \tag{5}$$

which consists of a regular component

$$\boldsymbol{E}_0=-\{[\boldsymbol{u}_0\boldsymbol{H}_0]+\langle[\boldsymbol{u}_1\boldsymbol{H}_1]\rangle\}/c, \tag{6}$$

and a random component

$$\varepsilon=-\{[\boldsymbol{u}_0\boldsymbol{H}_1]+[\boldsymbol{u}_1\boldsymbol{H}_0]+[\boldsymbol{u}_1\boldsymbol{H}_1]-\langle[\boldsymbol{u}_1\boldsymbol{H}_1]\rangle\}/c. \tag{7}$$

Therefore, the evolution of the distribution function $f_{\boldsymbol{p}}(\boldsymbol{r}, t)$ of CR particles propagating in these fields will be determined by a collision-free kinetic equation

$$(\partial_t+\boldsymbol{\nabla}\boldsymbol{v})f_{\boldsymbol{p}}(\boldsymbol{r}, t)+\frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{F}(\boldsymbol{r}, t)f_{\boldsymbol{p}}(\boldsymbol{r}, t)=0, \tag{8}$$

where

$$\boldsymbol{F}=\boldsymbol{F}_0+\boldsymbol{F}_1=e\boldsymbol{E}+\frac{e}{c}[\boldsymbol{v}\boldsymbol{H}] \tag{9}$$

is the force that acts on a particle with charge e velocity $\boldsymbol{v}$ and momentum $\boldsymbol{p}$;

$$\boldsymbol{F}_0=e\boldsymbol{E}_0+\frac{e}{c}[\boldsymbol{v}\boldsymbol{H}_0] \tag{10}$$

is the regular component of $\boldsymbol{F}$, and

$$\boldsymbol{F}_1=e\varepsilon+\frac{e}{c}[\boldsymbol{v}\boldsymbol{H}_1], \quad \langle \boldsymbol{F}_1\rangle=0, \tag{11}$$

the random component.

The exact distribution function $f_{\boldsymbol{p}}(\boldsymbol{r}, t)$ varies quickly because of a fluctuating random field. Obviously, the most interesting function is the averaged distribution function $\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r}, t)=\langle f_{\boldsymbol{p}}(\boldsymbol{r}, t)\rangle$. Let us now derive the kinetic equation for the mean distribution function $\mathscr{F}_{\boldsymbol{p}}$, starting from the collision-free kinetic equation (8). By averaging Eq. (8) over the statistical ensemble of random fields, we obtain

$$(\partial_t+\boldsymbol{\nabla}\boldsymbol{v})\mathscr{F}_{\boldsymbol{p}}+\frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{F}_0\mathscr{F}_{\boldsymbol{p}}=\mathrm{Col}\,\mathscr{F}_{\boldsymbol{p}}, \tag{12}$$

where

$$\mathrm{Col}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r}, t)=-\frac{\partial}{\partial \boldsymbol{p}}\langle \boldsymbol{F}_1(\boldsymbol{r}, t)f_{\boldsymbol{p}}(\boldsymbol{r}, t)\rangle \tag{13}$$

is the collision integral of the kinetic equation (12).

For computing the correlator in Eq. (13), we use the functional method (see Appendix). If $\boldsymbol{F}_1(\boldsymbol{r}, t)$ is a Gaussian random field with zero mean value, then we easily arrive at

$$\langle F_{1\alpha}(\boldsymbol{r}, t)f_{\boldsymbol{p}}(\boldsymbol{r}, t)\rangle=\int_0^t \mathrm{d}t_1\int \mathrm{d}^3r_1\mathscr{D}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)\times\langle\Psi_{\boldsymbol{p}\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)\rangle. \tag{14}$$

The repeated Greek indices here and below indicate summation.

$$\mathscr{D}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)=\langle F_{1\alpha}(\boldsymbol{r}, t)F_{1\lambda}(\boldsymbol{r}_1, t_1)\rangle \tag{15}$$

is the correlation tensor of random forces acting on the particles and

$$\Psi_{\boldsymbol{p}\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)=\delta f_{\boldsymbol{p}}(\boldsymbol{r}, t)/\delta F_{1\lambda}(\boldsymbol{r}_1, t_1) \tag{16}$$

is the functional derivative of the distribution function over the random field. We compute this derivative using Eq. (8):

$$\left(\partial_t+\boldsymbol{v}\nabla+\frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{F}\right)\Psi_{\boldsymbol{p}\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)=-\delta(\boldsymbol{r}-\boldsymbol{r}_1)\,\delta(t-t_1)\frac{\partial}{\partial p_\lambda}f_{\boldsymbol{p}}(\boldsymbol{r}, t). \tag{17}$$

The general solution of Eq. (17) is

$$\Psi_{\boldsymbol{p}\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)=-\int \mathrm{d}^3p_1\,\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)\frac{\partial}{\partial p_{1\lambda}}f_{\boldsymbol{p}_1}(\boldsymbol{r}_1, t_1), \tag{18}$$

where $\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)$ is Green function for Eq. (8). Substituting Eq. (18) into Eqs. (13) and (14) results in (cf. Dorman et al. 1977)

$$\mathrm{Col}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r}, t)=\frac{\partial}{\partial p_\alpha}\int_0^t \mathrm{d}t_1\int \mathrm{d}^3r_1\int \mathrm{d}^3p_1\,\mathscr{D}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)\times\langle\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)\frac{\partial}{\partial p_{1\lambda}}f_{\boldsymbol{p}_1}(\boldsymbol{r}_1, t_1)\rangle. \tag{19}$$

Equation (12), together with the collision integral (19), is not closed with respect to the mean distribution function $\mathscr{F}_{\boldsymbol{p}}$. If the random field should be δ-correlated in time, then the correlation tensor

$$\mathscr{D}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)=\mathscr{D}_{\alpha\lambda}(\boldsymbol{r}, \boldsymbol{r}_1, t)\,\delta(t-t_1)$$

and the integral (19) will take the standard Fokker–Planck form:

$$\mathrm{Col}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r},t)=\frac{\partial}{\partial p_\alpha}\mathscr{D}_{\alpha\lambda}(\boldsymbol{r},t)\frac{\partial}{\partial p_\lambda}\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r},t). \tag{20}$$

Note that the kinetic equation is exact in this case.

If, however, the correlation time of the random process is non-zero, then a closed-form equation can still be obtained on the basis of perturbation theory. For this purpose, we will follow the method by Dorman et al. (1977) and Dorman & Katz (1977). We will consider the situation involving a sufficiently high particle energy. If the correlation length L_c of the random field F_1 is about 10^{10-11} cm and the MF $H_1 \approx 5\,10^{-5}$ G in interplanetary space, the inequality $L_c/R \ll 1$ holds ($R=cp/e\sqrt{\langle H_1^2\rangle}$ is the mean gyroradius in the random field H_1). This condition means that the particle is scattered through a small angle $\varphi \sim L_c/R$ by each MF inhomogeneity.

Let G be Green function for the operator on the left-hand side of Eq. (12):

$$\left(\partial_t+\nabla \boldsymbol{v}+\frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{F}_0\right)G_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)=\delta(\boldsymbol{r}-\boldsymbol{r}_1)\delta(\boldsymbol{p}-\boldsymbol{p}_1)\delta(t-t_1). \tag{21}$$

Then $\delta\mathscr{G}\equiv\mathscr{G}-G$ is the part of the function $\mathscr{G}$ that includes the fluctuating changes caused by the stochastic field F_1, i.e. $\delta\mathscr{G}$ describes a variable δp of the particle momentum at the correlation length L_c (during the time $\tau \lesssim L_c/v$). Similarly, $\delta f\equiv f-\mathscr{F}$ describes rapid changes of $f_{\boldsymbol{p}}$ during $\tau\lesssim L_c/v$. As is evident, $\delta f_{\boldsymbol{p}}=\delta\mathscr{G}=0$ if $\boldsymbol{F}_1\equiv 0$, and the function $\mathscr{F}_{\boldsymbol{p}}$ would then describe an ensemble of free moving particles in the regular field $\boldsymbol{F}_0$. Substituting the expansions of $\mathscr{G}$ and $f_{\boldsymbol{p}}$ into Eq. (19), we arrive at

$$\begin{aligned}\mathrm{Col}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r},t)=\frac{\partial}{\partial p_\alpha}\int_0^t \mathrm{d}t_1\int \mathrm{d}^3r_1\int\mathrm{d}^3p_1\Big\{&G_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)\\ &\times\mathscr{D}_{\alpha\lambda}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)\frac{\partial}{\partial p_{1\lambda}}\mathscr{F}_{\boldsymbol{p}_1}(\boldsymbol{r}_1,t_1)+\mathscr{D}_{\alpha\lambda}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)\\ &\times\left\langle\delta\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)\frac{\partial}{\partial p_{1\lambda}}\delta f_{\boldsymbol{p}_1}(\boldsymbol{r}_1,t_1)\right\rangle\Big\}.\end{aligned} \tag{22}$$

The equations for the Green functions $\mathscr{G}$ and G yield

$$\begin{aligned}&\left(\partial_t+\nabla\boldsymbol{v}+\frac{\partial}{\partial\boldsymbol{p}}\boldsymbol{F}_0\right)\delta\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)\\ &\quad=-\frac{\partial}{\partial\boldsymbol{p}}\boldsymbol{F}_1(\boldsymbol{r},t)G_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)-\frac{\partial}{\partial\boldsymbol{p}}\boldsymbol{F}_1(\boldsymbol{r},t)\,\delta\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1).\end{aligned} \tag{23}$$

Taking into account the approximation following Eq. (21), we neglect the second term on the right-hand side of Eq. (23) in our perturbation theory, because the relative change of the particle momentum over one scale length (on the order of the correlation length L_c) is $\delta p/p\sim L_c/R\ll 1$.

The inequality $\sqrt{\langle H_1^2\rangle}\ll cp/eL_c$ must hold. Consequently.

$$\begin{aligned}\delta\mathscr{G}_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{r},t;\boldsymbol{r}_1,t_1)=-\int_0^t\mathrm{d}t_2\int\mathrm{d}^3r_2\int\mathrm{d}^3p_2\,&\boldsymbol{F}_1(\boldsymbol{r}_2,t_2)\\ &\times G_{\boldsymbol{p}\boldsymbol{p}_2}(\boldsymbol{r},t;\boldsymbol{r}_2,t_2)\frac{\partial}{\partial\boldsymbol{p}_2}G_{\boldsymbol{p}_2\boldsymbol{p}_1}(\boldsymbol{r}_2,t_2;\boldsymbol{r}_1,t_1).\end{aligned}$$

The second term of Eq. (22) is smaller than the first in the case of a weak random field. Other approaches (cf. Dolginov & Toptygin 1966; Dorman & Katz 1977) lead to the same result in this approximation.

Let us now choose the convenient Green function G. If the gyroradius $R_0=cp/eH_0$ in the regular MF also exceeds the correlation length L_c, $L_c\ll R_0$, then the influence of H_0 on the particle movement can also be neglected. It can readily be seen that the influence of the EF may also be neglected. In fact, the ratio of electric force to magnetic force is $F_{el}/F_{mag}\approx u/c\ll 1$. Even if hydromagnetic waves exist in the solar wind plasma, the EF is $v_A H/c\lesssim uH/c$ (v_A is the Alfvén velocity). Therefore, the Green function

$$G_{\boldsymbol{p}\boldsymbol{p}_1}(\boldsymbol{\rho},\tau)=\theta(\tau)\delta(\boldsymbol{\rho}-\boldsymbol{v}\tau)\delta(\boldsymbol{p}-\boldsymbol{p}_1) \tag{24}$$

($\tau=t-t_1$, $\boldsymbol{\rho}=\boldsymbol{r}-\boldsymbol{r}_1$, and $\theta(\tau)$ is the Heaviside function) describes the free movement of particles inside the range of field correlation. On the spatial and time scales considered, the mean function $\mathscr{F}_{\boldsymbol{p}}$ depends weakly on $\boldsymbol{r}$ and t. Therefore, Eq. (23) may now be written as

$$\left(\partial_t+\nabla\boldsymbol{v}+\frac{\partial}{\partial\boldsymbol{p}}\boldsymbol{F}_0\right)\mathscr{F}_{\boldsymbol{p}}=\frac{\partial}{\partial p_\alpha}\overline{\mathscr{D}_{\alpha\lambda}(\boldsymbol{p})}\frac{\partial}{\partial p_\lambda}\mathscr{F}_{\boldsymbol{p}}, \tag{25}$$

where

$$\overline{\mathscr{D}_{\alpha\lambda}(\boldsymbol{p})}=\int_0^\infty\mathrm{d}\tau\,\mathscr{D}_{\alpha\lambda}(\boldsymbol{v}\tau;\tau) \tag{26}$$

(cf. Toptygin 1985; Dorman & Katz 1977) is the diffusion coefficient of particles in momentum space. We have supposed in Eqs. (25) and (26) that the random field is homogeneous in space and stationary in time.

3. Diffusion approximation

Let us rewrite the kinetic equation (25) in the form of a particle conservation law in phase space,

$$\partial_t\mathscr{F}_{\boldsymbol{p}}+\nabla\boldsymbol{j}+\frac{\partial}{\partial\boldsymbol{p}}\boldsymbol{j}_p=0, \tag{27}$$

where

$$j_\alpha=v_\alpha\mathscr{F}_{\boldsymbol{p}}, \tag{28}$$

$$j_{p\alpha}=\left(F_{0\alpha}-\overline{\mathscr{D}_{\alpha\lambda}}\frac{\partial}{\partial p_\lambda}\right)\mathscr{F}_{\boldsymbol{p}}. \tag{29}$$

By integrating Eq. (27) over the angle variables in momentum space (Gleeson & Webb 1978; Dorman et al. 1978, 1980; Toptygin, 1985), we obtain the transport equation for particles at a given value of the momentum:

$$\partial_t N+\nabla\boldsymbol{J}+\frac{1}{p^2}\frac{\partial}{\partial p}p^2J_p=0. \tag{30}$$

Here

$$N(\boldsymbol{r},p,t)=\int\mathrm{d}\Omega_{\boldsymbol{p}}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r},t) \tag{31}$$

is the number density at a given value of the momentum modulus,

$$\boldsymbol{J}(\boldsymbol{r},p,t)=\int\mathrm{d}\Omega_{\boldsymbol{p}}\,\boldsymbol{v}\,\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r},t) \tag{32}$$

is the vector of particle flux density at a given value of the momentum modulus, and

$$J_p = \int \mathrm{d}\Omega_p \left(\boldsymbol{g} F_0 - g_\alpha \overline{\mathscr{D}_{\alpha\lambda}} \frac{\partial}{\partial p_\lambda} \right) \mathscr{F}_{\boldsymbol{p}} \tag{33}$$

is the flux density in the space of momentum modulus; $\mathrm{d}\Omega_{\boldsymbol{p}}$ is an infinitesimal solid angle in momentum space; $\boldsymbol{g} = \boldsymbol{p}/p$.

The first term in Eq. (33) is proportional to the scalar product $\boldsymbol{E}_0 \cdot \boldsymbol{J}$ [cf. Eq. (10)], and expresses a particle energy change in the regular EF $\boldsymbol{E}_0$. The second term describes a fluctuating change of particle energy caused by the stochastic EF $\varepsilon(\boldsymbol{r}, t)$. The expression (33) follows directly from the definition of particle flux in the space of momentum modulus, i.e. the number of particles in the unit volume passing over a given momentum modulus during the unit time:

$$J_p = \int \mathrm{d}\Omega_{\boldsymbol{p}} \left\langle \frac{\mathrm{d}p}{\mathrm{d}t} f_{\boldsymbol{p}}(\boldsymbol{r}, t) \right\rangle. \tag{34}$$

Using the equation of motion

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \boldsymbol{F} \tag{35}$$

we have

$$\frac{\mathrm{d}p}{\mathrm{d}t} = e\boldsymbol{g}\boldsymbol{E} = e\boldsymbol{g}(\boldsymbol{E}_0 + \varepsilon). \tag{36}$$

Therefore,

$$J_p = \frac{e}{v} \boldsymbol{E}_0 \boldsymbol{J} + e \int \mathrm{d}\Omega_{\boldsymbol{p}}\, \boldsymbol{g} \langle \varepsilon(\boldsymbol{r}, t) f_{\boldsymbol{p}}(\boldsymbol{r}, t) \rangle. \tag{37}$$

Using Eqs. (14) and (15) from Eq. (37), we arrive at Eq. (33). Because we want to calculate the fluxes, we need first to define the shape of the tensor $\mathscr{D}_{\alpha\lambda}$. This tensor given by Eq. (15) consists of the sum of the MF correlation tensor

$$\mathscr{B}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \langle H_{1\alpha}(\boldsymbol{r}, t) H_{1\lambda}(\boldsymbol{r}_1, t_1) \rangle, \tag{38}$$

the tensor of HD velocity fields

$$\mathscr{Q}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \langle u_{1\alpha}(\boldsymbol{r}, t) u_{1\lambda}(\boldsymbol{r}_1, t_1) \rangle, \tag{39}$$

the cross-correlation tensor

$$\mathscr{S}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \langle H_{1\alpha}(\boldsymbol{r}, t) u_{1\lambda}(\boldsymbol{r}_1, t_1) \rangle, \tag{40}$$

and the correlation tensors of the third and fourth rank:

$$\Gamma_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \frac{1}{C} \varepsilon_{\lambda\mu\nu} \langle H_{1\alpha}(\boldsymbol{r}, t) H_{1\mu}(\boldsymbol{r}_1, t_1) u_{1\nu}(\boldsymbol{r}_1, t_1) \rangle, \tag{41}$$

$$\Pi_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \frac{1}{C} \varepsilon_{\mu\lambda\nu} \langle u_{1\alpha}(\boldsymbol{v}, t) u_{1\mu}(\boldsymbol{r}_1, t_1) H_{1\nu}(\boldsymbol{r}_1, t_1) \rangle, \tag{42}$$

$$\begin{aligned} \mathscr{T}_{\alpha\lambda}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \frac{1}{C^2} \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} \{ & \langle u_{1\beta}(\boldsymbol{r}, t) H_{1\gamma}(\boldsymbol{r}, t) u_{1\mu}(\boldsymbol{r}_1, t_1) \\ & \times H_{1\nu}(\boldsymbol{r}_1, t_1) \rangle - \mathscr{S}_{\gamma\beta}(\boldsymbol{r}, t; \boldsymbol{r}, t) \mathscr{S}_{\nu\mu}(\boldsymbol{r}_1, t_1; \boldsymbol{r}_1, t_1) \}, \end{aligned} \tag{43}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the unit antisymmetric Levi-Civita tensor.

We now perform the diffusion approximation in Eq. (27), assuming that the particle distribution in the momentum space is close to isotropic. For that purpose, the distribution function $\mathscr{F}_{\boldsymbol{p}}$ should be expanded into a series of spherical functions (cf. Dolginov & Toptygin 1966; Earl 1974a; Dorman & Katz 1977), and we neglect all but the first two terms of the expansion:

$$\mathscr{F}_{\boldsymbol{p}}(\boldsymbol{r}, t) = (1/4\pi) \left\{ N(\boldsymbol{r}, p, t) + \frac{3}{v} \boldsymbol{g} \boldsymbol{J}(\boldsymbol{r}, p, t) \right\}, \tag{44}$$

where N and J are defined by expressions (31) and (32), respectively.

After multiplying the kinetic equation by the vector $\boldsymbol{g}$ and integrating the resulting equation over the angle variables, we obtain the equation for the vector of particle density flux in the form

$$\chi_{\alpha\lambda} I_\lambda = -\frac{1}{3} v R \nabla_\alpha N + \{ \langle [\boldsymbol{u}_1 \boldsymbol{H}_1]_\alpha \rangle / \sqrt{\langle H_1^2 \rangle} + b_\alpha \} \frac{p}{3} \frac{\partial N}{\partial p}, \tag{45}$$

$$\boldsymbol{I} = \boldsymbol{J} + \boldsymbol{u}_0 \frac{p}{3} \frac{\partial N}{\partial p}, \tag{46}$$

$$\chi_{\alpha\lambda} = \frac{3ev}{4\pi c p \sqrt{\langle H_1^2 \rangle}} \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} \int \mathrm{d}\Omega_{\boldsymbol{p}}\, g_\beta g_\mu \overline{\mathscr{B}_{\gamma\nu}(\boldsymbol{v})} - \frac{R}{R_0} \varepsilon_{\alpha\lambda\gamma} h_\gamma, \tag{47}$$

$$b_\alpha = \frac{3ev}{4\pi c p} \varepsilon_{\alpha\gamma\beta} \int \mathrm{d}\Omega_{\boldsymbol{p}}\, g_\beta g_\lambda \left\{ \frac{c}{\sqrt{\langle H_1^2 \rangle}} \overline{\Gamma_{\gamma\lambda}(\boldsymbol{v})} - \frac{R}{R_0} \varepsilon_{\lambda\mu\nu} h_\nu \overline{\mathscr{S}_{\gamma\mu}(\boldsymbol{v})} \right\},$$

$$\boldsymbol{h} = \boldsymbol{H}_0 / H_0. \tag{48}$$

The line above a tensor has the same meaning as in Eq. (26). Substituting the expansion (44) into Eq. (33), we arrive at the expression

$$\begin{aligned} J_p = {} & \frac{p}{3} (\boldsymbol{u}_0 \nabla) N - \frac{p}{Rv} \{ \langle [\boldsymbol{u}_1 \boldsymbol{H}_1] \rangle + \boldsymbol{b} \} \boldsymbol{I} \\ & - \frac{e^2}{4\pi} \frac{\partial N}{\partial p} \int \mathrm{d}\Omega_{\boldsymbol{p}}\, g_\alpha\ g_\lambda \left\{ \frac{H_0^2}{c} \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} h_\gamma h_\nu \overline{\mathscr{Q}_{\beta\mu}(\boldsymbol{v})} \right. \\ & \left. + \overline{\mathscr{T}_{\alpha\lambda}(\boldsymbol{v})} - \frac{2H_0}{c} \varepsilon_{\lambda\mu\nu} h_\nu \overline{\Pi_{\mu\alpha}(\boldsymbol{v})} \right\}. \end{aligned} \tag{49}$$

For the determination of the particle density flux, we need to find the inverse $\chi_{\zeta\alpha}^{-1}$ of the tensor $\chi_{\alpha\lambda}$ in (47), which is determined by the form of the correlation tensor of the random MF. In accordance with the experimental analysis by Belcher & Davis (1971), Matthaeus et al. (1981, 1980) and other authors, MF fluctuations are axially symmetric with regard to the direction of $\boldsymbol{h}$. The most general form of the second-rank correlation tensor, which is characterized by one preferred direction $\boldsymbol{h}$, can be defined by eight correlation functions (cf. Matthaeus & Goldstein 1982).

$$\begin{aligned} \mathscr{B}_{\alpha\lambda}(\boldsymbol{r}, t) = \frac{1}{3} \langle H_1^2 \rangle \{ & \Psi(\boldsymbol{r}, t) \delta_{\alpha\lambda} - \Psi_1(\boldsymbol{r}, t) n_\alpha n_\lambda \\ & + \Psi_2(\boldsymbol{r}, t) h_\alpha h_\lambda + \Psi_3(\boldsymbol{r}, t)(n_\alpha h_\lambda + n_\lambda h_\alpha) + \Psi_4(\boldsymbol{r}, t) \\ & \times ([\boldsymbol{nh}]_\alpha n_\lambda + [\boldsymbol{nh}]_\lambda n_\alpha) + \Psi_5(\boldsymbol{r}, t)([\boldsymbol{nh}]_\alpha h_\lambda \\ & + [\boldsymbol{nh}]_\lambda h_\alpha) + \Phi(\boldsymbol{r}, t) \varepsilon_{\alpha\lambda\gamma} n_\gamma + \Phi_1(\boldsymbol{r}, t) \varepsilon_{\alpha\lambda\gamma} h_\gamma \}, \end{aligned}$$

$$\boldsymbol{n} = \boldsymbol{r}/r. \tag{50}$$

Here Ψ, Ψ_1, Ψ_2, Ψ_4 and Φ are functions of even powers of $\boldsymbol{r}$, and $(\boldsymbol{nh})$. Ψ_3, Ψ_5 and Φ_1 are odd functions of $(\boldsymbol{nh})$ and contain the even powers of $\boldsymbol{r}$. These correlation functions are not independent, due to the solenoidal property of MF.

Using the general form (50) of MF, the correlation tensor will be

$$\chi_{\alpha\lambda} = q_1 \delta_{\alpha\lambda} + q_2 h_\alpha h_\lambda - \frac{R}{R_0} \varepsilon_{\alpha\lambda\gamma} h_\gamma, \tag{51}$$

where

$$q_1=\frac{1}{2R}\int_0^\infty \mathrm{d}x\int_0^1 \mathrm{d}y\{(1+y^2)\Psi(x,y)+(1-y^2)\Psi_2(x,y)\},$$

$$q_2=\frac{1}{2R}\int_0^\infty \mathrm{d}x\int_0^1 \mathrm{d}y\{(1-3y^2)\Psi(x,y)-(1-y^2)\Psi_2(x,y)\},$$

$$x=v\tau,\quad y=\boldsymbol{gh}. \tag{52}$$

The inverse tensor to $\chi_{\alpha\lambda}$ is

$$\chi_{\xi\alpha}^{-1}=\xi_1\delta_{\xi\alpha}+\xi_2 h_\xi h_\alpha+\xi_3\varepsilon_{\xi\alpha\gamma}h_\gamma, \tag{53}$$

$$\xi_1=q_1\left(q_1^2+\frac{R^2}{R_0^2}\right)^{-1};\qquad \xi_2=(q_1+q_2)^{-1}-\xi_1;$$

$$\xi_3=\frac{R}{R_0}\left(q_1^2+\frac{R^2}{R_0^2}\right)^{-1}. \tag{54}$$

For the final definition of the particle flux density vector, it is necessary to determine the quantity $\langle[\boldsymbol{u}_1\boldsymbol{H}_1]\rangle$ in Eq. (45). The presence of the random fields, together with the random mixing of media, is the primary feature characterizing the cosmic plasma. The stochastic MF requires special attention, because the hydromagnetic turbulence is reflective non-invariant. The appearance of both the large-scale MF and EF in this turbulence follows from turbulent dynamo theory (Vainstein et al. 1980; Moffat 1978; Krause & Radler 1980). The only vector available to construct the driven large-scale EF is $\boldsymbol{H}_0$, so a possible linear representation is

$$\langle[\boldsymbol{u}_1\boldsymbol{H}_1]\rangle=\alpha\boldsymbol{H}_0-\beta\,\mathrm{rot}\,\boldsymbol{H}_0, \tag{55}$$

where α is determined by the antisymmetric part of the tensor (50).

For a preliminary estimate of the value of both α and β in the high-conductivity limit, we have

$$\alpha=-\tfrac{1}{3}\tau_c h_u,\qquad \beta=\tfrac{1}{3}\tau_c\langle u_1^2\rangle,$$

where

$$h_u=\langle\boldsymbol{u}_1\cdot\mathrm{rot}\,\boldsymbol{u}_1\rangle$$

is the well-known helicity density. If we denote

$$R_H\approx H_0/|\mathrm{rot}\,H_0|,\qquad r_\mu\approx\langle u_1^2\rangle/|h_u|,$$

then $\alpha R_H/\beta\approx R_H/r_u\gg 1$ in interplanetary space; i.e. we neglect spatial inhomogeneity of the field H_0. This means the neglect of eddy viscosity. In fact, $R_H\lesssim 1$ AU, $\beta\lesssim 10^{12}$ cm^2 s^{-1}; consequently, the value $\alpha>0.1$ cm s^{-1} is sufficient for our approximation. (Even if $\beta\,\mathrm{rot}\,H_0\neq 0$, the vector rot $\boldsymbol{H}_0$ is radial, and it can weakly modify the direction of quantity $\boldsymbol{U}$, but the primary effect remains the same.)

Consequently, the large-scale EF is

$$\boldsymbol{E}_0=-\frac{\alpha}{c}\boldsymbol{H}_0. \tag{56}$$

The expression for the particle flux density vector follows from Eqs. (45)–(48) and (53)–(55):

$$J_\alpha=-\kappa_{\alpha\lambda}\nabla_\lambda N-(u_{0\alpha}-U_\alpha-W_\alpha)\frac{p}{3}\frac{\partial N}{\partial p}, \tag{57}$$

where

$$\kappa_{\alpha\lambda}=\tfrac{1}{3}v\Lambda_{\alpha\lambda}, \tag{58}$$

$$\Lambda_{\alpha\lambda}=\Lambda_{||}h_\alpha h_\lambda+\Lambda_1\Delta_{\alpha\lambda}(\boldsymbol{h})+\Lambda_2\varepsilon_{\alpha\lambda\gamma}h_\gamma),\quad \Delta_{\alpha\lambda}(\boldsymbol{h})=\delta_{\alpha\lambda}-h_\alpha h_\lambda, \tag{59}$$

$$\Lambda_1=\frac{R}{2}\Delta_{\alpha\lambda}(\boldsymbol{h})\chi_{\alpha\lambda}^{-1}=R\xi_1,\quad \Lambda_2=R\varepsilon_{\alpha\lambda\gamma}h_\gamma\chi_{\alpha\lambda}^{-1}=R\xi_2, \tag{60}$$

$$\Lambda_{||}=Rh_\alpha h_\lambda\chi_{\alpha\lambda}^{-1}=R(\xi_1+\xi_2)$$
$$=R^2\left[\int_0^\infty \mathrm{d}x\int_0^1 \mathrm{d}y(1-y^2)\Psi(x,y)\right]^{-1}. \tag{61}$$

The quantity $\Lambda_{||}$ denotes the particle transport path along the regular MF direction $\boldsymbol{h}$. One can find the concrete expressions for the diffusion tensor components calculated in various approximations in many papers: for example, Jokipii & Coleman (1968); Klimas & Sandri (1973); Earl (1974a); Dorman & Katz (1977); Kichatinov & Matyukhin (1981); Toptygin (1985); Bieber et al. (1987).

The quantities $\boldsymbol{U}$ and $\boldsymbol{W}$ in (57) possess the dimension of velocity and are directed along $\boldsymbol{h}$. Using Eqs. (56) and (61) we find:

$$\boldsymbol{U}=\alpha\frac{\Lambda_{||}}{R}\boldsymbol{h}, \tag{62}$$

$$\boldsymbol{W}=W\boldsymbol{h}, \tag{63}$$

$$W=\frac{3}{4\pi}\frac{ev\Lambda_{||}}{pc}\int_0^\infty \mathrm{d}\tau\int \mathrm{d}\Omega_{\boldsymbol{p}}\left\{R_0^{-1}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\lambda\mu\nu}h_\alpha h_\nu\mathscr{S}_{\gamma\mu}(\boldsymbol{v}\tau)\right.$$
$$\left.-\frac{e}{p}\varepsilon_{\alpha\beta\gamma}h_\alpha\Gamma_{\gamma\nu}(\boldsymbol{v}\tau)\right\}g_\beta g_\nu. \tag{64}$$

The particle flux in the momentum modulus space is

$$J_p=\frac{p}{3}(\boldsymbol{u}_0+\boldsymbol{U}-\boldsymbol{W})\nabla N-D_p\frac{\partial}{\partial p}N, \tag{65}$$

where

$$D_p=\frac{p^2}{3v\Lambda_{||}}(U^2-W^2)+D(p), \tag{66}$$

$$D(p)=\frac{1}{4\pi}\int_0^\infty \mathrm{d}\tau\int \mathrm{d}\Omega_{\boldsymbol{p}}g_\alpha g_\lambda\left\{\frac{cp^2}{R_0^2}\varepsilon_{\alpha\beta\gamma}h_\gamma\varepsilon_{\lambda\mu\nu}h_\nu\mathscr{Q}_{\beta\mu}(\boldsymbol{v}\tau)\right.$$
$$\left.-\frac{2ep}{R_0}\varepsilon_{\alpha\beta\gamma}h_\gamma\Pi_{\beta\lambda}(\boldsymbol{v}\tau)+e^2\mathscr{T}_{\alpha\lambda}(\boldsymbol{v}\tau)\right\}. \tag{67}$$

The random field is characterized by one preferred direction $\boldsymbol{h}$. Therefore, the correlation tensors (39)–(43) acquire tensor structure, which is the same as the structure of the MF correlation tensor (50). Then, the quantity W in (64) is determined by

$$W=\sqrt{\langle u_1^2\rangle}(\beta+\beta_1 H_0/\sqrt{\langle H_1^2\rangle}), \tag{68}$$

$$\left.\begin{matrix}\beta\\ \beta_1\end{matrix}\right\}=\beta_0\int_0^\infty \mathrm{d}x\int_{-1}^1 \mathrm{d}y(1-y^2)$$
$$\times\begin{cases}\Psi_4^\Gamma(x,y)+y\Psi_5^\Gamma(x,y)+\Phi_1^\Gamma(x,y),\\ \Psi^{\mathscr{S}}(x,y),\end{cases} \tag{69}$$

$$\beta_0=\left[\int_0^\infty \mathrm{d}x\int_{-1}^1 \mathrm{d}y(1-y^2)\Psi(x,y)\right]^{-1}. \tag{70}$$

The upper index denotes the tensor to which the considered correlation function belongs.

The coefficient $D(p)$ in Eq. (67) will be

$$D(p)=\frac{p^2}{3v\Lambda_{\|}}\left(\alpha_1+\frac{H_0}{\sqrt{\langle H_1^2\rangle}}\alpha_2+\frac{H_0^2}{\langle H_1^2\rangle}\alpha_3\right)\langle u_1^2\rangle, \tag{71}$$

where

$$\left.\begin{matrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{matrix}\right\}=\beta_0\int_0^\infty \mathrm{d}x\int_{-1}^{1}\mathrm{d}y$$

$$\times\left\{\begin{matrix}\Psi^{\mathscr{T}}(x,y)-\Psi_1^{\mathscr{T}}(x,y)+y^2\Psi_2^{\mathscr{T}}(x,y)+2y\Psi_3^{\mathscr{T}}(x,y),\\ [\Psi_4^{\Pi}(x,y)+y\Psi_5^{\Pi}(x,y)+\Phi_1^{\Pi}(x,y)](1-y^2),\\ (1-y^2)\Psi^{\mathscr{Q}}(x,y).\end{matrix}\right. \tag{72}$$

The effective velocity $\boldsymbol{U}$ depends only on the coefficient α characterizing the large-scale EF $\boldsymbol{E}_0$, which changes the particle energy. The coefficient α is usually written in terms of the antisymmetric part of the velocity field tensor $\mathscr{Q}_{\alpha\lambda}$ (cf. Krause & Radler 1980). For the investigation of particle acceleration effects, it is convenient to write the coefficient α using the correlator of the random MF, usually observed by experiment. The relation between α and the spectral characteristics of MF is conveniently established in the frame of the linearized MHD equations. Then

$$\alpha=\frac{\mathrm{i}}{32\pi^4\rho}\varepsilon_{\alpha\beta\gamma}h_\alpha\int_0^\infty \mathrm{d}\tau\int \mathrm{d}^3k(\boldsymbol{kh})\mathscr{B}_{\beta\gamma}(\boldsymbol{k},\tau), \tag{73}$$

where ρ is the plasma density, $\boldsymbol{k}$ the wave vector, and $\mathscr{B}_{\beta\gamma}(\boldsymbol{k},\tau)$ the MF spectral tensor. The MF correlation tensor is defined by Eq. (50); thus, after Matthaeus & Smith (1981), the spectral tensor is

$$\mathscr{B}_{\alpha\lambda}(\boldsymbol{k},\tau)=\frac{1}{3}\langle H_1^2\rangle\Big\{B(\boldsymbol{k},\tau)\Delta_{\alpha\lambda}(\boldsymbol{k})+B_1(\boldsymbol{k},\tau)(h_\alpha h_\lambda$$
$$+x^2\frac{k_\alpha k_\lambda}{k^2}-x\sigma_{\alpha\mu\lambda\nu}h_\mu k_\nu/k)+B_2(\boldsymbol{k},\tau)\sigma_{\alpha\mu\lambda\nu}h_\mu\left[\frac{\boldsymbol{k}}{k}\boldsymbol{h}\right]_\nu$$
$$+xB_2(\boldsymbol{k},\tau)\sigma_{\alpha\mu\lambda\nu}k_\mu[\boldsymbol{kh}]_\nu/k^2+\mathrm{i}B_3(\boldsymbol{k},\tau)\varepsilon_{\alpha\lambda\gamma}k_\gamma/k\Big\}, \tag{74}$$

where

$$x=\boldsymbol{kh}/k,\quad \Delta_{\alpha\lambda}(\boldsymbol{k})=\delta_{\alpha\lambda}-\frac{k_\alpha k_\lambda}{k^2},\ \sigma_{\alpha\mu\lambda\nu}=\delta_{\alpha\mu}\delta_{\lambda\nu}+\delta_{\alpha\nu}\delta_{\lambda\mu}.$$

The functions B, B_1 and B_3 are even under $\boldsymbol{k}\to-\boldsymbol{k}$. Only B_2 is odd. Substituting (74) into Eq. (73), we find that the non-zero contribution is given by only antisymmetric part of $\mathscr{B}_{\beta\gamma}$ (because of the existence of $\varepsilon_{\alpha\beta\gamma}$ in Eq. (73)). Therefore,

$$\alpha=-\frac{\langle H_1^2\rangle}{48\pi^4\rho}\int_0^\infty \mathrm{d}\tau\int \mathrm{d}^3k\left(\frac{1}{k}\right)(\boldsymbol{hk})^2B_3(\boldsymbol{k},\tau). \tag{75}$$

Note that in the case of statistical isotropic fluctuations,

$$\alpha=-\frac{\langle H_1^2\rangle}{36\pi^3\rho}\int_0^\infty \mathrm{d}\tau\int \mathrm{d}k\,k^3B_3(k,\tau), \tag{76}$$

because of $\int \mathrm{d}\Omega_{\boldsymbol{k}}(\boldsymbol{hk})^2=(4\pi/3)k^2$ and $B_3(\boldsymbol{k})=B_3(k)$. The coefficient α can be expressed in terms of the magnetic helicity density $H_\mathrm{A}=\langle\boldsymbol{H}_1\cdot\boldsymbol{A}_1\rangle$ ($\boldsymbol{H}_1=\mathrm{rot}\,\boldsymbol{A}_1$, $\boldsymbol{A}_1$ – the vector potential), which is measurable experimentally (Matthaeus & Goldstein 1982). If L_c and τ_c are the length and time of MF correlation, respectively, then (Vainstein 1980)

$$\alpha\approx\frac{H_\mathrm{A}\tau_\mathrm{c}}{12\pi\rho L_\mathrm{c}^2}. \tag{77}$$

An estimation of the dimensionless quantity $\delta=\alpha/\sqrt{\langle u_1^2\rangle}$ is

$$\delta=\frac{1}{3}S\frac{\mathscr{W}_\mathrm{M}}{\mathscr{W}_\mathrm{k}}\frac{H_\mathrm{A}}{L_\mathrm{c}\langle H_1^2\rangle}. \tag{78}$$

Here $S=\tau_\mathrm{c}\sqrt{\langle u_1^2\rangle}/L_\mathrm{c}$ is the Strouhal number; $\mathscr{W}_\mathrm{M}$ and $\mathscr{W}_\mathrm{k}$ are the magnetic and kinetic fluctuating energies, respectively.

Equation (78) creates the possibility of estimating the quantity δ from experimental data. Measurements of the magnetic helicity by Matthaeus & Goldstein (1982), from Voyager-1 and 2 provide us with the following values for δ: 0.085 S, 0.272 S and 0.0013 S at heliocentric distances 1.0, 2.8 and 5 AU, respectively. Assuming that the value of S is of order unity in the solar wind plasma, we arrive at the conclusion that $\alpha\approx(10^{-1}\text{–}10^{-2})\sqrt{\langle u_1^2\rangle}$ at moderate heliocentric distances (1 AU, 2.8 AU). Note that the measurements at a distance of about 2.8 AU, give a value of α three times larger than at 1 AU. This is probably because the measurements at 2.8 AU were made in the interaction region of plasma streams, which have different velocities. In the region of the streaming interaction, both the relative level of MF disturbance and magnetic helicity density are large, leading to a relatively large value of the large-scale EF $\boldsymbol{E}_0$. The observations at 5 AU show a relatively low level of MF fluctuations, as well as low magnetic helicity density.

4. Particle transport in a strong regular magnetic field

At this time, information about the correlation of tensors $\Gamma_{\alpha\lambda}$, $\Pi_{\alpha\lambda}$ and $\mathscr{T}_{\alpha\lambda}$, introduced in Eqs. (41)–(43), is insufficient. Therefore, it is difficult to estimate the diffusion coefficient $D(\varphi)$ in momentum space by Eq. (71) and/or the effective velocity W by Eq. (68) in the general case. In this section, we establish the case where the inequality $\langle H_1^2\rangle\ll H_0^2$ holds, i.e. when MF disturbances are weak. Thus, Eqs. (68) and (71) may be simplified:

$$W=\beta_1\sqrt{\langle u_1^2\rangle/\langle H_1^2\rangle}\,H_0, \tag{79}$$

$$D(p)=\frac{p^2}{3r\Lambda_{\|}}U_\mathrm{eff}^2, \tag{80}$$

$$U_\mathrm{eff}^2=\alpha_3\langle U_1^2\rangle H_0^2/\langle H_1^2\rangle. \tag{81}$$

Here β_1 and α_3 are determined by Eqs. (69) and (72). The coefficient α_3 represents the approximate ratio of the VF correlation length to the MF correlation length. Due to Matthaeus & Goldstein (1982), we can estimate the value of $\alpha_3\approx 1$. Consequently, Eq. (80) for the coefficient $D(p)$ can be rewritten to the form

$$D(p)\approx\frac{H_0^2}{\langle H_1^2\rangle}\frac{p^2}{3v\Lambda_{\|}}\langle u_1^2\rangle. \tag{82}$$

This value exceeds that estimated by Fermi (1949), Tverskoy (1967) and Toptygin (1985) for the classical mechanism of the

Fermi acceleration by MF inhomogeneities moving randomly in space by the factor $(H_0^2/\langle H_1^2\rangle)$. The estimation of W gives

$$W \approx \frac{H_0}{\langle H_1^2\rangle}\langle \boldsymbol{u}_1 \cdot \boldsymbol{H}_1\rangle. \quad (83)$$

By using the experimental data, Matthaeus & Goldstein (1982) concluded that velocity fluctuations were relatively weakly correlated with MF fluctuations in the solar wind (the value of $\langle \boldsymbol{u}_1 \boldsymbol{H}_1\rangle$ is about one order smaller than its maximal real value). Consequently, the velocity W contribution to both the CR anisotropy and the particle energy change is negligible.

The substitution of Eqs. (82) and (83) into (66) gives

$$D_p = \frac{p^2}{3v\Lambda_{\|}}U^2 + \frac{H_0^2}{\langle H_1^2\rangle}\left(1 - \frac{\langle \boldsymbol{u}_1 \boldsymbol{H}_1\rangle^2}{\langle u_1^2\rangle\langle H_1^2\rangle}\right)\frac{p^2}{3v\Lambda_{\|}}\langle u_1^2\rangle. \quad (84)$$

This expression shows that the existence of a cross-correlation of VF and MF causes a decrease in the rate of the Fermi acceleration. We can determine the value of $\langle \boldsymbol{u}_1 \boldsymbol{H}_1\rangle$ with a knowledge of the VF statistics of media. The calculation in the frame of linear MHD yields

$$\langle \boldsymbol{u}_1 \boldsymbol{H}_1\rangle \equiv \mathscr{S}_{\alpha\alpha}(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1) = \frac{H_0}{(2\pi)^3}\int_0^\infty d\tau \int_0^\tau d\tau' \int d^3k\, xkk_\mu \mathscr{Q}_{\mu\alpha,\alpha}(\boldsymbol{k}; -\tau, \tau'), \quad (85)$$

where $\mathscr{S}_{\alpha\alpha}$ is the trace of the cross-correlator and $\mathscr{Q}_{\mu\alpha,\alpha}(\boldsymbol{k}; -\tau, \tau')$ the Fourier transform of the third-rank correlation tensor of two-point VF moments (see Monin & Yaglom 1975). Note that, in addition, a linear theory gives $\langle \boldsymbol{u}_1 \cdot \boldsymbol{H}_1\rangle = 0$ in the case of an isotropic random field.

5. Particle transport in a weak regular magnetic field

Let us now consider the opposite limiting case of $H_0^2 \ll \langle H_1^2\rangle$, where the velocity W is defined by the tensor $\Gamma_{\alpha\lambda}$ in Eq. (41). From the MHD equations, it follows that

$$\Gamma_{\alpha\lambda}(\boldsymbol{r}) = -\frac{1}{c}\varepsilon_{\lambda\mu\nu}\int_0^\infty d\tau\{\mathscr{B}_{\beta\nu}(\boldsymbol{r}, -\tau)\nabla_\beta \mathscr{Q}_{\alpha\mu}(\boldsymbol{r}, -\tau) - \mathscr{Q}_{\beta\mu}(\boldsymbol{r}, -\tau)\nabla_\beta \mathscr{B}_{\alpha\nu}(\boldsymbol{r}, -\tau) + \mathscr{S}_{\beta\mu}(\boldsymbol{r}, -\tau)\nabla_\beta \mathscr{S}_{\nu\alpha}(-\boldsymbol{r}, \tau) - \mathscr{S}_{\nu\beta}(-\boldsymbol{r}, \tau)\nabla_\beta \mathscr{S}_{\alpha\mu}(\boldsymbol{r}, -\tau)\}. \quad (86)$$

If the fluctuations of both VF and MF are statistically anisotropic, (Eq. (50), for example), then by using Eq. (86) we obtain the complicated expression for W. A simple estimation provides the value of W, which is smaller than the Alfvén velocity. Thus, in the case of a weak regular MF, we can also neglect the contribution of W in processes of particle transport. The principal contribution to the diffusion coefficient $D(p)$, Eq. (71), is given by the correlation tensor $\mathscr{T}_{\alpha\lambda}$. Using the Millionschikov hypothesis (cf. Monin & Yaglom 1975), the estimation of the fourth moment in Eq. (43) yields.

$$\mathscr{T}_{\alpha\lambda}(\boldsymbol{r}) = \frac{1}{c^2}\varepsilon_{\alpha\beta\gamma}\varepsilon_{\lambda\mu\nu}\{\mathscr{Q}_{\beta\mu}(\boldsymbol{r})\mathscr{B}_{\gamma\nu}(\boldsymbol{r}) + \mathscr{S}_{\nu\beta}(-\boldsymbol{r})\mathscr{S}_{\gamma\mu}(\boldsymbol{r})\}. \quad (87)$$

In the result,

$$D(p) = p^2 u_{\rm eff}^2/3v\Lambda; \quad (88)$$

$$u_{\rm eff}^2 = \alpha_1\langle u_1^2\rangle = \frac{\langle u_1^2\rangle}{3}\beta_0\int_0^\infty dx\int_{-1}^1 dy\{\Psi^{\mathscr{Q}}(x,y)[2\Psi(x,y) + (1-y^2)\Psi_2(x,y)] + (1-y^2)\Psi_2^{\mathscr{Q}}(x,y)\Psi(x,y) - 2(1-y^2)^2 \times \Psi_5^{\mathscr{Q}}(x,y)\Psi_5(x,y) + 2[\Phi^{\mathscr{Q}}(x,y) + y\Phi_1^{Q}(x,y)][\Phi(x,y) + y\Phi_1(x,y)] - \Psi^{\mathscr{S}}(x,-y)[2\Psi^{\mathscr{S}}(x,y) + (1-y^2)\Psi_2^{\mathscr{S}}(x,y)] - (1-y^2)\Psi_2^{\mathscr{S}}(x,-y)\Psi^{\mathscr{S}}(x,y) - 2(1-y^2)^2\Psi_5^{\mathscr{S}}(x,-y) \times \Psi_5^{\mathscr{S}}(x,y) - 2[\Phi^{\mathscr{S}}(x,-y) - y\Phi_1^{\mathscr{S}}(x,-y)] \times [\Phi^{\mathscr{S}}(x,y) + y\Phi_1^{\mathscr{S}}(x,y)]\}. \quad (89)$$

In a situation where the turbulence is isotropic, the estimation of the effective velocity in Eq. (89) is determined by the coefficient

$$\alpha_1 = \left\{\int_0^\infty dx(\Psi\Psi^{\mathscr{Q}} - \Psi^{\mathscr{S}}\Psi^{\mathscr{S}} + \Phi\Phi^{\mathscr{Q}} - \Phi^{\mathscr{S}}\Phi^{\mathscr{S}})\right\}\Big/\int_0^\infty dx\,\Psi(x). \quad (90)$$

If we suppose that the correlation functions of both VF and MF ($\Psi^{\mathscr{Q}}$ and Ψ) have the same form, and both the cross-correlation tensor $\mathscr{S}_{\alpha\lambda}$ and antisymmetric part of MF tensor vanish, then according to Jokipii & Coleman (1968), Fisk et al. (1974) and Matthaeus & Goldstein (1982), the MF fluctuation spectrum is power law in the high-frequency range and depends weakly on the frequency in the low-frequency range. Thus we can use the correlation function $\Psi(r)$ in the form

$$\Psi(r) = \frac{2^{(3-\nu)/2}}{\Gamma\left(\frac{\nu-1}{2}\right)}\left\{\left(\frac{r}{L_c}\right)^{(\nu-1)/2} K_{(\nu-1)/2}\left(\frac{r}{L_c}\right) - \frac{1}{2}\left(\frac{r}{L_c}\right)^{(\nu+1)/2} K_{(3-\nu)/2}\left(\frac{r}{L_c}\right)\right\}, \quad (91)$$

where $K_\mu(x)$ is the McDonald function and $\Gamma(x)$ is the Gamma function. Then

$$\alpha_1 = \left(1 + \frac{3}{8}\frac{\nu}{(\nu+1)}\right)\Gamma\left(\nu - \frac{1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\Big/\Gamma\left(\frac{\nu-1}{2}\right)\Gamma(\nu). \quad (92)$$

If the spectral index $\nu = 5/3$ (the Kolmogorov spectrum), then $\alpha_1 \approx 0.53$.

Let the correlation length of VF sufficiently exceed the correlation length of MF, then the expression (90) results in $\alpha_1 \approx 1$, and $D(p)$ acquires the well-known form (cf. Fermi 1949; Toptygin 1985):

$$D(p) \approx \frac{p^2}{3v\Lambda}\langle u_1^2\rangle,$$

which defines the acceleration effectiveness of the Fermi mechanism.

6. Discussion

The diffusion coefficient D_p of particles in Eq. (66) can be rewritten in the form

$$D_p = \frac{p^2}{3v\Lambda_{\|}}U^2 + \frac{p^2}{3v\Lambda_{\|}}\langle u_1^2\rangle\left(\gamma_1 + \gamma_2\frac{H_0}{\sqrt{\langle H_1^2\rangle}} + \gamma_3\frac{H_0^2}{\langle H_1^2\rangle}\right), \quad (93)$$

where

$$\gamma_1 = \alpha_1 - \beta^2; \quad \gamma_2 = \alpha_2 - 2\beta\beta_1; \quad \gamma_3 = \alpha_3 - \beta_1^2. \quad (94)$$

The quantities $\alpha_1, \alpha_2, \alpha_3, \beta$ and β_1 are defined by Eqs. (69), (70), (72) and (90). However, as we have mentioned in Sect. 4, the detailed information about all of the cited correlators is presently insufficient. Therefore, we restrict ourselves to estimations.

The first term in Eq. (93) defines the particle acceleration effect under the action of the large-scale EF, introduced in Eq. (56). The second term is connected with the effect of the stochastic EF. The presence of effective velocity W [Eqs. (68) and (94)] leads to a decrease in the stochastic particle acceleration rate, which is connected with the cross-correlation of VF and MF [see Eq. (84)]. According to Matthaeus & Goldstein (1982), the cross-correlation coefficient of VF and MF has a typical value of 0.1, but it can also achieve a large value (Burlaga & Turner 1976; Leorat et al. 1981). Let us consider a simplified case where we may neglect the spectral functions $\Phi^{\mathscr{Q},\mathscr{S}}$ in Eq. (90) and in which the triple correlators vanish. Then the quantities β, α_2 and γ_2 have values much less than 1, $\gamma_3 = \alpha_3 - \beta_1^2$, and $\gamma_1 = \alpha_1$.

Now we distinguish two situations: low and high cross-correlations of VF and MF. If the cross-correlation is low, then $\beta_1 \ll 1$ and the values of γ_1 and γ_3 are probably in the range 0.5–1 [Eq. (92)], and Eq. (93) yields

$$D_p \approx \frac{p^2}{3v\Lambda_{\|}} U^2 + \frac{p^2}{3v\Lambda_{\|}} \langle u_1^2 \rangle \left(1 + \frac{H_0^2}{\langle H_1^2 \rangle}\right) a, \quad a \approx 0.5\text{–}1.$$

The value of the second term depends on the value of $H_0^2/\langle H_1^2 \rangle$. Smith (1974) mentions a measured value $H_0^2/\langle H_1^2 \rangle \approx 3\text{–}10$ and Grappin et al. (1982) choose a value $H_0^2/\langle H_1^2 \rangle = 4$, in agreement with the HELIOS 1 and 2 observations. Matthaeus et al. (1986) analyzed ISEE-3 magnetic field data at 1 AU. They found that $(H_0^2/\langle H_1^2 \rangle)^{1/2}$ may achieve a value 1.05–0.2. Consequently, the value of $a(1 + H_0^2/\langle H_1^2 \rangle)$ is approximately 1–5.

If the cross-correlation of VF and MF is high, then $\beta_1 \approx 1$ and $\gamma_3 = \alpha_3 - \beta_1^2 \ll 1$, because $\alpha_3 \approx 1$. In a situation where the cross-correlation function of both VF and MF ($\Psi^{\mathscr{Q}}$ and Ψ) have the same form,

$$\alpha_1 \approx 0.53 - \left(\int_0^\infty \mathrm{d}x [\Psi^{\mathscr{S}}(x)]^2\right) \Big/ \int_0^\infty \mathrm{d}x \Psi(x) \ll 1,$$

and $\gamma_1 = \alpha_1 \ll 1$. Then the second term in brackets in Eq. (93) has value much less than 1, and the Fermi acceleration seems to be damped. However, in more realistic situations where the correlation length of VF sufficiently exceeds the correlation length of MF, $\gamma_1 = \alpha_1 \approx 1$.

Note that both non-zero triple correlators (see β and α_2) and the functions Φ^i (see α_1) lead to similar estimations.

Thus, the mean value of the coefficient in brackets in Eq. (93), is approximately 1, and the relative effectiveness of the particle acceleration is determined by the parameter

$$\eta^2 = U^2/\langle u_1^2 \rangle = \delta^2 \left(\frac{\Lambda_{\|}}{R}\right)^2; \quad \delta = \frac{\alpha}{\sqrt{\langle u_1^2 \rangle}}. \tag{95}$$

For estimating the value of η, it is necessary to know the longitudinal (with respect to $\boldsymbol{H}_0$) transport path dependence on the particle energy. Observations of solar CR propagation (cf. Urch & Gleeson 1972; Zwickl & Webber 1977a, b); Miroshnichenko 1980) show weak $\Lambda_{\|}$ dependence on the particle energy in the range of about ten to a few hundred MeV for protons, and $\Lambda_{\|} \approx 10^{12}$ cm. The transport path increases with the growth of particle energy, but the dependence $\Lambda_{\|}(p) \sim p^2$ (cf. Dorman & Katz 1977; Toptygin 1985) holds only for sufficiently high energies up to 10 GeV. Consequently, the above longitudinal transport path $\Lambda_{\|}$ can considerably exceed the gyroradius R in H_0, if the particle energy is small. We choose the following simple $\Lambda_{\|}$ dependence on particle momentum:

$$\Lambda_{\|}(p) = \Lambda_0 (1 + p/p_0 + p^2/p_1^2). \tag{96}$$

Here $\Lambda_0 = 10^{12}$ cm, and the momentum p_0 and p_1 correspond to the proton kinetic energy $E_{\mathrm{kin}} \approx 1$ and 10 GeV, respectively. This simple dependence describes a relatively constant $\Lambda_{\|}$ in the low-energy range and a quadratic dependence in the high-energy range. Thus, we can approximate the character of $\Lambda_{\|}$ dependence on the momentum (energy) of particles.

The dependence of the quantity η on the energy, see Eq. (95), is shown in Fig. 1 for various values of the parameter δ. The curve (a) shows the ratio $(\Lambda_{\|}/R)$ dependence on the energy, when the gyroradius is computed for $\sqrt{\langle H_1^2 \rangle} = 5\,10^{-5}$ G. We see that $\Lambda_{\|} \gg R$ in the given energy interval, and that $\Lambda_{\|}$ exceeds the gyroradius by at least two orders in the proton energy range of up to 10 MeV. The curve (b) (for $\delta = 0.1$) shows that $\eta > 1$ in all energy intervals. Therefore, in this case the particle acceleration by the large-scale EF, Eq. (56), prevails for protons of all energies. Namely, the parameter $\eta \geqslant 10$ in the low-energy range (up to 10 MeV), and the relative effectiveness of particle acceleration by the large-scale EF exceeds the effectiveness of the Fermi acceleration mechanism ($\eta^2 \geqslant 100$) by two orders. The curve (c) corresponds to $\delta = 0.05$. In this case, the parameter $\eta \leqslant 1$ for energies of 800 MeV up to 90 GeV, i.e. the Fermi mechanism predominates for these particles. In the low-energy interval of a few tens up to hundreds of MeV, the acceleration by the large-scale EF is more effective. If $\delta = 0.01$ (curve d), then the acceleration by the large-scale EF is essential only in the very low-energy range up to 10 MeV. The acceleration effectiveness of these particles exceeds the statistical acceleration by the Fermi mechanism by approximately one order.

The above estimations show that the particle acceleration by the large-scale EF, which is caused by the existence of the non-mirrorsymmetric stochastic MF in a moving conductive fluid, is most effective for low-energy particles. If the quantity α in Eq. (56) has sufficient value ($\delta \approx 10^{-1}\text{–}10^{-2}$), as occurs in a cosmic plasma, then the acceleration of the charged CR particles by the large-scale EF prevails in the energy gain over Fermi's mechanism in a wide energy interval up to 1 GeV.

In addition, it follows from Eq. (57) for the vector of particle flux density J, that the large-scale EF contributes changes in CR

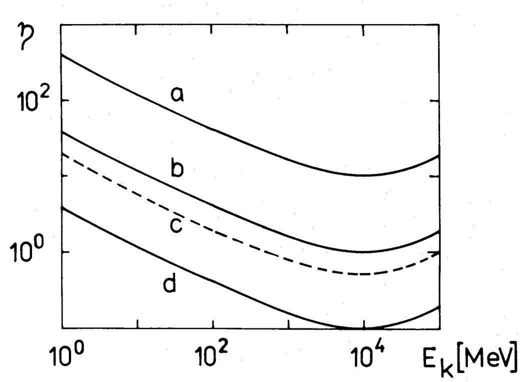

Fig. 1. Plot of the relative effectiveness η [Eq. (95)], vs E_{kin} for different values of the parameter δ [Eq. (78)]

anisotropy, because the convective component of particle flux density depends on the effective velocity U and on α. Substituting Eqs. (57) and (65) into (30), we obtain the explicit form of the particle transport equation:

$$\partial_t N = \nabla_\alpha \kappa_{\alpha\lambda} \nabla_\lambda N - (\boldsymbol{S}\cdot\nabla)N + \nabla(\boldsymbol{u}_0 - \boldsymbol{U} - \boldsymbol{W}) \frac{p}{3}\frac{\partial N}{\partial p} - \frac{2}{3} p(\boldsymbol{U}\nabla)\frac{\partial N}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_p \frac{\partial N}{\partial p}, \tag{97}$$

where

$$\boldsymbol{S} = \boldsymbol{u}_0 - \boldsymbol{W} + \frac{1}{3p^2}\frac{\partial}{\partial p} p^3 \boldsymbol{U}. \tag{98}$$

As noted above, the velocity W does not exceed the Alfvén velocity. The effective velocity U is also small in comparison with the HD velocity u_0. Thus Eq. (97) takes the form

$$\partial_t N = \nabla_\alpha \kappa_{\alpha\lambda} \nabla_\lambda N - (\boldsymbol{u}_0\cdot\nabla)N + (\nabla\cdot\boldsymbol{u}_0)\frac{p}{3}\frac{\partial N}{\partial p} - \frac{2p}{3}(\boldsymbol{U}\cdot\nabla)\frac{\partial N}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_p \frac{\partial N}{\partial p}, \tag{99}$$

which differs from the well-known transport equation (Dolginov & Toptygin 1966; Gleeson & Axford 1967; Dorman & Katz 1977), by the presence of the mixed derivative component, and also by the expression for the diffusion coefficient D_p in momentum space [defined in Eq. (93)], which includes the particle acceleration by the large-scale electric field.

7. Conclusion

Our result shows that the transport of high-energy charged particles up to 1 GeV in the medium of a reflective non-invariant magnetic field is accompanied by an additional particle acceleration. This acceleration is caused by the appearance of the large-scale electric field in the "magnetized" fluid, along with the classical effect of energy gain by multiple scattering in randomly moving magnetic field inhomogeneities.

The characteristic peculiarity of the considered acceleration mechanism consists of the fact that the mean electric field, which is connected with the gyrotropic turbulence, is directed along the large-scale magnetic field. This means that the electric field, simultaneously with the particle acceleration, leads to convective transport with effective velocity $\boldsymbol{U}$ [Eq. (62)] along the direction of the regular MF. This $\boldsymbol{U}$ determines the contribution of the large-scale electric field to the CR anisotropy of particles. The estimations show that the considered acceleration mechanism is especially effective for strongly "magnetized" particles, for which the inequality $(\Lambda_{\|}/R) \gg 1$ holds. This condition is well satisfied by the CR particles of energy up to 1 GeV.

Appendix

In this appendix, we briefly describe the functional method for computing correlators that figure in this discussion (cf. Rytov et al. 1978; Stehlík et al. 1983).

Let us consider a cross-correlation function (or a tensor) $\langle R[\boldsymbol{\kappa}] f_t[\boldsymbol{\kappa}]\rangle$, where $\boldsymbol{\kappa}$ is a $3n$-dimensional vector $\{\boldsymbol{H}_1, \boldsymbol{u}_1, \ldots\}$; $f_t[\boldsymbol{\kappa}]$ is a functional of stochastic fields and depends parametrically on the time t, and the functional $R[\boldsymbol{\kappa}]$ depends on the quantities κ_α explicitly. For example, in $\langle H_{1\alpha}(\boldsymbol{r}_1, t_1) f(\boldsymbol{r}, \boldsymbol{p}, t)\rangle$ is $R[\boldsymbol{\kappa}] \equiv H_{1\alpha}(\boldsymbol{r}_1, t_1)$, and $f_t[\boldsymbol{\kappa}] \equiv f[\boldsymbol{r}, \boldsymbol{p}, t; \boldsymbol{\kappa}]$ is the functional of the fields $\boldsymbol{H}_1, \boldsymbol{u}_1, \ldots$ In this work, the distribution function satisfies the kinetic equation (8) and thus depends on the form of the random function, $\boldsymbol{\kappa} \equiv \boldsymbol{F}_1$ for example.

From the principle of causality, it is evident that $f(\boldsymbol{r}, \boldsymbol{p}, t)$ depends on $\kappa(\boldsymbol{r}_1, t_1)$ only for $t_1 \le t$. The kinetic equation is a first-order differential equation with initial conditions at $t_0 = 0$. Subsequently, the functional $f[\boldsymbol{r}, \boldsymbol{p}, t; \boldsymbol{\kappa}]$ depends on $\kappa(\boldsymbol{r}_1, t_1)$ only for $t_1 \in \langle 0, t\rangle$:

$$\delta f_t[\boldsymbol{\kappa}]/\delta\kappa_\alpha(\boldsymbol{r}_1, t_1) = 0 \quad \text{for} \quad t_1 \notin \langle 0, t\rangle.$$

Let $\boldsymbol{\eta}(\boldsymbol{r}, t)$ be an arbitrary, but determined, vector function in 3n-dimensional space, defined on the interval $\langle 0, t\rangle$. The functional $f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]$ may be expanded in series around the function $\boldsymbol{\eta}(\boldsymbol{r}, t)$, i.e.

$$f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}] = \exp\left\{\int_0^t \mathrm{d}\tau \int \mathrm{d}^3\rho\, \kappa_\alpha(\boldsymbol{\rho}, \tau)\left(\frac{\delta}{\delta\eta_\alpha(\boldsymbol{\rho}, \tau)}\right)\right\} f_t[\boldsymbol{\eta}].$$

If the functional Ω_t is defined as

$$\Omega_t[\zeta] \equiv \frac{\left\langle R[\boldsymbol{\kappa}] \exp\left\{\mathrm{i}\int_0^t \mathrm{d}\tau \int \mathrm{d}^3\rho\, \kappa_\alpha\zeta_\alpha\right\}\right\rangle}{\left\langle \exp\left\{\mathrm{i}\int_0^t \mathrm{d}\tau \int \mathrm{d}^3\rho\, \kappa_\alpha\zeta_\alpha\right\}\right\rangle}, \tag{A1}$$

a simple calculation leads to an expression for the considered correlation tensor of the form

$$\langle R[\boldsymbol{\kappa}] f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\rangle = \Omega_t\left[\frac{\delta}{\mathrm{i}\delta\boldsymbol{\eta}}\right]\langle f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\rangle, \tag{A2}$$

where the relation $f_t[\boldsymbol{\eta}] = \langle f_t[\boldsymbol{\eta}]\rangle$ has been used. The relation (A2) was obtained first by Klyatskin & Tatarski (1973). It is evident from the expression (A1) that the functional Ω_t depends on both the character of the random field $\boldsymbol{\kappa}$ and the concrete form of the functional $R[\boldsymbol{\kappa}]$.

Let us consider in the following a random field $\boldsymbol{\kappa}$ having a gaussian probability distribution, and specify its characteristic functional in the interval $\langle 0, t\rangle$. Note that, by the characteristic functional of the random field (cf. Monin & Yaglom 1975), we mean the functional

$$\Phi_{\boldsymbol{\kappa}}[\boldsymbol{\eta}(\boldsymbol{r}, t)] \equiv \left\langle \exp\left\{\mathrm{i}\int \mathrm{d}t \int \mathrm{d}^3 r\, \eta_\alpha(\boldsymbol{r}, t)\kappa_\alpha(\boldsymbol{r}, t)\right\}\right\rangle.$$

From the theory of random functions, it follows that

$$\langle \exp \mathrm{i}\kappa_\alpha\rangle = \exp\{-\tfrac{1}{2}\langle\kappa_\alpha^2\rangle\} \tag{A3}$$

and the integral $A \equiv \int_0^t \mathrm{d}\tau \int \mathrm{d}^3\rho\, \eta_\alpha\kappa_\alpha$ has a Gaussian distribution with mean $\langle A\rangle = 0$, if $\langle\kappa\rangle = 0$. In this case

$$\langle A^2\rangle = \int_0^t \mathrm{d}\tau_1 \int \mathrm{d}^3\rho_1 \int_0^t \mathrm{d}\tau_2 \int \mathrm{d}^3\rho_2\, \mathscr{D}_{\alpha\beta}(\boldsymbol{\rho}_1, \tau_1; \boldsymbol{\rho}_2, t_2) \times \zeta_\alpha(\boldsymbol{\rho}_1, \tau_1)\zeta_\beta(\boldsymbol{\rho}_2, \tau_2).$$

Now $\mathscr{D}_{\alpha\beta}$ is the correlation tensor of the random field $\boldsymbol{\kappa}$. The characteristic functional of the field $\boldsymbol{\kappa}$ equals, by (A3),

$$\Phi_{t,\boldsymbol{\kappa}}[\zeta] = \exp\left\{-\frac{1}{2}\int_0^t \mathrm{d}\tau_1 \int \mathrm{d}^3\rho_1 \int_0^t \mathrm{d}\tau_2 \int \mathrm{d}\rho_2 \times \mathscr{D}_{\alpha\beta}(\boldsymbol{\rho}_1, \tau_1; \boldsymbol{\rho}_2, \tau_2)\zeta_\alpha(\boldsymbol{\rho}_1, \tau_1)\zeta_\beta(\boldsymbol{\rho}_2, \tau_2)\right\}. \tag{A4}$$

Let us now take a fairly general form of the functional

$$R[\boldsymbol{\kappa}]=R_{\boldsymbol{\kappa}}[\xi]=\exp\left\{\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\kappa_\alpha(\boldsymbol{\rho},\tau)\xi_\alpha(\boldsymbol{\rho},\tau)\right\}$$

and assume that $\zeta(\boldsymbol{r}_1,t_1)\equiv 0$ for $t_1\notin\langle 0,t\rangle$. We obtain successively

$$\Omega_t[\zeta]=\left\langle\exp\left\{\mathrm{i}\int_0^t\mathrm{d}\tau\int\mathrm{d}^3\rho\,\kappa_\alpha\zeta_\alpha+\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\kappa_\alpha\left(\frac{\delta}{\delta\eta_\alpha}\right)\right\}\right\rangle$$
$$\times\left\langle\exp\left\{\mathrm{i}\int_0^t\mathrm{d}\tau\int\mathrm{d}^3\rho\,\kappa_\alpha\zeta_\alpha\right\}\right\rangle^{-1}$$
$$\times\left\langle\exp\left\{\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\xi_\alpha\eta_\alpha\right\}\right\rangle\Bigg|_{\boldsymbol{\eta}=0}=\Phi_{\boldsymbol{\kappa}}\left[\zeta+\frac{\delta}{\mathrm{i}\delta\boldsymbol{\eta}}\right]$$
$$\times\Phi_{\boldsymbol{\kappa}}^{-1}[\zeta]\,\Phi_{\boldsymbol{\kappa}}^{-1}\left[\frac{\delta}{\mathrm{i}\delta\boldsymbol{\eta}}\right]\Phi_\xi[\boldsymbol{\eta}+\boldsymbol{\kappa}]_{\boldsymbol{\eta}=0}. \tag{A5}$$

From relations (A4) and (A5), we obtain

$$\Omega_t[\zeta]=\left\langle\exp\left\{\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\xi_\alpha(\boldsymbol{\rho},\tau)\left[\kappa_\alpha(\boldsymbol{\rho},\tau)\right.\right.\right.$$
$$\left.\left.\left.+\mathrm{i}\int_0^t\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,\mathscr{D}_{\alpha\beta}(\boldsymbol{\rho},\tau;\boldsymbol{\rho}_1,t_1)\zeta_\beta(\boldsymbol{\rho}_1,\tau_1)\right]\right\}\right\rangle.$$

Finally expression sought for the correlation tensor is

$$\langle R_{\boldsymbol{\kappa}}[\xi]f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\rangle$$
$$=\left\langle R_\xi\left[\boldsymbol{\kappa}(\boldsymbol{\rho},\tau)+\int_0^t\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,e_\alpha\mathscr{D}_{\alpha\beta}(\boldsymbol{\rho},\tau;\boldsymbol{\rho}_1,t_1)\right.\right.$$
$$\left.\left.\times\frac{\delta}{\delta\eta_\beta(\boldsymbol{\rho}_1,\tau_1)}\right]\right\rangle\langle f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\rangle, \tag{A6}$$

where e_α is the unit vector on the α-axis in $3n$-dimensional space. Because $R[a+b]=R[a]\cdot R[b]$, the relation (A6) can also be written in a form more convenient for use:

$$\langle R_{\boldsymbol{\kappa}}[\xi]f_t[\boldsymbol{\kappa}]\rangle=\left\langle\exp\left\{\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\kappa_\alpha(\boldsymbol{\rho},\tau)\xi_\alpha(\boldsymbol{\rho},\tau)\right\}\right\rangle$$
$$\times\langle\exp\left\{\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau\int\mathrm{d}^3\rho\,\xi_\alpha(\boldsymbol{\rho},\tau)\int_0^t\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,\mathscr{D}_{\alpha\beta}\right.$$
$$\left.\times(\boldsymbol{\rho},\tau;\boldsymbol{\rho}_1,t_1)\times\frac{\delta}{\delta\eta_\beta(\boldsymbol{\rho}_1,t_1)}\right\}f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\Big\rangle\Bigg|_{\boldsymbol{\eta}=0}. \tag{A7}$$

The latter correlator on the right-hand side of Eq. (A7) may be expressed as

$$\left\langle f_t\left[\boldsymbol{\kappa}(\boldsymbol{\rho},\tau)+\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,\xi_\alpha(\boldsymbol{\rho}_1,\tau_1)\mathscr{D}_{\alpha\beta}(\boldsymbol{\rho},\tau;\boldsymbol{\rho}_1,\tau_1)e_\alpha\right]\right\rangle.$$

Let us now calculate the simplest correlation function. From Eq. (A7), it follows that

$$\langle\kappa_\alpha(\boldsymbol{r},t)f(\boldsymbol{r},t)\rangle$$
$$=\left\langle\frac{\delta}{\mathrm{i}\delta\xi_\alpha(\boldsymbol{r},t)}R[\boldsymbol{\kappa}]f[\boldsymbol{r},t;\boldsymbol{\kappa}]\right\rangle\Bigg|_{\xi=0}$$
$$=\int_0^t\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,\mathscr{D}_{\alpha\beta}(\boldsymbol{r},t;\boldsymbol{\rho}_1,\tau_1)\frac{\delta}{\delta\eta_\beta(\boldsymbol{\rho}_1,t_1)}\langle f_t[\boldsymbol{\kappa}+\boldsymbol{\eta}]\rangle\Bigg|_{\boldsymbol{\eta}=0}$$
$$=\int_0^t\mathrm{d}\tau_1\int\mathrm{d}^3\rho_1\,\mathscr{D}_{\alpha\beta}(\boldsymbol{r},t;\boldsymbol{\rho}_1,t_1)\left\langle\frac{\delta f[\boldsymbol{r},t;\boldsymbol{\kappa}]}{\delta\kappa_\beta(\boldsymbol{\rho}_1,\tau_1)}\right\rangle. \tag{A8}$$

This relation is known as the Furutsu–Noviskov formula (cf. Furutsu 1963; Novikov 1964). Equation (A8) is Eq. (14) in the text.

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