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SPACE PLASMA

Generation of Alfvén Waves by a Plasma Inhomogeneity Moving in the Earth's Magnetosphere

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Abstract—The generation of an Alfvén wave by an azimuthally drifting cloud of high-energy particles injected in the Earth's magnetosphere is studied analytically. In contrast to the previous studies where the generation mechanisms associated with the resonant wave–particle interaction were considered, a nonresonant mechanism is investigated in which the wave is excited by the alternating current produced by drifting particles. It is shown that, at a point with a given azimuthal coordinate, a poloidally polarized wave, in which the magnetic field lines oscillate predominantly in the radial direction, is excited immediately after the passage of the particle cloud through this point. As the cloud moves away from that point, the wave polarization becomes toroidal (the magnetic field lines oscillate predominantly in the azimuthal direction). The azimuthal wavenumber m is defined as the ratio of the wave eigenfrequency to the angular velocity of the cloud (the drift velocity of the particles). It is shown that the amplitudes of the waves so generated are close to those obtained under realistic assumptions about the density and energy of the particles.

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1. INTRODUCTION

Among the vast variety of Alfvén waves in the Earth's magnetosphere, there are two limiting cases: toroidally and poloidally polarized waves. In the first case, the magnetic field lines oscillate in the azimuthal direction (along the binormal to them), whereas in the second case, they oscillate in the radial direction (across the magnetic shells). Accordingly, the electric field oscillates in the radial direction in the first case and in the azimuthal direction in the second. The question arises of what are the sources of such waves. It is suggested that toroidally polarized Alfvén waves are generated resonantly by fast magnetosonic waves coming from the outer regions of the magnetosphere [1]. As for the poloidally polarized Alfvén waves, they are thought to be excited by the particles injected into the magnetosphere during magnetic substorms. There is some experimental evidence in support of this hypothesis [2, 3]. It is widely believed that the waves are excited by unstable populations of 10- to 150-keV protons via the bounce-drift resonance mechanism (the bounce-drift instability) [4, 5].

In terms of this mechanism, however, it is impossible to explain a number of important features of the waves under discussion. First, the observed waves have quite definite azimuthal wavenumbers m. For the most commonly observed unstable high-energy proton distributions, however, the instability growth rate weakly depends on the azimuthal wavenumber [6]. Therefore, the instability cannot select a narrow range of azimuthal wavenumbers. Second, the direction of the phase velocity of the observed poloidal Alfvén waves usually coincides with that of proton drift in a nonuniform magnetic field. However, the instability can equally well generate waves propagating in the opposite direction [6]. Finally, as was shown in [7, 8], the poloidal waves are rapidly converted into toroidal ones, so the instability amplifies toroidal, rather than poloidal, waves.

It is therefore necessary to consider not only resonant but also nonresonant mechanisms for generating poloidal Alfvén waves. The excitation of MHD waves by an external alternating current was studied in [9, 10]. For magnetospheric conditions, this mechanism was considered in [11], where Alfvén waves were assumed to be generated by the current of particles injected into the magnetosphere. In the nonuniform magnetic field of the Earth, such particles drift azimuthally, producing an alternating current, because they are injected in the form of a cloud that has a finite size in the azimuthal direction. This alternating current generates an Alfvén wave. The clouds can also be regarded as nonuniformities of the magnetospheric ring current. The excitation of Alfvén waves by an unsteady external current was considered in [12], where additional arguments in support of this mechanism were presented.

In [9–12], the plasma was assumed to be homogeneous (although, in [11], it was additionally assumed to be bounded along the magnetic field lines). However, the magnetospheric conditions are inherently nonuniform: in particular, the plasma density and magnetic field strength vary both along the magnetic field lines

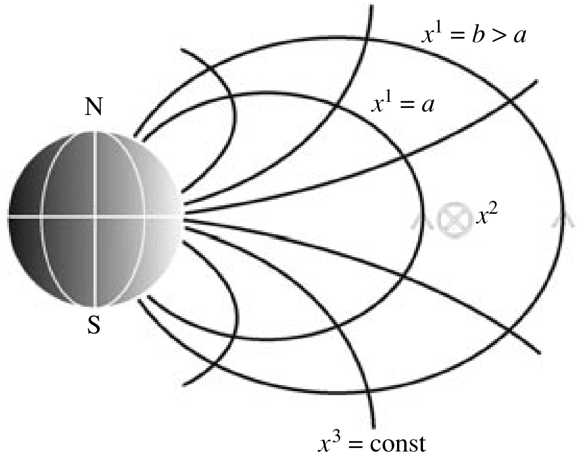

Fig. 1. Coordinate system.

and across the magnetic shells; moreover, the field lines are curved substantially. In the present paper, we investigate the excitation of Alfvén waves in an axisymmetric magnetosphere model in which these factors are taken into account. The problem is treated in the following formulation. At time $t = 0$, a cloud of high-energy particles is injected into an axisymmetric zero-pressure magnetospheric plasma. The particles then drift in the azimuthal direction. The aim is to analyze the spatiotemporal structure and polarization of the excited waves and to obtain the expressions for their amplitudes.

In studying the generation mechanism proposed here, we use the general approach of [9, 10] and utilize as a basis the theory of Alfvén eigenmodes in an axisymmetric magnetosphere that was constructed in [13, 14], as well as the method for solving the Alfvén wave equation developed there. In our analysis, the right-hand side of the Alfvén wave equation is a source term describing the generation of the wave field by an azimuthally moving nonuniformity—a cloud of drifting protons that were injected into the magnetosphere. In order to consider the generated Alfvén waves in the linear approximation, the particle density within the nonuniformity is assumed to be low in comparison with the density of the background plasma. The external current is assumed to be prescribed; i.e., the inverse effect of the waves on the current is ignored. This approach can well be applied to study the initial stage of the wave field evolution (see, e.g., [15]).

2. WAVE EQUATION

We will use an axisymmetric magnetosphere model in which the background plasma is inhomogeneous along the magnetic field lines and across the magnetic shells and the lines themselves are curved. We introduce a curvilinear coordinate system $\{x^1, x^2, x^3\}$ (see Fig. 1) in which the coordinate surfaces x^1 = const coincide with the magnetic shells, the coordinate x^2 labels the magnetic field lines on a magnetic surface, and the coordinate x^3 gives the position of a point on the line. The field lines are coordinate lines such that x^1 = const and x^2 = const. The coordinate x^1 corresponds to the radial coordinate and the coordinate x^2 is an analogue of the azimuthal coordinate (whose role can be played, e.g., by the azimuthal angle φ). The physical length along a field line is expressed in terms of the length element along the third coordinate as $dl_3 = \sqrt{g_3}dx^3$, where g_3 is the metric tensor component and $\sqrt{g_3}$ is the Lamé coefficient. Analogously, we have $dl_1 = \sqrt{g_1}dx^1$ and $dl_2 = \sqrt{g_2}dx^2$. The determinant of the metric tensor is $g = g_1g_2g_3$. We also introduce the following notation: B and ρ are the equilibrium values of the magnetic field and plasma density, ξ is the plasma displacement from the equilibrium position, E and b are the electric and magnetic fields of the wave, and j is the wave current density. The wave source is the unsteady transverse (azimuthal) current (with the density $\mathbf{j}_{\text{ext}}$) of the drifting particles in a cloud injected into the magnetosphere [9]. In the approximate model at hand, the steady current is zero.

In the cold background plasma approximation, the linearized equation of small oscillations has the form

$$\rho\frac{\partial^2\boldsymbol{\xi}}{\partial t^2} - \frac{1}{c}\mathbf{j}\times\mathbf{B} = 0. \tag{1}$$

The electrodynamic quantities are related by the equations

$$\nabla\times\mathbf{b} = \frac{4\pi}{c}\mathbf{j} + \frac{4\pi}{c}\mathbf{j}_{\text{ext}} \tag{2}$$

(Ampère's law),

$$\nabla\times\mathbf{E} = \frac{1}{c}\frac{\partial\mathbf{b}}{\partial t} \tag{3}$$

(Maxwell's equation), and

$$\mathbf{E} = \frac{1}{c}\frac{\partial\boldsymbol{\xi}}{\partial t}\times\mathbf{B} \tag{4}$$

(the condition that the magnetic field is frozen in the plasma). Note that the external current enters only into Eq. (2) [10]. In the perfectly conducting plasma approximation, the longitudinal component of the wave electric field is zero, so the electric field is two-dimensional and its components are tangential to the surfaces orthogonal to the magnetic field lines. From Eqs. (1)–(4), we obtain the following equation for the electric field $\mathbf{E}$:

$$\frac{1}{A^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla\times(\nabla\times\mathbf{E}) = -\frac{4\pi}{c^2}\frac{\partial\mathbf{j}_{\text{ext}}}{\partial t}, \tag{5}$$

where $A = B/\sqrt{4\pi\rho}$ is the Alfvén speed. The electric field of an Alfvén wave can be represented as

$$\mathbf{E} = -\nabla_{\perp}\Phi, \tag{6}$$

where Φ is a scalar function (potential) and $\nabla_{\perp}$ is the nabla operator in the transverse coordinates. Substituting expression (6) into Eq. (5) and acting on the resulting equation by the operator $\nabla_{\perp}$, we arrive at the equation

$$\mathcal{L}_A\Phi = -\frac{4\pi}{c^2}\sqrt{g}\frac{\partial}{\partial x^2}\frac{\partial}{\partial t}j^2_{\text{ext}}, \tag{7}$$

where $j^2_{\text{ext}} = j_{\text{ext}}/\sqrt{g_2}$ is the contravariant component of the vector $\mathbf{j}_{\text{ext}}$ along the x^2 coordinate and

$$\mathcal{L}_A = \frac{\partial}{\partial x^1}\left[-\frac{\sqrt{g}}{g_1}\frac{1}{A^2}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3}\right]\frac{\partial}{\partial x^1} + \frac{\partial}{\partial x^2}\left[-\frac{\sqrt{g}}{g_2}\frac{1}{A^2}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x^3}\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}\right]\frac{\partial}{\partial x^2}$$

is the Alfvén differential operator. We have thus derived a nonuniform partial differential equation describing Alfvén waves generated by the current of the azimuthally drifting particles. We choose the boundary conditions

$$\Phi|_{x^1, x^2 \to \pm\infty} = 0, \quad \Phi|_{x^3_\pm} = 0,$$

the second of which corresponds to a complete reflection of the wave from the ionosphere (the points $x^3_\pm$ are the intersection points of a magnetic field line with the upper boundary of the ionosphere). In what follows, the damping due to the finite conductivity of the ionospheric plasma will be ignored.

The cloud of the drifting particles that produce the azimuthal current is assumed to be highly localized in the azimuthal direction; i.e.,

$$j^2_{\text{ext}} = en_0\Omega\delta(\varphi - \Omega t)\Theta(t). \tag{8}$$

Here, Ω is the angular drift velocity in a nonuniform magnetic field; e is the charge of a particle; n_0 is the particle density; the theta function $\Theta(t)$ determines the time at which the source is switched on (i.e., at which the particles begin to be injected into the magnetosphere); and φ is the azimuthal angle, which will be used below as the azimuthal coordinate x^2. The physical current can be obtained by replacing the angular velocity in expression (8) with the linear velocity $V = \sqrt{g_2}\Omega$, which depends on the x^1 and x^3 coordinates.

3. SOLUTION OF THE WAVE EQUATION

We solve wave equation (7) by applying the Fourier transformation in φ and t. The transformation yields the following differential equation in the two variables x^1 and x^3:

$$\hat{L}_A\Phi_{m\omega} = \tilde{q}_{m\omega}, \tag{9}$$

where $\hat{L}_A$ is the Fourier transformed Alfvén operator $\mathcal{L}_A$, which is analogous to the Alfvén operator for a monochromatic wave with the frequency ω and azimuthal wavenumber m and is defined as

$$\hat{L}_A \equiv \frac{\partial}{\partial x^1}\hat{L}_T(\omega)\frac{\partial}{\partial x^1} - m^2\hat{L}_P(\omega).$$

Here,

$$\hat{L}_T(\omega) = \frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3} + \frac{\sqrt{g}}{g_1}\frac{\omega^2}{A^2}$$

is the toroidal mode operator;

$$\hat{L}_P(\omega) = \frac{\partial}{\partial x^3}\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3} + \frac{\sqrt{g}}{g_2}\frac{\omega^2}{A^2}$$

is the poloidal mode operator; ω and m are the parameters of the Fourier transformation in time (the frequency) and in the azimuthal angle (the azimuthal wavenumber), respectively; and

$$\tilde{q}_{m\omega} = -2m\omega\sqrt{g}\frac{en_0\Omega}{c^2}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\Theta(t')\exp(i\omega t' - im\Omega t')dt'.$$

Knowing the solution to Eq. (9), we can find a solution to wave equation (7) by applying the inverse Fourier transformation,

$$\Phi(x^1, x^2, x^3, t) = \int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}dm\Phi_{m\omega}\exp(im\varphi - i\omega t). \tag{10}$$

The method for solving Eq. (9) was developed in [14]. In that paper, it was shown that the function $\Phi_{m\omega}$ can be represented as

$$\Phi_{m\omega} \approx R_N(x^1)T_N(x^1, x^3), \tag{11}$$

where $T_N(x^1, x^3)$ is the eigenfunction of the toroidal operator $\hat{L}_T$. This function describes the longitudinal structure of the Nth longitudinal standing wave mode and is normalized so that

$$\left\langle\frac{\sqrt{g}}{g_1}\frac{T_N^2}{A^2}\right\rangle = 1,$$

where the angle brackets denote integration along a magnetic field line between two magnetically conjugate points of the ionosphere, $\langle\ldots\rangle = \int_{x^3_-}^{x^3_+}(\ldots)dx^3$.

The function $R_N(x^1)$, which describes the structure of this mode across the magnetic shells, is determined from the solution to the differential equation (see also [13])

$$\frac{\partial}{\partial x^1}(x^1 - x^1_{TN}(\omega))\frac{\partial}{\partial x^1}R_N - \frac{m^2}{L^2}(x^1 - x^1_{PN}(\omega))R_N = mq(\omega, m), \tag{12}$$

where

$$q(\omega, m) = \frac{q_0}{2\pi}\int_{-\infty}^{+\infty}\Theta(t')\exp(i\omega t' - im\Omega t')dt',$$
$$q_0 = -\frac{el}{c^2}\frac{\Omega}{\Omega_0}\langle n_0\sqrt{g}T_N\rangle. \tag{13}$$

Equation (12) was derived by using the following linear expansions of the eigenfrequencies of the toroidal and poloidal operators in the vicinity of a certain magnetic shell (at a distance L from it) in the equatorial plane:

$$\Omega_{TN}(x^1) = \Omega_0\left(1 - \frac{x^1}{l}\right)$$

for the toroidal frequency and

$$\Omega_{PN}(x^1) = \Omega_0\left(1 - \frac{x^1 + \Delta}{l}\right)$$

for the poloidal frequency. Here, the functions $\Omega_{TN}(x^1)$ and $\Omega_{PN}(x^1)$ are monotonically decreasing over most of the magnetosphere. In Eq. (12), the frequency-dependent functions $x^1_{TN}(\omega)$ and $x^1_{PN}(\omega)$ represent the coordinates of the magnetic surfaces on which the wave frequency is equal to the toroidal and the poloidal frequency, i.e., $x^1_{TN}(\omega)$ and $x^1_{PN}(\omega)$ are solutions to the equations $\omega = \Omega_{TN}(x^1)$ and $\omega = \Omega_{PN}(x^1)$:

$$x^1_{TN}(\omega) = l\left(1 - \frac{\omega}{\Omega_0}\right),$$
$$x^1_{PN}(\omega) = x^1_{TN} - \Delta_N, \tag{14}$$

where Δ_N is the distance between the toroidal and poloidal surfaces (in a cold plasma, the poloidal surface is nearer to the Earth than the toroidal one).

Equation (12) has the solution [16]

$$R_N(x^1, \omega, m) = iq(\omega, m)L\int_0^{+\infty}\frac{d\kappa}{\sqrt{\kappa^2 + m^2\frac{l^2}{L^2}}} \tag{15}$$
$$\times\exp\left[ik(\xi - \xi_T(\omega)) + im\delta\frac{l}{L}\arctan\frac{\kappa L}{ml}\right],$$

where we have introduced the notation

$$\xi = x^1/l, \quad \xi_T = x^1_{TN}/l, \quad \xi_P = x^1_{PN}/l, \quad \delta = \Delta_N/l.$$

Hence, according to expressions (10) and (11), the solution to wave equation (7) has the form

$$\Phi(x^1, x^2, x^3, t) = \mathcal{R}_N(x^1, x^2, t)T_N(x^1, x^3), \tag{16}$$

where the function

$$\mathcal{R}_N(x^1, x^2, t) = \int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}dmR_N(m, \omega)\exp(im\varphi - i\omega t) \tag{17}$$

describes the transverse structure and temporal evolution of the wave. With allowance for relationships (14) and expression (15), the expression for $\mathcal{R}_N$ can be reduced to (see the Appendix)

$$\mathcal{R}_N = iq_0L\int_{-\infty}^{+\infty}dm\int_0^{+\infty}d\kappa\frac{\exp[i\Psi(m, \kappa)]}{\sqrt{\kappa^2 + m^2\frac{l^2}{L^2}}}, \tag{18}$$

where

$$\Psi(m, \kappa) = m(\varphi - \Omega t) + \kappa\left(\xi - 1 + \frac{m\Omega}{\Omega_0}\right) + m\delta\frac{l}{L}\arctan\frac{\kappa L}{ml}. \tag{19}$$

Solution (18) is valid for steady waves on time scales such that $\Omega_0 t \gg 1$, i.e., many wave periods after the injection of the particles into the magnetosphere.

4. STRUCTURE AND EVOLUTION OF THE WAVE FIELD

Let us consider a source moving at a velocity much lower than the Alfvén speed. In this case, the double integral in expression (18) can be calculated by the stationary-phase method under the assumption that the parameter $\mu = \Omega_0/\Omega$ is large. The stationary-phase point (m_0, κ_0) can be found from the conditions

$$\left.\frac{\partial\Psi}{\partial\kappa}\right|_{m_0,\kappa_0} = 0, \quad \left.\frac{\partial\Psi}{\partial m}\right|_{m_0,\kappa_0} = 0.$$

As a result, the stationary-phase point is determined by the following two equations:

$$m_0 + (\xi - 1)\mu + \mu\delta\left[1 + \left(\frac{\kappa_0 L}{m_0 l}\right)^2\right]^{-1} = 0, \tag{20}$$

$$\kappa_0 + \mu(\varphi - \Omega t) + \mu\delta\frac{l}{L}\arctan\frac{\kappa_0 L}{m_0 l} + \mu\delta\frac{\kappa_0}{m_0}\left[1 + \left(\frac{\kappa_0 L}{m_0 l}\right)^2\right]^{-1} = 0. \quad (21)$$

From Eq. (20) we find the relationship between κ_0 and m_0:

$$\kappa_0^2 = m_0^2\frac{l^2}{L^2}\frac{m_0\Omega - \Omega_{PN}(\xi)}{\Omega_{TN}(\xi) - m_0\Omega}, \quad (22)$$

where

$$\Omega_{TN}(\xi) = \Omega_0(1-\xi), \quad \Omega_{PN}(\xi) = \Omega_0(1-\xi-\delta).$$

From Eq. (21) we find κ_0:

$$\kappa_0 = \mu(\Omega t - \phi). \quad (23)$$

In Eq. (21), we have omitted the last two terms because they are proportional to the small parameter $\delta = \Delta_N/l \ll 1$ (in a cold plasma, the distance Δ_N between the toroidal and poloidal surfaces is much less than the typical scale lengths of the magnetosphere and is finite at $|\kappa_0| \longrightarrow \infty$ and $m_0 \longrightarrow \infty$). Thus, the approximation at hand is valid for $|\varphi - \Omega t| \gg \delta$, i.e., for distances from the source that are much larger than Δ_N. From relationships (22) and (23) we obtain the equation for m_0:

$$\mu^2(\Omega t - \varphi)^2 = m_0^2\frac{l^2}{L^2}\frac{m_0\Omega - \Omega_{PN}(\xi)}{\Omega_{TN}(\xi) - m_0\Omega}. \quad (24)$$

Let us consider two limiting cases:

(i) at short distances from the source, $\Omega t - \varphi \longrightarrow 0$, we have

$$m_0 \longrightarrow m_P = \frac{\Omega_{PN}}{\Omega};$$

(ii) at long distances from the source, $|\Omega t - \varphi| \longrightarrow \infty$, we have

$$m_0 \longrightarrow m_T = \frac{\Omega_{TN}}{\Omega}.$$

We thus see that m_0 changes within a relatively narrow interval of width $\mu\delta$, from m_P (near the source) to m_T (far from the source or an infinitely long time after the passage of the source). Taking into account this circumstance, we represent m_0 as a sum of two terms:

$$m_0 = m_T + m'\delta.$$

Substituting this representation into Eq. (24) and ignoring the terms proportional to δ^2 and δ^3, we find m' to arrive at the following expression for m_0:

$$m_0 = m_T - \mu\delta\left[1 + \frac{\mu^2 L^2}{m_T^2 l^2}(\Omega t - \varphi)^2\right]^{-1}. \quad (25)$$

In the vicinity of the stationary-phase point (m_0, κ_0), phase expansion (19) yields

$$\Psi(\kappa, m) = \Psi(\kappa_0, m_0) + \frac{1}{2}A_0(\kappa - \kappa_0)^2 + \frac{1}{2}B_0(m - m_0)^2 + C_0(\kappa - \kappa_0)(m - m_0), \quad (26)$$

where

$$A_0 = \left.\frac{\partial^2\Psi}{\partial\kappa^2}\right|_{m_0,\,\kappa_0} = -\kappa_0^{-1}U,$$

$$B_0 = \left.\frac{\partial^2\Psi}{\partial m^2}\right|_{m_0,\,\kappa_0} = -\kappa_0^{-1}U\left(\frac{\kappa_0}{m_0}\right)^2,$$

$$C_0 = \left.\frac{\partial^2\Psi}{\partial\kappa\partial m}\right|_{m_0,\,\kappa_0} = \mu^{-1} + m_0^{-1}U,$$

$$U = 2\delta\left(\frac{\kappa_0 L}{m_0 l}\right)^2\left[1 + \left(\frac{\kappa_0 L}{m_0 l}\right)^2\right]^{-2}.$$

The quantity U, which enters into expansion (26), is proportional to the small parameter δ and is finite. The quantities m_0 and κ_0 are proportional to the large parameter μ, so A_0 and B_0 are proportional to δ/μ and C_0 is proportional to $1/\mu$. As a result, expansion (26) for the phase takes the form

$$\Psi(\kappa, m) = \Psi(\kappa_0, m_0) + \mu^{-1}(\kappa - \kappa_0)(m - m_0). \quad (27)$$

According to the stationary-phase method, we reduce expression (18) for $\mathcal{R}_N$ to

$$\mathcal{R}_N = \frac{iq_0 L\exp[i\Psi(\kappa_0, m_0)]}{\sqrt{\kappa_0^2 + m_0^2\dfrac{l^2}{L^2}}} \times \int_0^{+\infty} d\kappa \int_{-\infty}^{+\infty} dm \exp[i\mu^{-1}(\kappa - \kappa_0)(m - m_0)].$$

After integration, we obtain the following final approximate formula for $\mathcal{R}_N$:

$$\mathcal{R}_N = \mathcal{A}_0 e^{i\Psi_0}, \quad (28)$$

where

$$\Psi_0 \equiv \Psi(\kappa_0, m_0) = (\mu\varphi - \Omega_0 t)(1-\xi) + m_0\delta\frac{l}{L}\arctan\frac{(\Omega t - \varphi)\mu L}{m_0 l} \quad (29)$$

is the phase of the wave and

$$\mathcal{A}_0 = i2\pi q_0 L\left[(\Omega t - \varphi)^2 + \frac{m_0^2 l^2}{\mu^2 L^2}\right]^{-1/2}\Theta(\Omega t - \varphi) \quad (30)$$

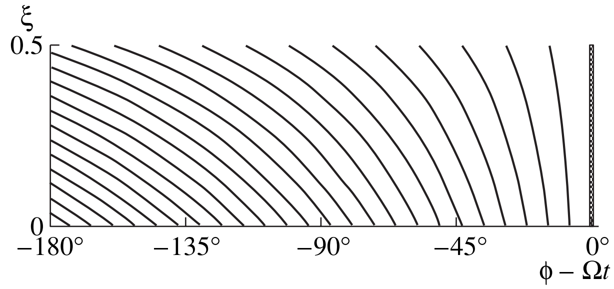

Fig. 2. Contour lines of the phase. The contours correspond to the phases that are multiples of 2π. The source, which is at $\varphi - \Omega t = 0$, moves from left to right. The ordinate is the dimensionless radial coordinate $\xi = x^1/l$.

is its amplitude. As is seen, expression (30) contains the theta function $\Theta(\Omega t - \varphi)$. This indicates that, in the approximation at hand, there is no wave field ahead of the source.

From expression (29) for the phase, we can readily determine the frequency of the wave and its lengths in the azimuthal and radial directions. These wave parameters are described by the following approximate expressions:

$$\omega = \Omega_{TN} - \Omega_0\delta\left[1 + \frac{\mu^2L^2}{m_0^2l^2}(\Omega t - \varphi)^2\right]^{-1} \tag{31}$$

for the wave frequency,

$$\lambda_\varphi = \frac{2\pi}{k_\varphi} = \frac{2\pi L}{m_0}, \quad k_\varphi = \frac{m_0}{L} \tag{32}$$

for the azimuthal wavelength and the azimuthal component of the wave vector, and

$$\lambda_r = \frac{2\pi}{k_r} = \frac{2\pi l}{\mu(\Omega t - \varphi)}, \quad k_r = \frac{\kappa_0}{l} \tag{33}$$

for the radial wavelength and the radial component of the wave vector. Consequently, the frequency of the wave and its length in the azimuthal direction depend on the radial coordinate x^1. They also depend on time: as the source moves farther and farther away, the frequency ω changes from Ω_{PN} to Ω_{TN} and λ_φ changes from $2\pi L/m_P$ to $2\pi L/m_T$. However, the time dependence is very weak because, in a cold plasma, $\Omega_{PN} \approx \Omega_{TN}$.

What is even more important is that the radial component of the wave vector depends strongly on time. For $\Omega t - \varphi \simeq 0$, the radial wave vector component is very small, $k_r \ll k_\varphi$. As the source moves away from a point at a given azimuthal position, the radial component increases (see Fig. 2), i.e., $k_r \longrightarrow \infty$ for $\Omega t - \varphi \longrightarrow \infty$. From expressions (6) and (28) it follows, however, that the wave polarization is expressed through the ratio between k_r and k_φ: $|E_\varphi/E_r| = k_\varphi/k_r$. Therefore, the generated wave has an initially poloidal polarization, $E_r \ll E_\varphi$, and, as time elapses, it is converted into a wave with a toroidal polarization, $E_r \gg E_\varphi$. As the distance from the source increases, the azimuthal component of the electric field, E_φ, decreases as $(\Omega t - \varphi)^{-1}$, while the radial component, E_r, approaches a constant value, so the amplitude of oscillations of the wave electric field $\mathbf{E}$ always remains constant. The characteristic transformation time is equal to

$$\tau = \frac{m_0 l}{\omega L} \sim m_0\omega^{-1} = \Omega^{-1}, \tag{34}$$

which corresponds to sufficiently long angular distances from the source, $\phi = \Omega\tau \sim 1$. An initially poloidal pulse-excited Alfvén wave transforms into a toroidal wave in a similar way (see, e.g., [8]) on the same time scale, $\tau \sim m/\omega$ (where m is the azimuthal wavenumber).

Let us estimate the amplitude of the generated wave. From expressions (13), (16), and (30), we obtain

$$\Phi \sim \frac{2\pi eL^2}{c^2\mu}\langle n_0\sqrt{g}T_N\rangle T_N \sim \frac{2\pi en_0L^2A^2}{c^2\mu},$$

where we have used the normalization condition for T_N, which implies that $T_N \sim A/L$ and $\langle n_0\sqrt{g}T_N\rangle \sim n_0AL$. From expression (6) we find the wave electric field,

$$E \sim \frac{\mu}{L}\Phi \sim \frac{2\pi en_0LA^2}{c^2}, \tag{35}$$

and, consequently, the wave magnetic field,

$$b \sim \frac{c}{A}E \sim \frac{2\pi en_0LA}{c}. \tag{36}$$

Using expression (36), we can determine the proton density in the drifting cloud, n_0, that is required for the generation of Alfvén waves with amplitudes consistent with the observed magnetic field strengths b in the magnetosphere:

$$n_0 \sim \frac{bc}{2\pi eLA}.$$

The waves that are generated on small azimuthal scales in the magnetosphere have amplitudes of up to $b \sim 40 \times 10^{-5}$ G, their periods and azimuthal wavenumbers being about 100 s and $m \sim 20$–100, respectively. Such waves are most often observed in the vicinity of a magnetic shell with a radius of about six Earth radii [3]. The characteristic Alfvén speed is ~1000 km/s. In order for Alfvén waves with such parameters to be generated, the proton density in the drifting cloud should be $n_0 \sim 10^{-2}$ cm^{-3}, which is much lower than the density of the background (cold) plasma. The energy ϵ of protons, which determines their drift velocity in a nonuniform magnetic field (the source velocity Ω), should be about 50 keV (the azimuthal wavenumber of the generated wave is $m \sim \omega/\Omega$). Protons with such energies are often observed simultaneously with waves having large azimuthal wavenumbers m.

5. CONCLUSIONS

The general pattern of the generation of an Alfvén wave by a moving plasma nonuniformity (a cloud of high-energy particles drifting in the azimuthal drift direction) is as follows. At a point with a given azimuthal coordinate, the wave is excited immediately after the passage of the cloud. The wave propagates in the source propagation direction and initially has a poloidal polarization. As the source moves away from that point, the wave polarization gradually changes from poloidal to toroidal. Under realistic assumptions of the particle density and energy, the calculated amplitudes of the generated waves are close to the observed ones.

The mechanism studied in the present paper makes it possible to explain the distinctive features of the azimuthally small-scale waves in the magnetosphere that were mentioned in the Introduction:

(i) The azimuthal wavenumber m is completely determined by the eigenfrequency $\omega \sim \Omega_{PN}(x^1)$ of the longitudinal wave mode and by the propagation velocity Ω of the source, $m \sim \omega/\Omega$. This explains why the observed waves have definite azimuthal wavenumbers m.

(ii) According to observations, the phase velocity of the poloidal Alfvén waves has the same direction as the proton drift.

(iii) Although the wave at a point with a given azimuthal coordinate is converted into a toroidal wave, it remains poloidally polarized over a fairly long time, $\tau \sim m/\omega$. This is the case for sufficiently long angular distances from the source, $\phi = \Omega\tau \sim 1$. Moreover, the source continues to move in the azimuthal direction, exciting a poloidal wave at newer and newer points in space; thus, in a certain sense, the wave always remains poloidal. If we take into account the wave damping (e.g., due to the finite conductivity of the ionospheric plasma), then we can see that the wave does not have enough time to be converted from a poloidal to a toroidal one, so it is a poloidally polarized wave that has the maximum amplitude.

As was mentioned in the Introduction, these characteristic features of the waves with large azimuthal wavenumbers cannot be adequately explained in terms of the bounce–drift resonance mechanism. It should be noted, however, that the wave excited by a moving plasma nonuniformity can resonantly exchange energy with the particles; as a result, the wave will undergo collisionless damping or growth. In addition, the wave field excited by the beam of injected particles can exert a ponderomotive force on it, thereby leading to the formation of a steady ring current. These questions, however, are beyond the scope of our study, which is devoted to rather early stages of the wave field evolution.

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APPENDIX

Derivation of Expression (18)

With allowance for relationships (14), solution (15), and the expression for $q(\omega, m)$ (see the notation in Eq. (12)), function (17) can be written as

$$\mathcal{R}_N = \frac{iq_0L}{2\pi}\int_{-\infty}^{+\infty} dm \int_0^{+\infty} d\kappa$$

$$\times \frac{\exp\left[im\varphi + ik(\xi - 1) + im\delta\frac{l}{L}\arctan\frac{\kappa L}{ml}\right]}{\sqrt{\kappa^2 + m^2\frac{l^2}{L^2}}}$$

$$\times \int_{-\infty}^{+\infty} dt'\Theta(t')\exp(-im\Omega t')$$

$$\times \int_{-\infty}^{+\infty} d\omega \exp[i\omega(t - t' + \kappa/\Omega_0)].$$

The last two integrals in this expressions are taken as follows:

$$\int_{-\infty}^{+\infty} dt' \ldots \int_{-\infty}^{+\infty} d\omega \ldots$$

$$= 2\pi\int_{-\infty}^{+\infty} dt'\Theta(t')\exp(-im\Omega t')\delta(t' - t + \kappa/\Omega_0)$$

$$= 2\pi\Theta(t - \kappa/\Omega_0)\exp\left(-im\Omega t + \kappa\frac{m\Omega}{\Omega_0}\right).$$

As a result, the function becomes

$$\mathcal{R}_N = iq_0L\int_{-\infty}^{+\infty} dm \int_0^{\Omega_0 t} d\kappa$$

$$\times \frac{\exp\left[m(\varphi - \Omega t) + \kappa\left(\xi - 1 + \frac{m\Omega}{\Omega_0}\right) + im\delta\frac{l}{L}\arctan\frac{\kappa L}{ml}\right]}{\sqrt{\kappa^2 + m^2\frac{l^2}{L^2}}}.$$

In the limit $\Omega_0 t \longrightarrow \infty$ (the upper limit in the integral over κ), this expression reduces to formula (18).

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