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Magnetospheric resonator for transverse-small-scale standing Alfven waves

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In a recent paper (Leonovich and Mazur, 1993) we have presented a theory of monochromatic standing Alfven waves with large azimuthal wave numbers ($m \gg 1$) in an axisymmetrical magnetosphere. It was found that an oscillation with a given frequency ω and wave number m, being a standing wave along the geomagnetic field, is a slowly running propagating wave across magnetic shells as a consequence of a specific weak dispersion caused by the curvature of the geomagnetic field lines. The oscillation is generated by external sources on a poloidal resonance surface, on which the frequency of poloidal oscillations Ω_{PN} ($N = 1, 2, \ldots$ is the harmonic number) coincides with frequency ω and is totally absorbed on the toroidal resonance surface, on which the toroidal frequency Ω_{TN} is equal to the oscillation frequency ω. While travelling between the resonance surfaces, the wave is slowly transformed from poloidal to toroidal.

The picture described here occurs if the frequencies Ω_{PN} and Ω_{TN} vary monotonically with a radial coordinate as is the case in most of the magnetosphere. But never extreme of the functions Ω_{PN} and Ω_{TN} the situation can be quite different. It is the goal of this paper to investigate such cases. It is a direct extension of the cited paper by these authors, and we shall be using the notations and symbols introduced previously without giving further explanations to them here.

Figure 1 is a schematic representation of the plots of the functions $\Omega_{PN}(x^1)$ and $\Omega_{TN}(x^1)$ which qualitatively represent their variation in the dayside part of the magnetosphere. Points of intersection of each of the horizontal straight lines on this plot with curves $\Omega_{PN}(x^1)$ and $\Omega_{TN}(x^1)$ define the coordinates of the turning point of the mode of a corresponding frequency, and the intersection with curve $\Omega_{PN}(x^1)$ and with curve $\Omega_{TN}(x^1)$ specifies, respectively, a regular (poloidal) and singular (toroidal) turning point. The transparency region of the mode is defined by the inequality $\Omega_{PN}(x^1) < \omega < \Omega_{TN}(x^1)$. Based on these considerations it is easy to analyse qualitatively the possibilities that present themselves.

The most typical case is the one when the transparency region of the mode is bounded, on the one hand, by a regular turning point x^1_{PN} and, on the other, by a singular turning point x^1_{TN}. If $\Omega_{PN}(x^1)$ and $\Omega_{TN}(x^1)$ decrease, as is the case with most of the magnetosphere, then $x^1_{TN} > x^1_{PN}$. It is this possibility (cases 1 in the figure correspond to it) that has been investigated in detail in Leonovich and Mazur (1993).

There exist also fundamentally different possibilities, however. To case 2 there corresponds the solution, whose transparency region is bounded on both sides by regular turning points. Since in this case the singular turning

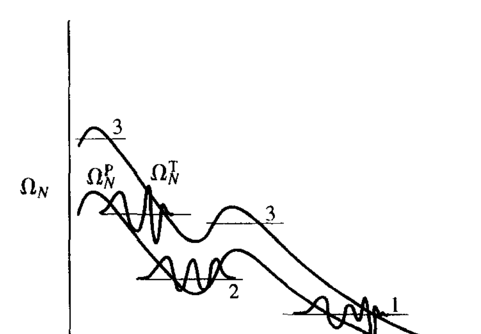

Fig. 1. Schematic plots of the functions $\Omega^P_N \equiv \Omega_{PN}(x^1)$ and $\Omega^T_N \equiv \Omega_{TN}(x^1)$. The role of the coordinate x^1 is played by the McIlwain parameter L. Extrema of these functions occur at $L \sim 1.3$ as well as on the inner and outer edges of plasmapause ($L \sim 4$–5). Points of intersection of the curves with the horizontal straight lines determine the position of the poloidal and toroidal resonance surfaces for a given frequency ω. In case 1 the transparency region lies between the poloidal and the toroidal turning points. In case 2 an Alfven resonator takes place. In case 3 the presence of two toroidal turning points makes the existence of an oscillation impossible

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point, at which a total absorption of the wave energy occurs, is absent, eigen-oscillations are possible, i.e. oscillations without sources. Frequencies of such eigen-oscillations are determined by the relevant quantization condition. If the WKB approximation is applicable, then it represents the well-known Bohr–Sommerfeld quantum condition

$$\oint k_{1N}(x^1,\omega)\,\mathrm{d}x^1 = 2\pi(n+1/2) \tag{1}$$

where $n = 0, 1, 2, \ldots$ is the transverse wave number. The domain where there exist solutions locked in coordinate x^1 by the turning points and in longitudinal coordinate by the ionospheric ends can be called the Alfvenic resonator. The quantum condition determines the spectrum of its frequencies $\omega = \omega_{Nn}$ that depend on two wave numbers, N and n. In the presence of a source, a forced oscillation with arbitrary frequency ω is possible. Note that the possibility that such a resonator for fast magnetosonic waves can exist in the magnetosphere, is rather well established to date (Gul'elmi, 1970, 1972; Zhu and Kivelson, 1989). The possibility of such a resonator for Alfven waves was discussed for the first time by Gul'elmi and Polyakov (1983) and Leonovich *et al.* (1983).

We shall now obtain explicit formulas for Alfvenic resonator oscillations, whose frequencies are close to the minimum of the function $\Omega_{PN}(x^1)$. Near this minimum we put

$$\Omega_{PN}(x^1) = \bar{\Omega}_{PN}\left[1+\frac{(x^1-\bar{x}^1_{PN})^2}{2a^2_{PN}}\right]. \tag{2}$$

Consider the oscillations with frequencies that satisfy the condition $\omega-\bar{\Omega}_{PN} \ll \Delta\Omega_N$. These oscillations are close to the poloidal mode throughout the region of their localization, and to describe them, it is possible to use equation (82) of the work of Leonovich and Mazur (1993):

$$W^P_N\nabla^2_1 U_N + k^2_2[(\omega+i\gamma_N)^2-\Omega^2_{PN}]U_N = k^2_2 I_\parallel. \tag{3}$$

Introduce the dimensionless variable

$$\zeta = (x^1-\bar{x}^1_{PN})/\bar{\lambda}_{PN},$$

by defining the constant $\bar{\lambda}_{PN}$ by the equality

$$\bar{\lambda}_{PN} = \left(\frac{W^P_N a_{PN}}{k^2_2\omega^2}\right)^{1/4}.$$

This constant represents the typical scale of the solution in coordinate x^1. On the order of magnitude, $\bar{\lambda}_{PN} \sim \alpha^{1/2}a/m^{1/2}$. For a minimum on the inner plasmapause edge and for the wave with $N = 1$ and $m = 20$–50, a numerical estimation yields $\bar{\lambda}_{PN} \sim 500$–$1000$ km. Using the new variable equation (3) can be represented as

$$\frac{\mathrm{d}^2U_N}{\mathrm{d}\zeta^2}+(\sigma-\zeta^2)U_N = \frac{a^2_{PN}}{\bar{\lambda}^2_{PN}}\frac{I_\parallel}{\omega^2} \tag{4}$$

where

$$\sigma = \frac{a^2_{PN}}{\bar{\lambda}^2_{PN}}\frac{(\omega+i\gamma_N)^2-\bar{\Omega}^2_{PN}}{\omega^2}.$$

If it is assumed that extraneous currents in the ionosphere are absent, i.e. the right-hand side of equation (4) is zero, then together with the requirement for the boundedness of the solution for $\zeta \to \pm\infty$, we arrive at the well-known problem for eigenvalues for a quantum oscillator. Its solutions are

$$\sigma = \sigma_n \equiv 2n+1, \quad U_N = Cy_n(\zeta) \tag{5}$$

where

$$y_n = \pi^{-1/4}2^{-n/2}(n!)^{-1/2}H_n(\zeta)\exp(-\zeta^2/2)$$

and $H_N(\zeta)$ are the Hermitian polynomials. The functions $y_n(\zeta)$ satisfy the orthonormalization condition:

$$\int_{-\infty}^{\infty} y_n(\zeta)y_{n'}(\zeta)\,\mathrm{d}\zeta = \delta_{nn'}.$$

The solution (5) defines the eigenmodes and eigenfrequencies of the Alfvenic resonator:

$$\Phi = CP_N(x^1,l)\,y_n\!\left(\frac{x^1-x^1_{PN}}{\bar{\lambda}_{PN}}\right); \quad \omega = \omega_{Nn}-i\gamma_N, \tag{6}$$

$$\omega_{Nn} = \bar{\Omega}_{PN}\left[1+\frac{\bar{\lambda}^2_{PN}}{a^2_{PN}}(n+\tfrac{1}{2})\right].$$

Note that the eigenfrequencies in (6) can be obtained from the quantum condition (1), and even for small $n \sim 1$, a well-known fact for a quantum oscillator.

It is easy to obtain the solution of the inhomogeneous equation (4) with the help of Green's function by its left-hand side:

$$G(\zeta,\zeta',\sigma) = \sum_{n=0}^{\infty}\frac{y_n(\zeta)y_n(\zeta')}{\sigma-\sigma_n}$$

which satisfies the equation

$$\frac{\mathrm{d}^2G}{\mathrm{d}\zeta^2}+(\sigma-\zeta^2)G = \delta(\zeta-\zeta').$$

We have

$$\Phi = U_NP_N = P_N(x^1,l)\sum_{n=0}^{\infty}\frac{c_n}{(\omega+i\gamma_N)^2-\omega^2_{Nn}}\,y_n\!\left(\frac{x^1-\bar{x}^1_{PN}}{\bar{\lambda}_{PN}}\right) \tag{7}$$

where

$$c_n = \int_{-\infty}^{\infty} y_n(\zeta)I_\parallel(\bar{x}^1_{PN}+\bar{\lambda}_{PN}\zeta)\,\mathrm{d}\zeta.$$

The solution represents a superposition of eigenmodes of the Alfvenic resonator. If frequency ω is close to one of the eigenfrequencies ω_{Nn} so that the difference $\omega-\omega_{Nn}$ in absolute value is much smaller than the splitting of the eigenfrequencies $(\bar{\lambda}^2_{PN}/a^2_{PN})\bar{\Omega}_{PN}$, and the damping decrement γ_N is also much smaller than this splitting, then one term corresponding to the resonance eigenfrequency will be dominant in the sum (7).

As far as cases 3 in Fig. 1 are concerned, however, simple considerations show that there exist no corresponding solutions. Indeed, transparency regions of such solutions would be bounded on both sides by singular turning points. We know that, in the presence of a singular turning point, the solution that is bounded in the opacity region, is a wave travelling toward that point. But a wave travelling toward two opposite turning points cannot exist.

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