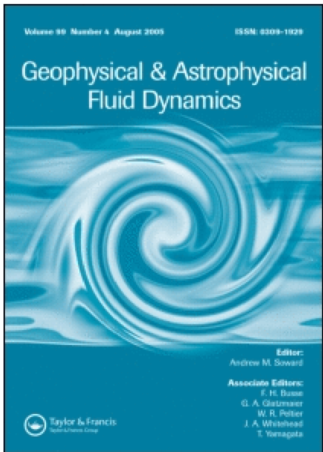

Geophysical & Astrophysical Fluid Dynamics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713642804

The mean electro-motive force and current helicity under the influence of rotation, magnetic field and shear

V. V. Pipin [a]
[a] Institute Solar-Terrestrial Physics, Irkutsk, Russia

First Published on: 21 December 2007

To cite this Article: Pipin, V. V. (2007) 'The mean electro-motive force and current helicity under the influence of rotation, magnetic field and shear', Geophysical & Astrophysical Fluid Dynamics, 102:1, 21 — 49

To link to this article: DOI: 10.1080/03091920701374772
URL: http://dx.doi.org/10.1080/03091920701374772

Geophysical and Astrophysical Fluid Dynamics,
Vol. 102, No. 1, February 2008, 21–49

The mean electro-motive force and current helicity under the influence of rotation, magnetic field and shear

V. V. PIPIN*

Institute Solar-Terrestrial Physics, Irkutsk, Russia

(*Received 18 April 2006; in final form 20 March 2007*)

The mean electromotive force (MEMF) in a rotating stratified magnetohydrodynamical turbulence is studied. Our study is based on the mean-field magnetohydrodynamics framework and τ approximation. We compute the effects of the large-scale magnetic fields (LSMF), global rotation and large-scale shear flow on the different parts of the MEMF (such as α-effect, turbulent diffusion, turbulent transport, etc.) in an explicit form. The influence of the helical magnetic fluctuations which stem from the small-scale dynamo is taken into account, as well. In the article, we derive the equation governing the current helicity evolution. It is shown that the joint effect of the differential rotation and magnetic fluctuations in the stratified media can be responsible for the generation, maintenance and redistribution of the current helicity. The implication of the obtained results to astrophysical dynamos is considered.

Keywords: Mean electro-motive force; Current helicity; Magnetic fluctuations

1. Introduction

The mean-field magnetohydrodynamics presents one of the most powerful tools for exploring the nature of the large-scale magnetic activity in cosmic bodies (Moffatt 1978, Parker 1979, Krause and Rädler 1980). It is widely believed that there magnetic field generation is governed by interplay between turbulent motions of electrically conductive fluids and global rotation. The growth and evolution of the large-scale magnetic fields (LSMF) in cosmic plasma depends on the mean electromotive force, $\boldsymbol{\mathcal{E}} = \langle \mathbf{u} \times \mathbf{b} \rangle$, which is given by the correlation between the fluctuating components of the velocity field of plasma, $\mathbf{u}$, and the fluctuating magnetic fields, $\mathbf{b}$.

The global rotation, stratification and the strong LSMF can substantially modify the structure and amplitude of the mean electromotive force (hereafter, MEMF) leading to the rich variety of the turbulence effects driving the evolution of the LSMF in cosmic bodies, e.g., the α-effect (Roberts and Soward 1975, Moffatt 1978, Krause and Rädler 1980, Parker 1979, Rüdiger and Kichatinov 1993), the rotationally-induced anisotropy of turbulent diffusion and effective drift of LSMF (Roberts and Soward 1975, Krause and Rädler 1980, Kichatinov *et al.* 1994), etc. Broadly speaking,

*Corresponding author. Email: pip@iszf.irk.ru

Geophysical and Astrophysical Fluid Dynamics
ISSN 0309-1929 print/ISSN 1029-0419 online © 2008 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/03091920701374772

the nonlinear effects of the small-scale Lorentz forces on the MEMF and LSMF evolution stem from two sources. One is driven by perturbations of the LSMF due to turbulent motions and another is due to magnetic fluctuations, which stem from the small-scale dynamo. For the time being, the role of the small-scale dynamo in the LSMF evolution is poorly understood. Numerous contributions to this subject can be found in the modern literature, (e.g., Moffatt 1978, Frisch *et al.* 1975, Pouquet *et al.* 1975, Brandenburg and Subramanian 2005). According to the mentioned studies the most important effect of the growing magnetic fluctuations on the LSMF evolution is caused by the helical part of magnetic fluctuations. The magnetic helicity conservation law, if applied to the mean-field magnetohydrodynamics, requires that the amount of helicity contained in the LSMF (controlled mostly by α-effect) should be roughly the same and opposite in sign to its counterpart in the small scales, (see Kleeorin and Ruzmaikin 1982). In this way the helical part of magnetic fluctuations, which is excited both due to shredding the LSMF by turbulent motions and due to small-scale dynamo, effectively saturates the generation of the LSMF by α-effect (Vainshtein and Kitchatinov 1983, Brandenburg 2001, Field and Blackman 2002, Blackman and Brandenburg 2002). Further discussions on this subject can be found in above cited articles. Their main lesson is that the construction of the realistic mean-field dynamo theory requires the evolution of the small-scale magnetic (or current-) helicity to be taken into account.

Currently, there are two basic schemes for computing the MEMF of turbulent fields. One is the quasi-linear approximation (the same approximation is called the FOSA or SOCA in literature). A comprehensive discussion about its applicability and validity in astrophysics can be found in (Moffatt 1978, Parker 1979, Krause and Rädler 1980 and Brandenburg and Subramanian 2005). This scheme remains one of the main tools of the mean-field magnetohydrodynamics. However, one of unfortunate problem of SOCA is that the contribution of the magnetic fluctuations (and the corresponding magnetic helicity) driven by the small-scale dynamo is hardly possible to include in the theory in self-consistent way. The third order closure scheme based on τ-approximation (Orszag 1970, Vainshtein and Kitchatinov 1983, Rädler *et al.* 2003, Brandenburg and Subramanian 2005) gives a chance to consider, roughly, the effects of the small-scale dynamo on the MEMF. Following Brandenburg and Subramanian (2005) (hereafter BS05), I will call it MTA (minimal tau approximation). Different kinds of this approximation are used in the literature, see (Vainshtein 1983, Vainshtein and Kitchatinov 1983, Rädler *et al.* 2003, Brandenburg and Subramanian 2005, Rogachevskii and Kleeorin 2003, Blackman and Field 2002, Field and Blackman 2002). In the article we follow teh procedure described in BS05. Furthermore, the variant of tau approximation with a scale-independent relaxation time, τ, is applied. For this reason, some results obtained in the article can be different form those given in Rogachevskii and Kleeorin (2003, 2004b,a), Rädler *et al.* (2003).

The main purpose of this article is to compute the MEMF *via* MTA taking into account the influence of the global rotation and LSMF on the turbulence. The stratification of the medium and the large-scale shear are taken into account as well. The influence of rotation, LSMF and uniform shear on the different parts of the MEMF (such as α-effect, turbulent diffusion, turbulent transport and etc.) is explicitly defined via factors describing the efficiency of rotational and LSMF feedback on the turbulent flows. The influence of rotation is measured by the Coriolis number, $\Omega^* = 2\Omega\tau_c$, where Ω is the solid body angular velocity and τ_c – the typical correlation time of turbulent flows. The influence of LSMF is measured by $\beta = \bar{B}/(u_c\sqrt{\mu\rho})$, where $\bar{B}$ is the strength of the LSMF, u_c is a typical rms velocity of turbulent flows and μ, ρ are the magnetic permeability and the density of the media, respectively. Following the basic approach developed in above cited articles we derive the equations governing the evolution of the current helicity both in rotating and in magnetized turbulent flows with imposed uniform shear.

The article is structured as follows. In the next section we shortly outline the basic equations, assumptions and the computational scheme for derivation of the MEMF and the evolutionary equation for current helicity. Section 3 is devoted to the results of calculations of the MEMF for different situations (slow rotation, strong LSMF, vice versa and etc.). In section 4 we derive the evolutionary equation for current helicity. In section 5 we summarize the main results of the article.

2. Basic equations

In the spirit of the mean-field magnetohydrodynamics, we split the physical quantities of the turbulent conducting fluid into the mean and randomly fluctuating part with the mean part defined as the ensemble average of the random fields. One assumes the validity of the Reynolds rules. The magnetic field $\mathbf{B}$ and velocity of motions $\mathbf{V}$ are decomposed as follows: $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$, $\mathbf{V} = \overline{\mathbf{V}} + \mathbf{u}$. Hereafter, everywhere, we use the small letters for the fluctuating part of the fields and capital letters with a bar above for the mean fields. The angle brackets are used for the ensemble average of products. Following the lines of two-scale approximation (Roberts and Soward 1975, Krause and Rädler 1980) we assume that the mean fields vary over the much larger scales (both in time and in space) than the fluctuating fields. The average effect of the MHD-turbulence on the LSMF evolution is described by the MEMF, $\boldsymbol{\mathcal{E}} = \langle \mathbf{u} \times \mathbf{b} \rangle$. The governing equations for fluctuating magnetic field and velocity are written in a rotating coordinate system as follows:

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times \left(\mathbf{u} \times \overline{\mathbf{B}} + \overline{\mathbf{V}} \times \mathbf{b}\right) + \eta\nabla^2\mathbf{b} + \mathfrak{G}, \tag{1}$$

$$\begin{aligned}\frac{\partial m_i}{\partial t} + 2(\boldsymbol{\Omega} \times \mathbf{m})_i = &-\nabla_i\left(p - \frac{2}{3}(\mathbf{G}\cdot\mathbf{m})\nu + \frac{(\mathbf{b}\cdot\overline{\mathbf{B}})}{2\mu}\right) + \nu\Delta m_i + \nu(\mathbf{G}\cdot\nabla)m_i \\ &+ \frac{1}{\mu}\nabla_j\left(\overline{B}_j b_i + \overline{B}_i b_j\right) - \nabla_j\left(\overline{V}_j m_i + \overline{V}_i m_j\right) + f_i + \mathfrak{F}_i,\end{aligned} \tag{2}$$

where $\mathfrak{G}$, $\mathfrak{F}$ are nonlinear contributions of fluctuating fields, $\mathbf{m} = \bar{\rho}\mathbf{u}$, $\mathbf{G} = \nabla \log \bar{\rho}$ is the density stratification scale of the media, p is the fluctuating pressure, $\boldsymbol{\Omega}$ is the angular velocity responsible for the Coriolis force, $\overline{\mathbf{V}}$ is mean flow which is a weakly variable in space, $\mathbf{f}$ is the random force driving the turbulence.

To compute $\boldsymbol{\mathcal{E}}$ it is convenient to write equations (1) and (2) in Fourier space:

$$\begin{aligned}\left(\frac{\partial}{\partial t} + \eta z^2\right)\hat{b}_j = {} & \mathrm{i}z_l \int\left[\widehat{m}_j(\mathbf{z}-\mathbf{q})\left(\widehat{\frac{\overline{B}_l}{\rho}}\right)(\mathbf{q}) - \widehat{m}_l(\mathbf{z}-\mathbf{q})\left(\widehat{\frac{\overline{B}_j}{\rho}}\right)(\mathbf{q})\right]\mathrm{d}\mathbf{q} \\ &+ \mathrm{i}z_l \int\left[\widehat{b}_l(\mathbf{z}-\mathbf{q})\hat{\overline{V}}_j(\mathbf{q}) - \widehat{b}_j(\mathbf{z}-\mathbf{q})\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q} + \widehat{\mathfrak{G}}_j,\end{aligned} \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\nu z^2+\mathrm{i}\nu(\mathbf{G}\cdot\mathbf{z})\right)\hat{m}_i=\hat{f}_i+\hat{\mathfrak{F}}_i-2(\boldsymbol{\Omega}\cdot\hat{\mathbf{z}})(\hat{\mathbf{z}}\times\hat{\mathbf{m}})_i$$
$$-\mathrm{i}\pi_{if}(\mathbf{z})z_l\int\left[\widehat{m}_l(\mathbf{z}-\mathbf{q})\hat{\overline{V}}_f(\mathbf{q})+\widehat{m}_f(\mathbf{z}-\mathbf{q})\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$+\frac{\mathrm{i}}{\mu}\pi_{if}(\mathbf{z})z_l\int\left[\widehat{b}_l(\mathbf{z}-\mathbf{q})\hat{\overline{B}}_f(\mathbf{q})+\widehat{b}_f(\mathbf{z}-\mathbf{q})\hat{\overline{B}}_l(q)\right]\mathrm{d}\mathbf{q}, \tag{4}$$

where the turbulent pressure was excluded from (2) by convolution with the projection tensor $\pi_{ij}(\mathbf{z})=\delta_{ij}-\hat{z}_i\hat{z}_j$, δ_{ij} is the Kronecker symbol and $\hat{\mathbf{z}}$ is a unit wave vector. The equations for the second-order moments which make contributions to the MEMF can be found from (3,4). As the preliminary step we write the equations for the second-order products of the fluctuating fields, and make the ensemble averaging of them:

$$\frac{\partial}{\partial t}\left\langle\hat{m}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle=Th^{\varkappa}_{ij}(\mathbf{z},\mathbf{z}')-\left(\eta z'^2+\nu z^2+\mathrm{i}\nu(\mathbf{G}\cdot\mathbf{z})\right)\left\langle\hat{m}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle$$
$$\times\,\mathrm{i}z'_l\int\left[\langle\hat{m}_i(\mathbf{z})\hat{m}_j(\mathbf{z}'-\mathbf{q})\rangle\left(\frac{\hat{\overline{B}}_l}{\rho}\right)(\mathbf{q})-\langle\hat{m}_i(\mathbf{z})\hat{m}_l(\mathbf{z}'-\mathbf{q})\rangle\left(\frac{\hat{\overline{B}}_j}{\rho}\right)(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$-2(\boldsymbol{\Omega}\cdot\hat{\mathbf{z}})\varepsilon_{iln}\hat{z}_l\left\langle\hat{m}_n(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle+\mathrm{i}z'_l\int\left[\left\langle\hat{m}_i(\mathbf{z})\widehat{b}_l(\mathbf{z}'-\mathbf{q})\right\rangle\hat{\overline{V}}_j(\mathbf{q})\right.$$
$$\left.-\left\langle\hat{m}_i(\mathbf{z})\widehat{b}_j(\mathbf{z}'-\mathbf{q})\right\rangle\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$-\mathrm{i}\pi_{if}(\mathbf{z})z_l\int\left[\left\langle\widehat{m}_l(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\hat{\overline{V}}_f(\mathbf{q})+\left\langle\widehat{m}_f(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$+\frac{\mathrm{i}}{\mu}z_l\pi_{if}(\mathbf{z})\int\left[\left\langle\widehat{b}_l(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\overline{B}_f(\mathbf{q})+\left\langle\widehat{b}_f(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\overline{B}_l(q)\right]\mathrm{d}\mathbf{q}, \tag{5}$$

$$\frac{\partial}{\partial t}\langle\hat{m}_i(\mathbf{z})\hat{m}_j(\mathbf{z}')\rangle=-2(\boldsymbol{\Omega}\cdot\hat{\mathbf{z}})\varepsilon_{iln}\hat{z}_l\langle\hat{m}_n(\mathbf{z})\hat{m}_j(\mathbf{z}')\rangle-2(\boldsymbol{\Omega}\cdot\hat{\mathbf{z}}')\varepsilon_{jln}\hat{z}'_l\langle\hat{m}_i(\mathbf{z})\hat{m}_n(\mathbf{z}')\rangle$$
$$-\mathrm{i}\pi_{if}(\mathbf{z})z_l\int\left[\langle\widehat{m}_l(\mathbf{z}-\mathbf{q})\hat{m}_j(\mathbf{z}')\rangle\hat{\overline{V}}_f(\mathbf{q})+\langle\widehat{m}_f(\mathbf{z}-\mathbf{q})\hat{m}_j(\mathbf{z}')\rangle\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$-\mathrm{i}\pi_{jf}(\mathbf{z}')z'_l\int\left[\langle\hat{m}_i(\mathbf{z})\widehat{m}_l(\mathbf{z}-\mathbf{q})\rangle\hat{\overline{V}}_f(\mathbf{q})+\langle\hat{m}_i(\mathbf{z})\widehat{m}_f(\mathbf{z}-\mathbf{q})\rangle\hat{\overline{V}}_l(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$+\frac{\mathrm{i}}{\mu}\pi_{if}(\mathbf{z})z_l\int\left[\left\langle\widehat{b}_l(\mathbf{z}-\mathbf{q})\hat{m}_j(\mathbf{z}')\right\rangle\hat{\overline{B}}_f(\mathbf{q})+\left\langle\widehat{b}_f(\mathbf{z}-\mathbf{q})\hat{m}_j(\mathbf{z}')\right\rangle\hat{\overline{B}}_l(q)\right]\mathrm{d}\mathbf{q}$$
$$+\frac{\mathrm{i}}{\mu}\pi_{jf}(\mathbf{z}')z'_l\int\left[\left\langle\hat{m}_i(\mathbf{z})\widehat{b}_l(\mathbf{z}-\mathbf{q})\right\rangle\hat{\overline{B}}_f(\mathbf{q})+\left\langle\hat{m}_i(\mathbf{z})\widehat{b}_f(\mathbf{z}-\mathbf{q})\right\rangle\hat{\overline{B}}_l(q)\right]\mathrm{d}\mathbf{q}$$
$$+Th^{\nu}_{ij}(\mathbf{z},\mathbf{z}')-\nu\left(z'^2+z^2+i(\mathbf{G}\mathbf{z})+i(\mathbf{G}\mathbf{z}')\right)\langle\hat{m}_i(\mathbf{z})\hat{m}_j(\mathbf{z}')\rangle, \tag{6}$$

$$\frac{\partial}{\partial t}\left\langle\hat{b}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle=Th^{h}_{ij}(\mathbf{z},\mathbf{z}')-\left(\eta z'^2+\eta z^2\right)\left\langle\hat{b}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle$$
$$+\mathrm{i}z'_l\int\left[\left\langle\hat{b}_i(\mathbf{z})\hat{m}_j(\mathbf{z}'-\mathbf{q})\right\rangle\left(\frac{\hat{\overline{B}}_l}{\rho}\right)(\mathbf{q})-\left\langle\hat{b}_i(\mathbf{z})\hat{m}_l(\mathbf{z}'-\mathbf{q})\right\rangle\left(\frac{\hat{\overline{B}}_j}{\rho}\right)(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$
$$+\mathrm{i}z_l\int\left[\left\langle\hat{m}_i(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\left(\frac{\hat{\overline{B}}_l}{\rho}\right)(\mathbf{q})-\left\langle\hat{m}_l(\mathbf{z}-\mathbf{q})\hat{b}_j(\mathbf{z}')\right\rangle\left(\frac{\hat{\overline{B}}_i}{\rho}\right)(\mathbf{q})\right]\mathrm{d}\mathbf{q}$$

$$+\mathrm{i}z'_l\int\Big[\Big\langle\hat{b}_i(\mathbf{z})\widehat{b_l}(\mathbf{z}'-\mathbf{q})\Big\rangle\hat{\overline{V}}_j(\mathbf{q})-\Big\langle\hat{b}_i(\mathbf{z})\widehat{b_j}(\mathbf{z}'-\mathbf{q})\Big\rangle\hat{\overline{V}}_l(\mathbf{q})\Big]\mathrm{d}\mathbf{q}$$
$$+\mathrm{i}z_l\int\Big[\Big\langle\hat{b}_l(\mathbf{z}-\mathbf{q})\widehat{b_j}(\mathbf{z}')\Big\rangle\hat{\overline{V}}_i(\mathbf{q})-\Big\langle\hat{b}_i(\mathbf{z}-\mathbf{q})\widehat{b_j}(\mathbf{z}')\Big\rangle\hat{\overline{V}}_l(\mathbf{q})\Big]\mathrm{d}\mathbf{q},\tag{7}$$

where, the terms $Th_{ij}^{(\varkappa,v,h)}$ involve the third-order moments of fluctuating fields and second-order moments of them with the forcing term.

To proceed further, it is convenient to introduce some notations which are used in the literature. The double Fourier transformation of an ensemble average of two fluctuating quantities, say f and g, taken at equal times and at the different positions $\mathbf{x}$, $\mathbf{x}'$, is given by

$$\langle f(\mathbf{x})g(\mathbf{x}')\rangle=\int\int\Big\langle\hat{f}(\mathbf{z})\hat{g}(\mathbf{z}')\Big\rangle e^{\mathrm{i}(\mathbf{z}\cdot\mathbf{x}+\mathbf{z}'\cdot\mathbf{x}')}\mathrm{d}^3\mathbf{z}\mathrm{d}^3\mathbf{z}'.\tag{8}$$

Let us define the "fast" spatial variable $\mathbf{r}$ by the relative difference of $\mathbf{x},\mathbf{x}'$ coordinates, $\mathbf{r}=\mathbf{x}-\mathbf{x}'$. The "slow" spatial variable $\mathbf{R}$ is $\mathbf{R}=(\mathbf{x}+\mathbf{x}')/2$. Then, equation (8) can be written in the form

$$\langle f(\mathbf{x})g(\mathbf{x}')\rangle=\int\int\Big\langle\hat{f}\Big(\mathbf{k}+\frac{1}{2}\mathbf{K}\Big)\hat{g}\Big(-\mathbf{k}+\frac{1}{2}\mathbf{K}\Big)\Big\rangle e^{\mathrm{i}(\mathbf{K}\cdot\mathbf{R}+\mathbf{k}\cdot\mathbf{r})}\mathrm{d}^3\mathbf{K}\mathrm{d}^3\mathbf{k},\tag{9}$$

where I have introduced two wave vectors: $\mathbf{k}=(\mathbf{z}-\mathbf{z}')/2$ and $\mathbf{K}=\mathbf{z}+\mathbf{z}'$. Following BS05, we define the correlation function of $\hat{\mathbf{f}}$ and $\hat{\mathbf{g}}$ obtained from (7) by integration with respect to $\mathbf{K}$:

$$\Phi\Big(\hat{f},\hat{g},\mathbf{k},\mathbf{R}\Big)=\int\Big\langle\hat{f}\Big(\mathbf{k}+\frac{1}{2}\mathbf{K}\Big)\hat{g}\Big(-\mathbf{k}+\frac{1}{2}\mathbf{K}\Big)\Big\rangle e^{\mathrm{i}(\mathbf{K}\cdot\mathbf{R})}\mathrm{d}^3\mathbf{K}.\tag{10}$$

For further convenience I introduce the following notations for the second order correlations of momentum density, magnetic fluctuations and the cross-correlations of momentum and magnetic fluctuations,

$$\hat{v}_{ij}(\mathbf{k},\mathbf{R})=\Phi(\hat{m}_i,\hat{m}_j,\mathbf{k},\mathbf{R}),\bar{\rho}^2\langle u^2\rangle(\mathbf{R})=\int\hat{v}_{ii}(\mathbf{k},\mathbf{R})\mathrm{d}^3\mathbf{k},\tag{11}$$

$$\hat{h}_{ij}(\mathbf{k},\mathbf{R})=\Phi(\hat{b}_i,\hat{b}_j,\mathbf{k},\mathbf{R}),\langle b^2\rangle(\mathbf{R})=\int\hat{h}_{ii}(\mathbf{k},\mathbf{R})\mathrm{d}^3\mathbf{k},\tag{12}$$

$$\hat{\varkappa}_{ij}(\mathbf{k},\mathbf{R})=\Phi(\hat{m}_i,\hat{b}_j,\mathbf{k},\mathbf{R}),\bar{\rho}\mathcal{E}_i(\mathbf{R})=\varepsilon_{ijk}\int\hat{\varkappa}_{jk}(\mathbf{k},\mathbf{R})\mathrm{d}^3\mathbf{k}.\tag{13}$$

Let us now return to equations (5)–(7). As the first step, we approximate the $Th_{ij}^{(\varkappa,v,h)}$ terms by the corresponding τ relaxation terms of the second-order contributions,

$$Th_{ij}^{(\varkappa)}\to-\frac{\Big\langle\hat{m}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\Big\rangle}{\tau_c},\tag{14}$$

$$Th^{(v)}_{ij} \to -\frac{\langle \hat{m}_i(\mathbf{z})\hat{m}_j(\mathbf{z}')\rangle - \langle \hat{m}_i(\mathbf{z})\hat{m}_j(\mathbf{z}')\rangle^{(0)}}{\tau_c}, \tag{15}$$

$$Th^{(h)}_{ij} \to -\frac{\left\langle \hat{b}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle - \left\langle \hat{b}_i(\mathbf{z})\hat{b}_j(\mathbf{z}')\right\rangle^{(0)}}{\tau_c}, \tag{16}$$

where the superscript (0) denotes the moments of the background turbulence. Here, τ_c is independent on $\mathbf{k}$ and it is independent on the mean fields as well. Furthermore, for the sake of simplicity, we restrict ourselves to the high Reynolds numbers limit and discard the microscopic diffusion terms. As the next step we make the Taylor expansion with respect to the "slow" variables and take the Fourier transformation, (10), about them. The details of this procedure can be found in BS05. In result, we obtain equations for the second order correlations of momentum density, magnetic fluctuations and the cross-correlations of momentum and magnetic fluctuations,

$$\begin{aligned}
\frac{\partial \hat{\varkappa}_{ij}}{\partial t} &= -\mathrm{i}\left(\overline{\mathbf{B}}\cdot\mathbf{k}\right)\left(\frac{\hat{v}_{ij}}{\rho} - \frac{\hat{h}_{ij}}{\mu}\right) + \frac{\left(\overline{\mathbf{B}}\cdot\nabla\right)}{2}\left(\frac{\hat{v}_{ij}}{\rho} + \frac{\hat{h}_{ij}}{\mu}\right) + \frac{\left(\overline{\mathbf{B}}\cdot\mathbf{k}\right)}{2\rho}G_s\frac{\partial \hat{v}_{ij}}{\partial k^s} - \frac{\left(\mathbf{G}\cdot\overline{\mathbf{B}}\right)}{2\rho}\hat{v}_{ij} \\
&+ \frac{1}{\rho}G_l\hat{v}_{il}B_j + \frac{\hat{h}_{lj}\overline{B}_{i,l}}{\mu} - \frac{\hat{v}_{il}\overline{B}_{j,l}}{\rho} - \frac{k_l\overline{B}_{l,f}}{2}\frac{\partial}{\partial k_f}\left[\frac{\hat{v}_{ij}}{\rho} + \frac{\hat{h}_{ij}}{\mu}\right] - \frac{2}{\mu}\hat{k}_i\hat{k}_f\overline{B}_{f,l}\hat{h}_{lj} \\
&+ \overline{V}_{j,l}\hat{\varkappa}_{il} - \overline{V}_{i,l}\hat{\varkappa}_{lj} + 2\hat{k}_i\hat{k}_f\hat{\varkappa}_{lj}\overline{V}_{f,l} + k_l\overline{V}_{f,l}\frac{\partial \hat{\varkappa}_{ij}}{\partial k_f} - \frac{\hat{\varkappa}_{ij}}{\tau_c} - 2\left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\hat{k}_p\varepsilon_{ipl}\hat{\varkappa}_{lj} \\
&- 2\frac{\mathrm{i}}{k}\left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\hat{k}_p\varepsilon_{ipl}\left(\hat{\mathbf{k}}\cdot\nabla\right)\hat{\varkappa}_{lj} + \frac{\mathrm{i}}{k}\varepsilon_{ipl}\left(\left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\nabla_p\hat{\varkappa}_{lj} + \hat{k}_p(\boldsymbol{\Omega}\cdot\nabla)\hat{\varkappa}_{lj}\right),
\end{aligned} \tag{17}$$

$$\begin{aligned}
\frac{\partial \hat{v}_{ij}}{\partial t} &= -2\left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\hat{k}_p\left(\varepsilon_{ipl}\hat{v}_{lj} + \varepsilon_{jpl}\hat{v}_{il}\right) - \frac{\hat{v}_{ij} - \hat{v}^{(0)}_{ij}}{\tau_c} - \hat{v}_{lj}\overline{V}_{i,l} - \hat{v}_{il}\overline{V}_{j,l} + 2\hat{k}_f\overline{V}_{f,l}\left(\hat{k}_i\hat{v}_{lj} + \hat{k}_j\hat{v}_{il}\right) \\
&+ k_l\overline{V}_{f,l}\frac{\partial \hat{v}_{ij}}{\partial k_f} - \mathrm{i}\left(\overline{\mathbf{B}}\cdot\mathbf{k}\right)\left(\hat{\varkappa}_{ij} - \hat{\varkappa}^*_{ji}\right) + \frac{1}{2}\overline{B}_l\left(\hat{\varkappa}_{ij,\,l} + \hat{\varkappa}^*_{ji,\,l}\right) + \overline{B}_{i,l}\hat{\varkappa}^*_{jl} + \overline{B}_{j,l}\hat{\varkappa}_{i,l} \\
&- 2\hat{k}_f\overline{B}_{f,l}\left(\hat{k}_i\hat{\varkappa}^*_{jl} + \hat{k}_j\hat{\varkappa}_{i,l}\right) - \frac{\overline{B}_{l,f}}{2}k_l\frac{\partial}{\partial k_f}\left(\hat{\varkappa}_{ij} + \hat{\varkappa}^*_{ji}\right) \\
&+ \frac{\mathrm{i}}{k}\varepsilon_{ipl}\left[\hat{k}_p\left((\boldsymbol{\Omega}\cdot\nabla) - 2\left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\left(\hat{\mathbf{k}}\nabla\right)\right) + \left(\boldsymbol{\Omega}\cdot\hat{\mathbf{k}}\right)\nabla_p\right]\left(\varepsilon_{ipl}\hat{v}_{lj} - \varepsilon_{jpl}\hat{v}_{il}\right),
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\partial \hat{h}_{ij}}{\partial t} &= -\frac{\hat{h}_{ij} - \hat{h}^{(0)}_{ij}}{\tau_c} + \hat{h}_{il}\overline{V}_{j,l} + \hat{h}_{lj}\overline{V}_{i,l} + k_l\overline{V}_{f,l}\frac{\partial \hat{h}_{ij}}{\partial k_f} + \frac{\mathrm{i}\left(\overline{\mathbf{B}}\cdot\mathbf{k}\right)}{\rho}\left(\hat{\varkappa}_{ij} - \hat{\varkappa}^*_{ji}\right) \\
&+ \left\{\frac{\left(\overline{\mathbf{B}}\cdot\nabla\right)}{2\rho} - \frac{\left(\overline{\mathbf{B}}\cdot\mathbf{G}\right)}{2\rho}\right\}\left(\hat{\varkappa}_{ij} + \hat{\varkappa}^*_{ji}\right) - \left(\frac{\overline{B}_j}{\rho}\right)_{,l}\hat{\varkappa}^*_{li} - \left(\frac{\overline{B}_i}{\rho}\right)_{,l}\hat{\varkappa}_{lj} \\
&- \frac{1}{2}\left(\frac{\overline{B}_l}{\rho}\right)_{,f}k_l\frac{\partial\left(\hat{\varkappa}_{ij} + \hat{\varkappa}^*_{ji}\right)}{\partial k_f},
\end{aligned} \tag{19}$$

where $\hat{\varkappa}^*_{ji} = \Phi(\hat{b}_j, \hat{m}_i, \mathbf{k}, \mathbf{R})$, $\hat{\mathbf{k}}$ is the unit wave vector, the indexes behind the comma stand for the spatial derivatives. Equations (17)–(19) are in agreement with those considered in the paper by Rogachevskii and Kleeorin (2004a).

To solve (17)–(19) we neglect the time derivatives at the left-hand-side of equations and apply the perturbation method with respect to the large-scale field inhomogeneities (associated with nonuniform LSMF and shear) and stratification scales of turbulence. We shall not reproduce explicitly the rather bulky derivations which are explained elsewhere: (Rogachevskii and Kleeorin 2003, 2004b). The solution of (17)–(19) will be given for two specific cases. In the first case we apply no restriction to the angular velocity (the Coriolis number, $\Omega^* = 2\Omega\tau_c$, is arbitrary) and LSMF is assumed to be weak. In the second case we keep the linear terms in angular velocity and solve equation (17)–(19) for the case of arbitrary $\beta = \bar{B}/\left(u_c\sqrt{\mu\rho}\right)$, where $\bar{B}$ is the strength of the LSMF. In all derivations we keep contributions which are the first order in the shear. Furthermore, for the contributions involving the shear we make two additional simplifications. The first one is that we neglect the density stratification, but leave the contributions of the turbulence intensity stratification. Additionally, we discard the joint effect of the Coriolis force and the shear to the MEMF. In the present study I consider an intermediate nonlinearity which implies that effect of the mean magnetic field and global rotation is not enough strong in order to affect the correlation time of turbulent velocity field.

For integration in **k**-space I adopt the quasi-isotropic form of the spectra (Roberts and Soward 1975, Rüdiger and Kichatinov 1993) for the background turbulence. Additionally, the background magnetic fluctuations are helical, while there is no prescribed kinetic helicity in the background turbulence:

$$\hat{v}_{ij}^{(0)} = \left\{\pi_{ij}(\mathbf{k}) + \frac{\mathrm{i}}{2k^2}\left(k_i\nabla_j - k_j\nabla_i\right)\right\}\frac{\rho^2 E(k,\mathbf{R})}{8\pi k^2}, \tag{20}$$

$$\hat{h}_{ij}^{(0)} = \left\{\left(\pi_{ij}(\mathbf{k}) + \frac{\mathrm{i}}{2k^2}\left(k_i\nabla_j - k_j\nabla_i\right)\right)\frac{\mathcal{B}(k,\mathbf{R})}{8\pi k^2} - \mathrm{i}\varepsilon_{ijp}k_p\frac{\mathcal{N}(k,\mathbf{R})}{8\pi k^4}\right\}, \tag{21}$$

where, the spectral functions $E(k,\mathbf{R}), \mathcal{B}(k,\mathbf{R}), \mathcal{N}(k,\mathbf{R})$ define, respectively, the intensity of the velocity fluctuations, the intensity of the magnetic fluctuations and the amount of current helicity in the background turbulence. They are defined via

$$\left\langle u^{(0)2}\right\rangle = \int\frac{E(k,\mathbf{R})}{4\pi k^2}\mathrm{d}^3\mathbf{k},\ \left\langle b^{(0)2}\right\rangle = \int\frac{\mathcal{B}(k,\mathbf{R})}{4\pi k^2}\mathrm{d}^3\mathbf{k},\ h_C^{(0)} = \frac{1}{\mu\rho}\int\frac{\mathcal{N}(k,\mathbf{R})}{4\pi k^2}\mathrm{d}^3\mathbf{k}, \tag{22}$$

where $h_C^{(0)} = \left\langle\mathbf{b^{(0)}}\cdot\nabla\times\mathbf{b^{(0)}}\right\rangle/(\mu\rho)$. In final results we use the relation between intensities of magnetic and kinetic fluctuations which is defined via $\mathcal{B}(k,\mathbf{R}) = \varepsilon\mu\bar{\rho}E(k,\mathbf{R})$. The state with $\varepsilon = 1$ means equipartition between energies of magnetic and kinetic fluctuations in the background turbulence. The point to note is that inconsistency between (20) and (21) does not influence the final results. The general structure of the mean electromotive force vector obtained within the given framework are in agreement with the known results from the literature (Rädler, *et al.* 2003; Rogachevskii and Kleeorin 2003). We keep the current helicity contribution in the background turbulence to investigate the nonlinear saturation phase of the helical large-scale dynamo.

The final remarks in this section concern with discussion given in the article by Rädler and Rheinhardt (2006). There, authors argue that τ approximation may lead to results which are in conflict with those of SOCA. One difference is apparent between the two approaches: there is no overlap in applicability limits of SOCA and τ approximation. The given scheme to obtain (17)–(19) is hardly justified for small hydrodynamic Reynolds numbers. The same is true in a highly conductivity limit, where SOCA can be valid only for the small Strouhal numbers. Currently, the range of τ approximation validity is purely understood. This problem requires further careful study.

There is another reason for difference between results presented in the article and those of SOCA. In the given variant of τ approximation the relaxation time τ_c is independent of $\mathbf{k}$. This issue is especially important in computing effects of the nonuniform LSMF and shear. Perhaps, the spectral τ-approximation can correct this defect. For more detail, see (Rädler *et al.* 2003, Rogachevskii and Kleeorin 2007). Hense, in confronting MTA and SOCA, it is of some use to simplify the expressions obtained within SOCA by applying the mixing-length approximation. The transition from SOCA to MLT can be done by replacing the spectrum of turbulent fields by the single-scaled function of the form $\delta\left(k-\ell_c^{-1}\right)\delta(\omega)$, and applying $\eta k^2 = \nu k^2 = \tau_c^{-1}$, here ℓ_c is the correlation length of the turbulence. For more details, see Kichatinov (1991) and Kichatinov *et al.* (1994).

3. Results

3.1. *Weak LSMF, arbitrary Coriolis number*

3.1.1. Spatially uniform LSMF. I decompose the electromotive force into different contributions, in particular, $\boldsymbol{\mathcal{E}}^{(a)}$ contains the effects of stratification, and $\boldsymbol{\mathcal{E}}^{(s)}$ gives contributions due to shear, which are computed only in slow rotation limit. We find the following expression for $\boldsymbol{\mathcal{E}}^{(a)}$:

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}^{(a)} = \Big\{&(\varepsilon-1)\Big(f_2^{(a)}\big(\mathbf{U}\times\overline{\mathbf{B}}\big)+f_1^{(a)}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)(\mathbf{e}\times\mathbf{U})\Big)+f_3^{(a)}\big(\mathbf{G}\times\overline{\mathbf{B}}\big)\\
&+f_1^{(a)}\big((\mathbf{e}\cdot\mathbf{G})\big(\mathbf{e}\times\overline{\mathbf{B}}\big)+(\varepsilon-2)\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)(\mathbf{e}\times\mathbf{G})\big)\\
&+f_4^{(a)}\mathbf{e}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)(\mathbf{e}\cdot\mathbf{U})+f_{11}^{(a)}\overline{\mathbf{B}}(\mathbf{e}\cdot\mathbf{U})+f_5^{(a)}\mathbf{e}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)(\mathbf{e}\cdot\mathbf{G})\\
&+f_8^{(a)}\big(\mathbf{e}\big(\overline{\mathbf{B}}\cdot\mathbf{U}\big)+\mathbf{U}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)\big)+f_6^{(a)}\big(\mathbf{e}\big(\overline{\mathbf{B}}\cdot\mathbf{G}\big)+\mathbf{G}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)\big)+f_{10}^{(a)}\overline{\mathbf{B}}(\mathbf{e}\cdot\mathbf{G})\\
&+f_9^{(a)}\big(\mathbf{e}\big(\overline{\mathbf{B}}\cdot\mathbf{U}\big)-\mathbf{U}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)\big)+f_7^{(a)}\big(\mathbf{e}\big(\overline{\mathbf{B}}\cdot\mathbf{G}\big)-\mathbf{G}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)\big)\Big\}\langle u^{(0)2}\rangle\tau_c\\
&+2\Big\{f_2^{(a)}\overline{\mathbf{B}}-f_1^{(a)}\mathbf{e}\big(\mathbf{e}\cdot\overline{\mathbf{B}}\big)\Big\}\tau_c h_C^{(0)},
\end{aligned}
\tag{23}
$$

where functions $f_{\{n\}}^{(a)}=f_{\{n\}}^{(a)}(\Omega^*,\varepsilon)$ (and all which are used below) are given in Appendix A, $\mathbf{U}=\nabla\log\langle u^{(0)2}\rangle$ is a scale of the turbulence intensity stratification, $\mathbf{e}=\boldsymbol{\Omega}/|\Omega|$ is a unit vector in direction of global rotation. For the slow rotation limit ($\Omega^*\to 0$) we get

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}^{(a)}|_{\Omega^*\to 0} &= \alpha\circ\overline{\mathbf{B}}+\langle u^{(0)2}\rangle\tau_c\left\{\frac{(\varepsilon-1)}{6}\big(\mathbf{U}\times\overline{\mathbf{B}}\big)+\frac{\varepsilon}{6}\big(\mathbf{G}\times\overline{\mathbf{B}}\big)\right\}\\
&\quad+\langle u^{(0)2}\rangle\tau_c\frac{\Omega^*}{12}\left\{(\varepsilon+2)\big(\big((\mathbf{G}\times\mathbf{e})\times\overline{\mathbf{B}}\big)\big)+(\varepsilon+1)\big(\big((\mathbf{U}\times\mathbf{e})\times\overline{\mathbf{B}}\big)\big)\right\},
\end{aligned}
\tag{24}
$$

$$\alpha_{ij} = \delta_{ij}\tau_c\left(\langle u^{(0)2}\rangle\left\{\frac{2\varepsilon((\mathbf{e}\cdot\mathbf{U})+(\mathbf{e}\cdot\mathbf{G}))\Omega^*}{15}-\frac{2(\mathbf{e}\cdot\mathbf{U})\Omega^*}{5}-\frac{4(\mathbf{e}\cdot\mathbf{G})\Omega^*}{5}\right\}+\frac{h_{\mathcal{C}}^{(0)}}{3}\right)$$
$$+\tau_c\langle u^{(0)2}\rangle\frac{\Omega^*}{20}\left\{\left(e_iG_j+e_jG_i\right)(\varepsilon+4)+\left(e_iU_j+e_jU_i\right)\left(\varepsilon+\frac{11}{3}\right)\right\}, \quad (25)$$

where only linear terms in Ω are kept. Except contributions due to $\mathbf{G}$ equations (24) and (25) are in agreement with results by Rädler *et al.* (2003) and Brandenburg and Subramanian (2005). The mean transport of the LSMF due to stratification of turbulence is given by second term in (24). They are in agreement with the mixing-length expressions obtained by Kichatinov (1991). Note that, additional components of the turbulent transport may be excited due to the antisymmetric part of $\boldsymbol{\alpha}$-tensor in (25).

For the fast rotation limit ($\Omega^* \to \infty$) of (23) we get

$$\boldsymbol{\mathcal{E}}^{(a)}|_{\Omega^*\to\infty} \to \frac{\pi\tau_c}{2}\left(\frac{h_{\mathcal{C}}^{(0)}}{2\Omega^*}-\langle u^{(0)2}\rangle\left(\frac{(\mathbf{e}\cdot\mathbf{U})}{2}+(\mathbf{e}\cdot\mathbf{G})\right)\right)\left(\overline{\mathbf{B}}-\mathbf{e}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)\right), \quad (26)$$

where for the current helicity we also retain the terms of one higher order with respect to Ω^*. The reason for this will be clarified below in section 4. Except the helicity term, equation (26) is in identical agreement with the mixing-length approximation results obtained by Rüdiger and Kichatinov (1993) within SOCA.

In the case of the spatially uniform LSMF the shear contributions to the mean electromotive force are expressed as follows

$$\mathcal{E}_i^{(s)} = z\varepsilon_{inm}\left\{A_4U_k\overline{B}_n\overline{V}_{m,k}+A_2\overline{B}_k\overline{V}_{n,k}U_m+A_3\left(\overline{\mathbf{B}}\cdot\mathbf{U}\right)\overline{V}_{m,n}+A_1\overline{V}_{k,n}\overline{B}_kU_m\right\}\langle u^{(0)2}\rangle$$
$$+\tau_c^2\frac{h_{\mathcal{C}}^{(0)}}{2}\left(\mathbf{W}\times\overline{\mathbf{B}}\right)_i-\frac{13}{30}\tau_c^2h_{\mathcal{C}}^{(0)}\left\{\overline{V}_{n,i}+\overline{V}_{i,n}\right\}\overline{B}_n, \quad (27)$$

where $\mathbf{W} = \nabla\times\overline{\mathbf{V}}$, we assume that $(\mathbf{U}\cdot\nabla)\mathbf{V} = 0$ and $A_1 = (2\varepsilon-1)\tau_c^2/15$, $A_2 = -(3\varepsilon+1)\tau_c^2/15$, $A_3 = (\varepsilon+1)\tau_c^2/6$, $A_4 = -A_3$. Coefficients A_{1-3} correspond to those from Rüdiger and Kichatinov (2006) (hereafter RK06) and A_4 is corresponding to their A_5. Recently, similar contributions of the large-scale shear were calculated within SOCA by Rädler and Stepanov (2006) (RS06), as well. We have to note that both the RK06 and RS06 results are related with the case $\varepsilon = 0$. The (27) differs with results obtained in RK06 and RS06 articles. For example, after applying the mixing-length relations $\eta k^2 = \nu k^2 = \tau_c^{-1}$ to expressions given by RK06 we get $A_1 = \tau_c^2/3$ (in our case $-\tau_c^2/15$) and $A_2 = -\tau_c^2/60$ (compare to our $-\tau_c^2/15$). Unfortunately RK06 did not give the results for other coefficients. The comparison with RS06 is given in Appendix B. The difference between the given results and those by RK06 and RS06 can be explained, in part, by the crudeness of the given version of tau approximation. Here, we assume that τ_c is independent of $\mathbf{k}$. This especially influences the accuracy of calculations of the contributions due to shear because they involve the derivatives in $\mathbf{k}$ space.

According to (27) the joint effect of current helicity and shear contributes to pumping of LSMF. The interpretation of the effect is difficult to illustrate. To show the general

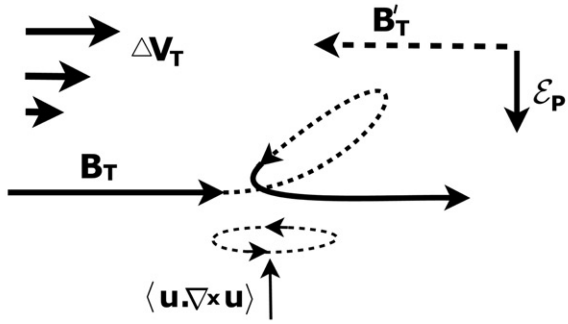

Figure 1. The modification of standard α-effect (cf. Krarad and Rädler 1980) due to shear. The helical motions (denoted with $\langle \mathbf{u}\cdot\nabla\times\mathbf{u}\rangle$) go up, drag and twist the LSMF $\mathbf{B}_T$, where index T denotes the toroidal component of LSMF. The shear,ΔV_T, additionally, folds the loop in direction of large-scale flow. The effect is equivalent to inducing the transversal large-scale electromotive force, $\boldsymbol{\mathcal{E}}_P$ (here index P denotes the poloidal component of the MEMF), and the magnetic field, $\mathbf{B}'_T$ parallel to original one. Direction of the induced field depends on the sign of the helicity. For the situation given on the picture, the induced field $\mathbf{B}'_T$ quenches the original LSMF in direction of the gradient of the mean flow. This means that the LSMF is effectively pumped in opposite direction.

idea we invoke an auxiliary illustration of effect for the helical turbulent motions. It is shown in figure 1.

3.1.2. Anisotropic diffusion, the $\boldsymbol{\Omega}\times\mathbf{J}$ and shear-current effects. In rotating turbulence the magnetic diffusivity become anisotropic (Kichatinov *et al.* 1994). The corresponding part of the MEMF reads,

$$\mathcal{E}_i^{(d)} = \Big\{ f_1^{(d)} e_n \overline{B}_{\mathrm{n,i}} + f_2^{(d)} \varepsilon_{\mathrm{inm}} \overline{B}_{\mathrm{m,n}} + \varepsilon f_3^{(d)} e_i e_n e_m \overline{B}_{\mathrm{m,n}} + f_1^{(a)} \varepsilon_{inm} e_n e_l \big(2\varepsilon \overline{B}_{l,m} - (\varepsilon+1)\overline{B}_{m,l}\big) + \varepsilon f_4^{(d)} e_n \overline{B}_{\mathrm{i,n}} \Big\} \big\langle u^{(0)2} \big\rangle \tau_c, \tag{28}$$

where functions $f_{\{n\}}^{(d)} = f_{\{n\}}^{(d)}(\Omega^*)$ are given in Appendix A. If we put the magnetic fluctuations in background turbulence equal to zero in (28) ($\varepsilon = 0$), we return to results obtained by Kichatinov *et al.* (1994). The magnetic fluctuation contributions in (28) give rise to the $\boldsymbol{\Omega}\times\mathbf{J}$ effect (terms related with $e_n\overline{B}_{\mathrm{n,i}}$ and $e_n\overline{B}_{\mathrm{i,n}}$) and to additions in anisotropic diffusion. In the slow-rotation limit equation (28) can be reduced to

$$\mathcal{E}_i^{(d)}|_{\Omega*\to 0} = \left\{ e_n\big((\varepsilon+5)\overline{B}_{\mathrm{n,i}} + 6\varepsilon\overline{B}_{\mathrm{i,n}}\big)\frac{\Omega^*}{10} - \varepsilon_{\mathrm{inm}}\overline{B}_{\mathrm{m,n}} \right\} \frac{\big\langle u^{(0)2}\big\rangle \tau_c}{3}. \tag{29}$$

Equation (29) corresponds to results by Subramanian (2005). Note, only magnetic fluctuations contribute to the induction term $(\mathbf{e}\cdot\nabla)\overline{\mathbf{B}}$. The $\boldsymbol{\Omega}\times\mathbf{J}$ effect was proposed originally by Rädler (1969) (see, also Krause and Rädler 1980, Rädler *et al.* 2003, Kichatinov 2003).

The physical interpretation of this effect is shown in figure 2. Lets consider the situation in disk geometry and the rotating media penetrated by the inhomogeneous toroidal LSMF. For simplicity, we assume that LSMF is nonuniform along the axis of rotation. Let the direction of LSMF will be opposite to direction of

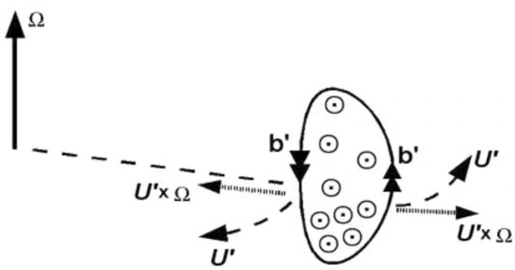

Figure 2. An illustration of $\mathbf{\Omega} \times \mathbf{J}$ effect in disk geometry. Direction of rotation is marked by $\mathbf{\Omega}$, the large-scale toroidal field has opposite direction to rotational velocity and it is marked by $\odot$, what means that LSMF is perpendicular to the figure's plane and it is directed to the reader. The loop of fluctuating magnetic field, $\mathbf{b}'$, comprises LSMF that is nonuniform along the axis of rotation. Its direction is marked by double arrows. The small-scale Lorentz forces induce the azimuthal fluctuations of velocity, $\mathbf{u}' \sim (\mathbf{b}' \cdot \nabla)\overline{\mathbf{B}}$. They are marked by dashed lines ending with arrows. The Coriolis force deflects these fluctuations to radial direction (this is marked by dotted lines). The resulting electromotive force has the same direction as the original LSMF and it is proportional to $\langle b'^2 \rangle (\mathbf{\Omega} \cdot \nabla)\overline{\mathbf{B}}$.

rotating plasma. If the loop of the small-scale fluctuating magnetic field comprises LSMF, it induces fluctuation of velocity in azimuthal direction. The influence of the Coriolis force declines the velocities in radial direction. The effective electromotive force is co-lined with original LSMF and is proportional to $\langle b^2 \rangle (\mathbf{\Omega} \cdot \nabla)\overline{\mathbf{B}}$, see figure 2.

The shear-current effect discussed by Rogachevskii and Kleeorin (2003) (hereafter RK03) is of similar nature, because the large-scale vorticity $\mathbf{W} = \nabla \times \overline{\mathbf{V}}$ and the Coriolis force act on the turbulent motions in a like manner. The additional contributions due to shear in the diffusion part of the mean electromotive force are expressed as follows,

$$\mathcal{E}_i^{(V)} = \varepsilon_{inm} \left\{ C_2 \overline{B}_{\mathrm{n},1} \overline{V}_{m,1} + C_1 \overline{V}_{l,m} \overline{B}_{n,l} + C_3 \overline{V}_{l,m} \overline{B}_{l,n} + C_4 \overline{B}_{l,n} \overline{V}_{m,l} \right\} \langle u^{(0)2} \rangle, \tag{30}$$

where $C_1 = (\varepsilon - 3/5)\tau_c^2/6$, $C_2 = (\varepsilon - 1)\tau_c^2/5$, $C_3 = (1 + \varepsilon)\tau_c^2/15$, $C_4 = -(7\varepsilon + 11)\tau_c^2/30$. Coefficients C_{1-4} correspond to those from RK06. After applying the mixing-length approximation to RK06's results we get $C_1 = -2\tau_c^2/5$, $C_2 = -4\tau_c^2/15$, $C_3 = 0$, $C_4 = -\tau_c^2/5$. In confronting these coefficients to ours, we see the difference. It can be explained, in part, by the crudeness of the given version of tau approximation. The comparison with RS06 is given in Appendix B. The given results are in agreement with those given in (Rogachevskii and Kleeorin 2007). As shown in the paper cited earlier, the spectral τ approximation is capable to give result in closer agreement with those of SOCA.

In the commonly accepted scheme of the solar α–Ω dynamo, the poloidal LSMF of the Sun is produced from the large-scale toroidal magnetic field via the alpha effect. Expressions (28,30) give contributions which are capable to induce the MEMF along the LSMF. Therefore, these terms are potentially very important for the solar dynamo, because they provides additional induction sources of the large-scale poloidal magnetic field of the Sun. subsequently, I consider the efficiency of induction effect along the nonuniform LSMF due to global rotation and shear. In (28,30) we leave only those terms that capable to induce the toroidal MEMF and skip the usual contributions due to turbulent diffusion.

For the sake of simplicity we restrict consideration to the axisymmetric LSMF, $\overline{\mathbf{B}} = B\mathbf{e}_\phi + \nabla(A\mathbf{e}_\phi)$, in the Keplerian disk. We assume that toroidal LSMF exceeds its poloidal counterpart, $\overline{\mathbf{B}} \approx B\mathbf{e}_\phi$. Suppose that a rotating frame of reference is defined by angular velocity Ω_0 in a given point $r = r_0$. The mean velocity in this rotating frame is $\overline{\mathbf{V}} = r\delta\Omega(r)\mathbf{e}_\phi$, where $\delta\Omega(r) = (\Omega(r) - \Omega_0)$ and $\Omega(r) = \Omega_0\sqrt{(r_0/r)^3}$. Computing (22) in cylindrical coordinates (r, ϕ, z) at the point $r = r_0$ we get

$$\mathcal{E}_\phi^{(V)} = \left(C_3 r\frac{\partial\Omega}{\partial r} + (C_3 - C_4)\delta\Omega\right)\frac{\partial B}{\partial z}\langle u^{(0)2}\rangle = -\frac{3}{2}\Omega_0 C_3\frac{\partial B}{\partial z}\langle u^{(0)2}\rangle. \tag{31}$$

Combining the given result with one from (21) we get,

$$\mathcal{E}_\phi \approx \Omega^*\frac{(3\varepsilon - 1)}{20}\langle u^{(0)2}\rangle\tau_c\frac{\partial B}{\partial z}, \tag{32}$$

where $\Omega^* = 2\Omega_0\tau_c$. If $\varepsilon > 1/3$ and provided the LSMF is concentrated to the plane of the disk, then in the upper half plane of the disk the induced MEMF will be in direction of the LSMF.

3.2. *Slow rotation, arbitrary LSMF*

3.2.1. Spatially uniform LSMF. In this part of the article we consider results obtained for the slow rotation limit. In what follows, no restriction is applied to the strength of the LSMF. The MEMF, that is induced due to influence of rotation and stratification on the turbulence, is described with expression

$$\begin{aligned}
\boldsymbol{\mathcal{E}}^{(a)} = \langle u^{(0)2}\rangle\tau_c\Big\{&\varphi_1^{(a)}(\mathbf{G}\times\overline{\mathbf{B}}) + \varphi_2^{(a)}(\mathbf{U}\times\overline{\mathbf{B}}) + \tau_c(\boldsymbol{\Omega}\cdot\overline{B})\left(\varphi_4^{(a)}\mathbf{G} + \varphi_{10}^{(a)}\mathbf{U}\right)\\
&+ \tau_\mathbf{c}\overline{\mathbf{B}}\left(\varphi_\mathbf{6}^{(\mathbf{a})}(\boldsymbol{\Omega}\cdot\mathbf{G}) + \varphi_\mathbf{8}^{(\mathbf{a})}(\boldsymbol{\Omega}\cdot\mathbf{U})\right) + \tau_c\boldsymbol{\Omega}\left(\varphi_5^{(a)}(\overline{\mathbf{B}}\cdot\mathbf{G}) + \varphi_9^{(a)}(\overline{\mathbf{B}}\cdot\mathbf{U})\right)\\
&+ \tau_\mathbf{c}\frac{(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}})\overline{\mathbf{B}}}{\overline{\mathbf{B}}^2}\left(\varphi_\mathbf{3}^{(\mathbf{a})}(\overline{\mathbf{B}}\cdot\mathbf{G}) + \varphi_7^{(\mathbf{a})}(\overline{\mathbf{B}}\cdot\mathbf{U})\right)\Big\} + \tau_c h_C^{(0)}\varphi_1^{(h)}\overline{\mathbf{B}},
\end{aligned} \tag{33}$$

where $\varphi_n^{(a)}$ are functions of β defined in the appendix. This formula generalizes the similar results by Rüdiger and Kichatinov (1993), Kichatinov and Rüdiger (1992) taking the density stratification, magnetic fluctuations and current helicity into account. The nonlinear MEMF of helical MHD turbulence was considered by Rogachevskii and Kleeorin (2004a), as well. For the strong LSMF limit we obtain

$$\begin{aligned}
\boldsymbol{\mathcal{E}}^{(a)}|_{\beta\to\infty} = \Big\{&\frac{\tau_c}{8}\left((\varepsilon+1)(\overline{\mathbf{B}}\cdot\mathbf{U}) + \frac{3(3\varepsilon+5)}{8}(\overline{\mathbf{B}}\cdot\mathbf{G})\right)\left(\boldsymbol{\Omega} - \frac{(\boldsymbol{\Omega}\cdot\overline{B})\overline{\mathbf{B}}}{\overline{B}^2}\right)\\
&+ \frac{3\varepsilon+1}{64}(\mathbf{G}\times\overline{\mathbf{B}})\Big\}\frac{\pi}{\beta}\langle u^{(0)2}\rangle\tau_c.
\end{aligned} \tag{34}$$

The results by Rüdiger and Kichatinov (1993) can be recovered from (34), if we put $\mathbf{G} = 0$ and $\varepsilon = 0$. Following to arguments given in the article cited above, we conclude that the MEMF like (34) does not produce a dynamo.

The first term at the upper line of (33) describes the so-called "turbulent buoyancy" (Kichatinov and Rüdiger 1992). The expression (34) shows that the transport of LSMF is downward for the strong magnetic field limit. For the case of the weak field we get $\varphi_1^{(a)} \approx \varepsilon/6 + (6\varepsilon - 8\beta^2)/15$. Then, if we neglect contributions due to small-scale magnetic fluctuations, we obtain that for the weak field transport is upward (opposite to direction of **G**). In this case the effective drift velocity is proportional to the LSMF's pressure (Kichatinov and Rüdiger 1992). In this aspect it is similar to the usual buoyancy of magnetic flux tubes (Parker 1979). Furthermore, we find that the large-scale inhomogeneity of magnetic fluctuations provide the downward drift of LSMF in the whole range of magnetic field strength.

The quenching functions for the isotropic components of α-effect are shown in figure 3. There, for comparison, the dash-dotted line indicates the curve corresponding to quenching of isotropic components of the α effect obtained within SOCA in (Rüdiger and Kichatinov 1993).

In the strong LSMF limit we found that α-effect is quenched as β^{-2} which is different from results by Rüdiger and Kichatinov (1993) and similar to findings by Rogachevskii and Kleeorin (2004a). Though, as seen from the figure, the numerical difference between the quenching curves obtained within SOCA (dash-dotted line) and MTA (dashed line) is within a few percents.

The nonlinear electromotive force induced by shear is expressed as follows,

$$\begin{aligned}\mathcal{E}_i^{(s)} &= \varepsilon_{inm}\left\{\varphi_1^{(s)}\frac{\overline{B}_l\overline{B}_k}{\overline{B}^2}\overline{V}_{l,k}U_n\overline{B}_m + \varphi_2^{(s)}\overline{B}_l\overline{V}_{l,m}U_n + \varphi_3^{(s)}(\mathbf{U}\cdot\overline{\mathbf{B}})\frac{\overline{B}_l\overline{B}_m}{\overline{B}^2}(\overline{V}_{l,n} - \overline{V}_{n,l})\right.\\ &\quad \left. + \varphi_4^{(s)}U_l\overline{V}_{n,l}\overline{B}_m + \varphi_5^{(s)}U_n\overline{V}_{l,m}\overline{B}_l\right\}\langle u^{(0)2}\rangle\tau_c^2\\ &\quad + \tau_c^2 h_C^{(0)}\left\{\varphi_4^{(h)}\overline{V}_{m,n}\frac{\overline{B}_m\overline{B}_n}{\overline{B}^2}\overline{B}_i + \varphi_3^{(h)}(\overline{V}_{n,i} + \overline{V}_{i,n})\overline{B}_n + \varphi_2^{(h)}\left(\mathbf{W}\times\overline{\mathbf{B}}\right)_i\right\}.\end{aligned} \tag{35}$$

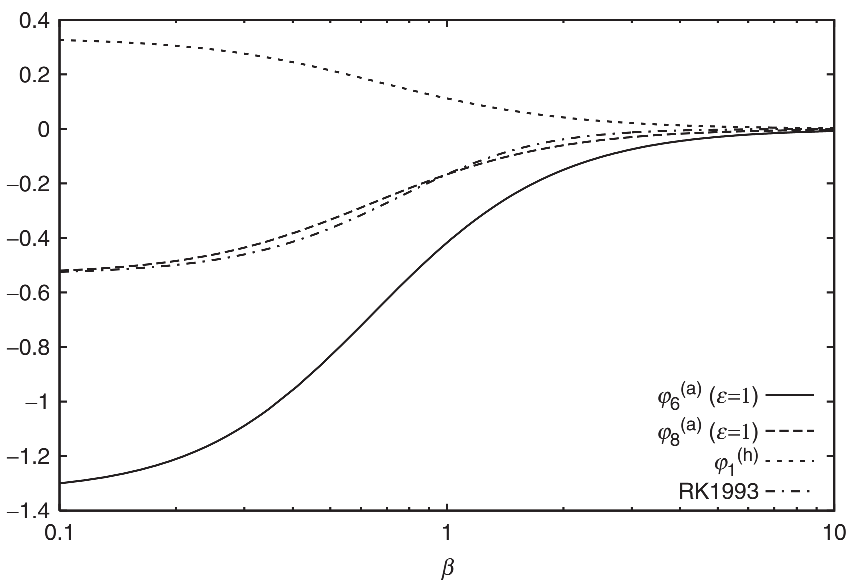

Figure 3. The quenching functions for isotropic components of α-effect.

From (35) we can find that the terms with $\varphi^{(s)}_{2,3,5}$ and the second term in brackets with $\varphi^{(5)}_5$ have components along the LSMF. Therefore, their effect to the MEMF is similar to the α effect. The term with $\varphi^{(h)}_2$ and $\varphi^{(s)}_{1,4}$ provide the pumping of LSMF. Surprisingly, the α-effect like terms survive even in the limit of the strong magnetic field. In this case we get

$$\begin{aligned}\mathcal{E}^{(s)}_i|_{\beta\to\infty} = \varepsilon_{inm}\Bigg\{&\frac{3}{4}(\varepsilon-1)\left(\frac{\overline{B}_l\overline{B}_k}{\overline{B}^2}\overline{V}_{l,k}U_n\overline{B}_m - \overline{B}_l\overline{V}_{l,m}U_n\right)\\ &+(\varepsilon+1)(\mathbf{U}\cdot\overline{\mathbf{B}})\frac{\overline{B}_l\overline{B}_m}{\overline{B}^2}(\overline{V}_{l,n}-\overline{V}_{n,l})\Bigg\}\frac{\pi}{16\beta}\langle u^{(0)2}\rangle\tau_c^2\\ &+\frac{3\pi}{64\beta}\tau_c^2 h_C^{(0)}\left\{\overline{V}_{m,n}\frac{\overline{B}_m\overline{B}_n}{\overline{B}^2}\overline{B}_i - (\overline{V}_{n,i}+\overline{V}_{i,n})\overline{B}_n + \left(\mathbf{W}\times\overline{\mathbf{B}}\right)_i\right\}. \end{aligned} \tag{36}$$

According to (27) and (36) the pumping of the LSMF due to joint effect of current helicity and shear have the same sign for the weak and strong LSMF.

3.2.2. Diffusion, $\mathbf{\Omega}\times\mathbf{J}$ and shear current effect. The results for nonlinear turbulent diffusion are similar to those found within SOCA by Kichatinov *et al.* (1994). We have

$$\boldsymbol{\mathcal{E}}^{(d)} = \left\{\varphi_3\nabla\times\overline{\mathbf{B}} + \left(\varphi_2\frac{((\nabla\times\overline{\mathbf{B}})\times\overline{\mathbf{B}})}{\overline{B}^2} + \varphi_1\nabla\log\left(\frac{\overline{B}^2}{2}\right)\right)\times\overline{\mathbf{B}}\right\}\langle u^{(0)2}\rangle\tau_c + \boldsymbol{\mathcal{E}}^{(w)}, \tag{37}$$

where $\boldsymbol{\mathcal{E}}^{(w)}$ stands for the contributions due to rotation. The corresponding quenching functions are given in figure 4.

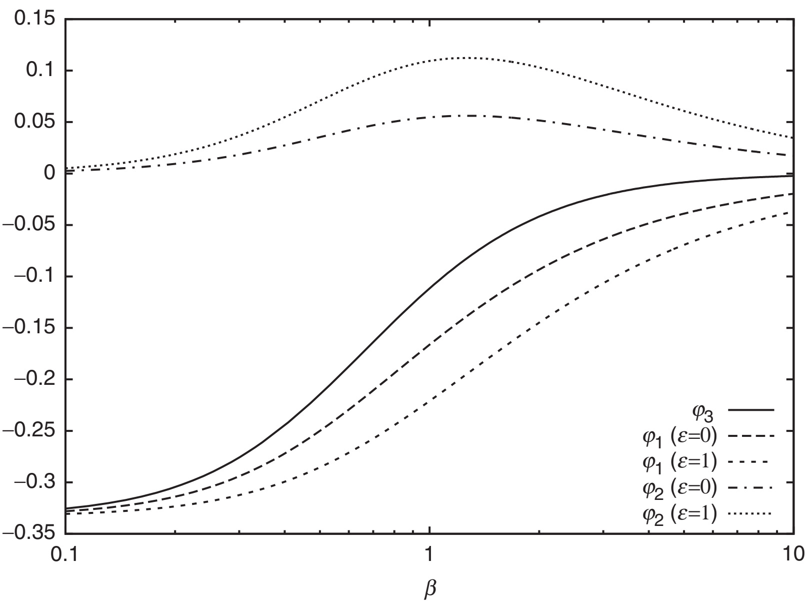

Figure 4. Functions defining the nonlinear turbulent diffusion of LSMF (see equation (29)).

The next formula generalizes the results for the nonlinear diffusion of LSMF to the case of the slowly rotating media:

$$\begin{aligned}\mathcal{E}_i^{(w)} = \Bigg\{ &\varphi_8^{(w)} \nabla_i \left(\boldsymbol{\Omega} \cdot \overline{\mathbf{B}}\right) + \varphi_1^{(w)} \frac{\left(\boldsymbol{\Omega} \cdot \overline{\mathbf{B}}\right)}{2} \nabla_i \log\left(\overline{B}^2\right) + \varphi_4^{(w)} \overline{B}_i \frac{\left(\overline{\mathbf{B}} \cdot \nabla\right)\left(\boldsymbol{\Omega} \cdot \overline{B}\right)}{\overline{B}^2} \\ &+ \varphi_5^{(w)} \Omega_i \frac{\left(\overline{\mathbf{B}} \cdot \nabla\right)}{2} \log\left(\overline{B}^2\right) + \varphi_3^{(w)} \overline{B}_i \frac{\left(\boldsymbol{\Omega} \cdot \overline{\mathbf{B}}\right)}{\overline{B}^2} \frac{\left(\overline{\mathbf{B}} \cdot \nabla\right)}{2} \log\left(\overline{B}^2\right) \\ &+ \varphi_6^{(w)} \frac{\left(\boldsymbol{\Omega} \cdot \overline{\mathbf{B}}\right)}{\overline{B}^2} \left(\overline{\mathbf{B}} \cdot \nabla\right) \overline{B}_i + \varphi_2^{(w)} \overline{B}_i \frac{\left(\boldsymbol{\Omega} \cdot \nabla\right)}{2} \log\left(\overline{B}^2\right) + \varphi_7^{(w)} (\boldsymbol{\Omega} \cdot \nabla) \overline{B}_i \Bigg\} \langle u^{(0)2} \rangle \tau_c^2. \quad (38)\end{aligned}$$

The last two terms at the third line in (38) are related with the induction of MEMF along the direction of LSMF. The corresponding functions $\varphi_2^{(w)}$ and $\varphi_7^{(w)}$ are shown in figure 5. As can be seen there, in the absence of the background magnetic fluctuations ($\varepsilon = 0$) the MEMF induction along the LSMF due to $\boldsymbol{\Omega} \times \mathbf{J}$-effect exists only in nonlinear regime.

If $\beta > 1$, functions $\varphi_2^{(w)}$ and $\varphi_7^{(w)}$ have opposite signs everywhere. Note, while the term $(\boldsymbol{\Omega} \cdot \nabla)\overline{B}_i$ induces MEMF in direction of LSMF's gradients along axis of rotation, and the term $\overline{B}_i(\boldsymbol{\Omega} \cdot \nabla)\log(\overline{B}^2)$ induces MEMF in opposite direction. Formally, the latter effect is similar to α-effect. The only difference with the standard α is that instead stratification parameters of turbulence we have a parameter which is related with nonuniform distribution of the LSMF's energy. For the solar magnetic fields the effect is antisymmetric about equator. Below, it is shown that in the strong LSMF this α is quenched by factor β^{-1} which is lesser than for the standard α-effect.

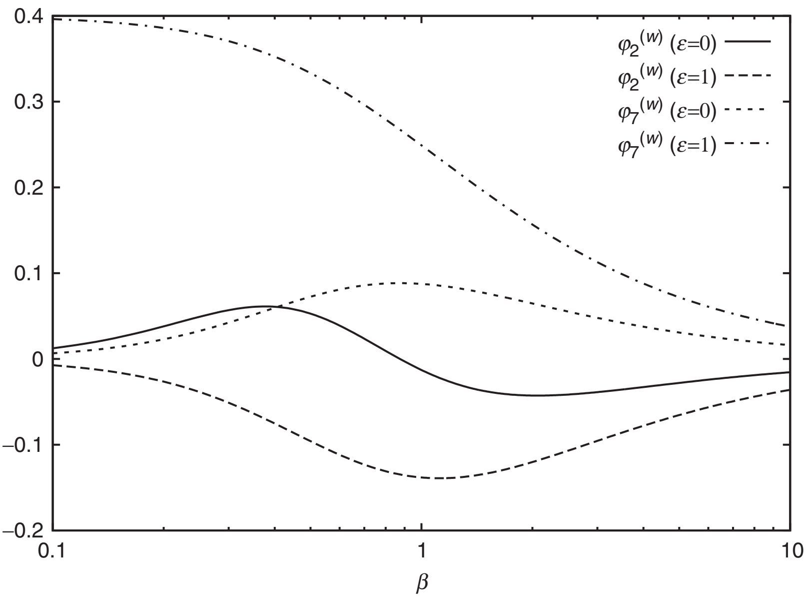

Figure 5. The quenching functions for "$\boldsymbol{\Omega} \times \mathbf{J}$" generation effect for different parameters.

For the limit of the strong LSMF we get

$$\begin{aligned}\mathcal{E}_i^{(w)}|_{\beta\to\infty} = &\left\{(17\varepsilon+47)\left(\nabla_i\left(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}}\right) - \frac{\left(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}}\right)}{2}\nabla_i\log\left(\overline{B}^2\right)\right)\right.\\ &-(21\varepsilon+43)\left(\Omega_i\frac{\left(\overline{\mathbf{B}}\cdot\nabla\right)}{2}\log\left(\overline{B}^2\right)+\overline{B}_i\frac{\left(\overline{\mathbf{B}}\cdot\nabla\right)\left(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}}\right)}{\overline{B}^2}+\frac{\left(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}}\right)}{\overline{B}^2}\left(\overline{\mathbf{B}}\cdot\nabla\right)\overline{B}_i\right)\\ &+3(21\varepsilon+43)\frac{\left(\boldsymbol{\Omega}\cdot\overline{\mathbf{B}}\right)\left(\overline{\mathbf{B}}\cdot\nabla\right)}{2\overline{B}^2}\log\left(\overline{B}^2\right)\overline{B}_i\\ &\left.-(37\varepsilon+27)\left(\overline{B}_i\frac{\left(\boldsymbol{\Omega}\cdot\nabla\right)}{2}\log\left(\overline{B}^2\right)-\left(\boldsymbol{\Omega}\cdot\nabla\right)\overline{B}_i\right)\right\}\frac{\pi}{512\beta}\langle u^{(0)2}\rangle\tau_c^2. \end{aligned} \tag{39}$$

From there we find that $\boldsymbol{\Omega}\times\mathbf{J}$-effect (terms in the first and in the last line of (39)) maintain the induction of the MEMF along direction of the LSMF even for the strong magnetic fields. The amplitude of effect tends to constant as the strength of LSMF is increased. It is hardly possible to make a definite conclusion about the dynamo effect in this case, because the generation part of (39) is contributed by terms with opposite signs.

The MEMF's contributions due to shear are defined by

$$\begin{aligned}\mathcal{E}_i^{(V)} = &\langle u^{(0)2}\rangle\tau_c^2\varepsilon_{inm}\left\{\varphi_1^{(V)}\overline{V}_{n,l}\overline{B}_{l,m}+\frac{\overline{B}_l}{\overline{B}^2}\overline{V}_{l,k}(\varphi_2^{(V)}\overline{B}_m\overline{B}_{k,n}+\varphi_3^{(V)}\overline{B}_n\overline{B}_{k,m})\right.\\ &+(\varphi_4^{(V)}\overline{V}_{n,l}+\varphi_5^{(V)}\overline{V}_{l,n})\overline{B}_{m,l}+\varphi_6^{(V)}\frac{\overline{B}_l\overline{B}_n}{\overline{B}^2}\overline{V}_{l,k}\overline{B}_{m,k}+\varphi_7^{(V)}\frac{\overline{B}_k\overline{B}_l}{\overline{B}^2}\overline{V}_{l,k}\overline{B}_{m,n}\\ &\left.+\varphi_8^{(V)}\frac{\overline{B}_k\overline{B}_l}{\overline{B}^2}\overline{V}_{m,n}\overline{B}_{l,k}+\varphi_9^{(V)}\overline{V}_{l,n}\overline{B}_{l,m}+\frac{\overline{B}_k\overline{B}_l}{\overline{B}^2}\overline{V}_{l,n}(\varphi_{10}^{(V)}\overline{B}_{k,m}+\varphi_{11}^{(V)}\overline{B}_{m,k})\right\}, \end{aligned} \tag{40}$$

where, for the sake of simplicity, we leave only the largest contributions and those which are important for the solar-type dynamo models, where the strength of LSMF component along direction of the large-scale flow dominates components directed along the shear. Reader can find the expressions for $\varphi_n^{(V)}$ in Appendix A. The full expression has a much more complicated tensorial structure than (40). In the case of the strong LSMF we get

$$\begin{aligned}\mathcal{E}_i^{(V)}|_{\beta\to\infty} = \frac{\tau_c^2}{6}\langle u^{(0)2}\rangle\varepsilon_{inm}&\left\{\left(\frac{\varepsilon+15}{20}\overline{V}_{n,l}-\frac{\varepsilon}{5}\overline{V}_{l,n}\right)\overline{B}_{m,l}+(\varepsilon+1)\frac{\overline{B}_l\overline{B}_n}{\overline{B}^2}\overline{V}_{l,k}\overline{B}_{m,k}\right.\\ &-\frac{3\varepsilon+13}{20}\overline{V}_{n,l}\overline{B}_{l,m}-\frac{\varepsilon+3}{2}\frac{\overline{B}_l\overline{B}_m}{\overline{B}^2}\overline{V}_{l,k}\overline{B}_{k,m}+\frac{\varepsilon+1}{10}\overline{V}_{l,n}\overline{B}_{l,m}\\ &\left.+\frac{\overline{B}_k\overline{B}_l}{\overline{B}^2}\overline{V}_{l,n}\left((\varepsilon+1)\overline{B}_{m,k}-\frac{\varepsilon+3}{4}\overline{B}_{k,m}\right)\right\}, \end{aligned} \tag{41}$$

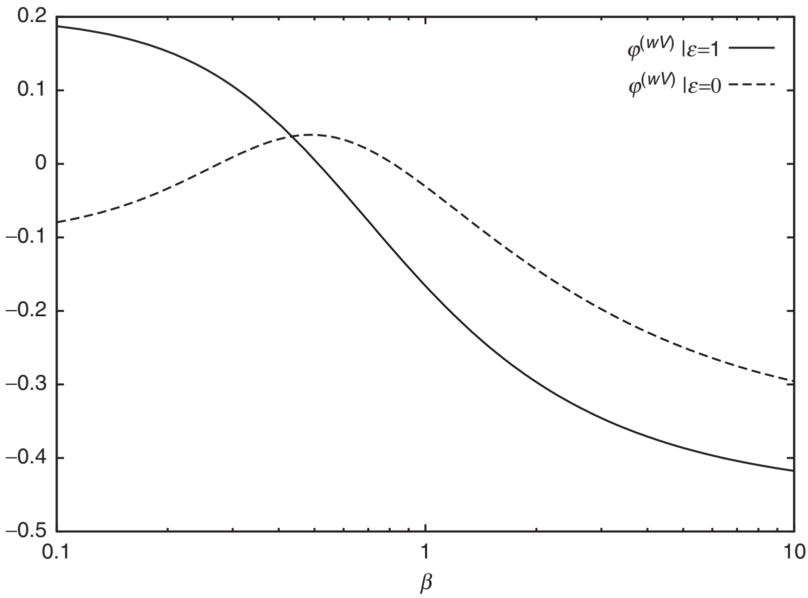

Figure 6. The dependence of induction effect along the nonuniform toroidal LSMF on the strength of magnetic field.

Now, we would like to consider efficiency of induction effect along the nonuniform LSMF due to global rotation and shear in nonlinear regimes for the Keplerian discs. As before, we assume a disc penetrated by the large-scale toroidal magnetic field that is nonuniform along the axis of rotation. From (39) and (40) we get

$$\mathcal{E}_{\phi} \approx \frac{\Omega^*}{2}\left\langle u^{(0)2}\right\rangle \tau_c \varphi^{(wV)} \frac{\partial B}{\partial z}, \tag{42}$$

where the quenching function is $\varphi^{(wV)} = \varphi_2^{(w)} + \varphi_7^{(w)} + 1.5(\varphi_9^{(V)} + \varphi_{10}^{(V)})$. Here, $\Omega^* = 2\Omega_0\tau_c$ and the MEMF is computed at $r = r_0$ in a rotating frame of reference which is defined by angular velocity Ω_0 in a given point $r = r_0$. Note, equation (42) transforms to equation (32) in limit $\beta \to 0$. The dependence of $\varphi^{(wV)}$ on the LSMF's strength is shown in the figure 6.

Results given in the figuere 6 show that the $\varphi^{(wV)}$ is positive for $\beta < 1$ and negative for $\beta > 1$ for all ε. This result supports an idea about the change of dynamo type in passing from linear to non-linear regime of the LSMF's generation by $\mathbf{\Omega} \times \mathbf{J}$ and shear-current effects. Previously we found that the induction term due to $\mathbf{\Omega} \times \mathbf{J}$ effect tends to constant when $\beta \to \infty$ (equation (39)) while the induction term due to shear-current effect is growing under $\beta \to \infty$ (equation (41)). Therefore, the primary nonlinear generation effect in the differentially rotating *uniform* MHD turbulence penetrated by the *nonuniform* toroidal LSMF may be due to shear-current effect. The same nonlinear dependence of shear-current effect was discovered in the article by Rogachevskii and Kleeorin (2004a) for the different kind of MTA. In the next section I show that the given sources of the MEMF ultimately result in current helicity generation. Therefore, the effect considered above is saturated dynamically due to magnetic helicity conservation law.

4. The current helicity evolution

As we have seen, the current helicity contributes to the different kind of MEMF's action, not only to the α-effect. The recent articles (Subramanian and Brandenburg 2004) show that the magnetic helicity conservation law can be described in terms of the current helicity evolution if the assumption of the scale separation is fulfilled. For the time being the redistribution of current helicity over the space scales is not satisfactorily understood. One attempt to describe the helicity evolution in turbulent media penetrated by LSMF was given in the articles by Brandenburg and Subramanian (2005); Subramanian and Brandenburg (2004). Here, we will follow their results and obtain the explicit evolutionary equation for the current helicity. The equation in question can be derived from (3) and (4). After integration over the large-scale variables we can get the general equation for the current helicity in the following form

$$\frac{\partial h_C}{\partial t} = -\frac{h_C}{\tau_h} + \frac{2}{\mu\rho}\varepsilon_{plm}\int\left[k^2\hat{\varkappa}_{lp}\frac{\overline{B}_m}{\rho} - \mathrm{i}\hat{\varkappa}_{lp}(\mathbf{k}\cdot\nabla)\left(\frac{\overline{B}_m}{\rho}\right) - \frac{\mathrm{i}}{2}(\mathbf{k}\cdot\nabla)\left(\hat{\varkappa}_{lp}\frac{\overline{B}_m}{\rho}\right)\right.$$
$$\left. + \mathrm{i}k_p\nabla_n\left(\hat{\varkappa}_{ln}\frac{\overline{B}_m}{\rho}\right) + \frac{1}{2}\overline{V}_{l,n}\left(\mathrm{i}k_p - \frac{1}{2}\nabla_p\right)\left(\hat{h}_{mn} - \hat{h}_{nm}\right) - \frac{1}{2}\overline{V}_{l,m}\nabla_n\hat{h}_{np}\right]\mathrm{d}\mathbf{k}. \quad (43)$$

The third order moments were replaced by $-h_C/\tau_h$, where τ_h is a relaxation time for the current helicity. This is a rather rough way because the triple correlations may give important contribution for the helicity redistribution over the space scales (Frisch *et al*. 1975, Kleeorin and Ruzmaikin 1982, Kleeorin and Rogachevskii 1999). Because of the very rough assumptions used in derivation of (43), it should be considered with caution. In spite of the latter, the equation (43) provides a useful tool for investigation the nonlinear saturation in helical mean-field dynamo (Brandenburg and Subramanian 2004). Except for contributions due to density stratification and shear, equation (43) can be reproduced from results of BS05 after substiuation identity $\varepsilon_{ijk}\varepsilon_{ipq}\varepsilon_{qlm} = \varepsilon_{lmk}\delta_{jp} - \varepsilon_{lmj}\delta_{kp}$ in equation (10.71) of BS05. Inspection of (43) shows that if we replace $k^2 \to \ell_c^{-2}$ and use (12), we can write the evolutionary equation in the following form,

$$\frac{\partial h_C}{\partial t} = -\frac{2\left(\boldsymbol{\mathcal{E}}\cdot\overline{\mathbf{B}}\right)}{\mu\rho\ell_c^2} - \frac{h_C}{\tau_h} + \frac{2}{\mu\rho}\varepsilon_{plm}\int\left[-\mathrm{i}\hat{\varkappa}_{lp}k^n\nabla_n\left(\frac{\overline{B}_m}{\rho}\right) - \frac{\mathrm{i}}{2}(\mathbf{k}\cdot\nabla)\left(\hat{\varkappa}_{lp}\frac{\overline{B}_m}{\rho}\right)\right.$$
$$\left. + \mathrm{i}k_p\nabla_n\left(\hat{\varkappa}_{ln}\frac{\overline{B}_m}{\rho}\right) + \frac{1}{2}\overline{V}_{l,n}\left(\mathrm{i}k_p - \frac{1}{2}\nabla_p\right)\left(\hat{h}_{mn} - \hat{h}_{nm}\right) - \frac{1}{2}\overline{V}_{l,m}\nabla_n\hat{h}_{np}\right]\mathrm{d}\mathbf{k}. \quad (44)$$

According to Frisch *et al*. (1975) Kleeorin and Ruzmaikin (1982), Vainshtein (1983), Brandenburg (2001), and Vishniac and Cho (2001) the first term in (44) is responsible for helicity generation in turbulent medium. The rest part of equation can be interpreted as the helicity fluxes (Vishniac and Cho 2001, Subramanian and Brandenburg 2004, 2005). The given expression for helicity fluxes is incomplete because the contribution of the third order moments is dropped in (44). As the first step we consider the case of the weak LSMF. From (44) and (18,17,19) we get

$$\begin{aligned}\frac{\partial h_{\mathcal{C}}}{\partial t}+\frac{1}{\tau_h}h_{\mathcal{C}} = &-\frac{2}{\mu\rho\ell_c^2}\left(\boldsymbol{\mathcal{E}}\cdot\overline{\mathbf{B}}\right)+\frac{(\varepsilon-1)}{\mu\rho\tau_c}\Big\{2f_1^{(a)}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)\left(\mathbf{e}\cdot\left(\mathbf{U}\times\overline{\mathbf{B}}\right)\right)\\ &+\frac{(\mathbf{e}\cdot\mathbf{G})}{3}\left(f_4^{(d)}\overline{B}^2+f_3^{(d)}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)^2\right)+2f_2^{(a)}\left(\overline{\mathbf{B}}\cdot\left(\nabla\times\overline{\mathbf{B}}\right)\right)\\ &+\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)\left(\frac{1}{3}f_4^{(d)}\left(\overline{\mathbf{B}}\cdot\mathbf{G}\right)+\frac{4f_9^{(a)}}{(\varepsilon+1)}\left(\overline{\mathbf{B}}\cdot\mathbf{U}\right)\right)-f_4^{(d)}\frac{(\mathbf{e}\cdot\nabla)}{6}\overline{B}^2\\ &-\frac{4}{3}f_1^{(a)}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)\left(\mathbf{e}\cdot\left(\nabla\times\overline{\mathbf{B}}\right)\right)-f_3^{(d)}\frac{(\mathbf{e}\cdot\nabla)}{6}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)^2-f_4^{(d)}\frac{(\mathbf{B}\cdot\nabla)}{3}\left(\mathbf{e}\cdot\overline{\mathbf{B}}\right)\Big\},\end{aligned}\tag{45}$$

where substitution $\langle u^{(0)2}\rangle\ell_c^{-2}\to\tau_c^{-2}$ was used, and $\mathbf{e}=\boldsymbol{\Omega}/|\Omega|$. Here, we dropped the contributions due to shear because their effect to the mean electromotive force was computed only to the zero order terms about angular velocity. Furthermore, in (45) we kept only those contributions which could be the most interesting from the stellar dynamo applications standpoint. Note, for the equipartition case, $\varepsilon=1$, helicity evolution satisfies the simple equation

$$\frac{\partial h_{\mathcal{C}}}{\partial t}+\frac{1}{\tau_h}h_{\mathcal{C}}=-\frac{2\left(\boldsymbol{\mathcal{E}}\cdot\overline{\mathbf{B}}\right)}{\mu\rho\ell_c^2}.\tag{46}$$

It is in accordance with equation for the magnetic helicity density obtained by Subramanian and Brandenburg (2005). As an example of application of (46) to the problem of the nonlinear saturation of alpha-effect, consider the α^2 dynamo in the fast rotation limit. For the sake of simplicity we restrict ourselves only with the isotropic components of α-effect and neglect the helicity loss due to $h_{\mathcal{C}}/\tau_h$. From (46) and (26) we get

$$\frac{\partial h_{\mathcal{C}}}{\partial t}=\frac{\pi\beta^2}{4\tau_c}\left(2\langle u^{(0)2}\rangle\Omega^*(\mathbf{e}\cdot\mathbf{G})-h_{\mathcal{C}}\right),\tag{47}$$

where we keep the contributions of order Ω^{*-1} for the current helicity, and drop the terms which are due to nonuniform LSMF. If L is the typical spatial scale of the LSMF then the equation (47) is justified when $LG\Omega^*\gg 1$ and $\mu\rho|h_{\mathcal{C}}|\gg\left|\overline{\mathbf{B}}\cdot\left(\nabla\times\overline{\mathbf{B}}\right)\right|$. The point to note that in (47) we implicitly assume that $h_{\mathcal{C}}^{(0)}\equiv h_{\mathcal{C}}$. It is a shortcoming of the theory. However, this procedure is widely used in the literature (Kleeorin and Ruzmaikin 1982, Vainshtein and Kitchatinov 1983, Vishniac and Cho 2001, Kleeorin *et al.* 2003, Brandenburg and Subramanian 2004). With initial condition, $t=0$, $h_{\mathcal{C}}=0$, we write, similar to Vainshtein (1983), the solution of equation (47) is as follows,

$$h_{\mathcal{C}}=2\Omega^*\langle u^{(0)2}\rangle(\mathbf{e}\cdot\mathbf{G})\left(1-\exp\left(-\frac{\pi}{4\tau_c}\int_0^t\beta^2 dt\right)\right).\tag{48}$$

The given solution shows that under $t\to\infty$ we get $h_{\mathcal{C}}\to 2\Omega^*\langle u^{(0)2}\rangle(\mathbf{e}\cdot\mathbf{G})\tau_c$. On this basis, and in taking into account (26), we can conclude that α-effect will saturates exponentially under the increase of the LSMF strength. Furthermore, this conclusion was confirmed with numerical dynamo model which is considered by author in the separate article (Pipin 2007).

Next, we consider the equation for the current helicity evolution for the slow rotation limit. No restriction is applied to the strength of LSMF. The contribution of shear to the transport and generation part of equation is described with a quite bulky tensor expressions and we decide to restrict ourselves with terms which have either a finite limit under $\beta \to 0$ or the amplitude functions that are greater than 0.1. We write the evolutionary equation for the current helicity as follows

$$\frac{\partial h_{\mathcal{C}}}{\partial t} + \frac{1}{\tau_h} h_{\mathcal{C}} = -\frac{2}{\mu\rho\ell_c^2}\left(\boldsymbol{\mathcal{E}} \cdot \overline{\mathbf{B}}\right) + \psi_1 \frac{\overline{B}_m \overline{B}_p}{\overline{B}^2} \overline{V}_{p,m} h_{\mathcal{C}} + (\psi_2 \mathbf{G} + \psi_3 \mathbf{U}) \cdot \mathbf{W}\left\langle u^{(0)2}\right\rangle$$
$$+ \frac{1}{\mu\rho} \nabla \cdot \left(\left[\psi_5 \nabla \times \overline{\mathbf{B}} + \psi_4\left(\mathbf{U} \times \overline{\mathbf{B}}\right)\right]\left(\overline{\mathbf{B}} \cdot \overline{\mathbf{V}}\right) + \psi_6 \mathbf{W} \overline{B}^2 \right) + (\varepsilon - 1)\{\ldots\}, \quad (49)$$

where $\mathbf{W} = \nabla \times \overline{\mathbf{V}}$. Quenching functions $\psi_{\{n\}}$ are given in Appendix A. Symbol $\{\ldots\}$ denotes those terms which are not important in the case $\varepsilon = 1$. Taking the Taylor expansion of (49) for the case $\overline{B} \to 0$ (keeping $\overline{B}^2$ terms) we get

$$\frac{\partial h_{\mathcal{C}}}{\partial t} + \frac{1}{\tau_h} h_{\mathcal{C}} = -\frac{2}{\mu\rho\ell_c^2}\left(\boldsymbol{\mathcal{E}} \cdot \overline{\mathbf{B}}\right) - \frac{4}{15} \frac{\overline{B}_m \overline{B}_p}{\mu\rho\left\langle u^{(0)2}\right\rangle} V_{p,m} h_{\mathcal{C}} - \frac{(\mathbf{G} \cdot \mathbf{W})}{6}\left\langle u^{(0)2}\right\rangle - \nabla \cdot \mathcal{F} \quad (50)$$

$$\mathcal{F} = \left(\frac{1}{6}\left\langle u^{(0)2}\right\rangle + \frac{2}{15}\frac{\overline{B}^2}{\mu\rho}\right)\mathbf{W} + \frac{2}{15\mu\rho}\left(\left[\nabla \times \overline{\mathbf{B}} - \left(\mathbf{U} \times \overline{\mathbf{B}}\right)\right]\left(\overline{\mathbf{B}} \cdot \overline{\mathbf{V}}\right)\right), \quad (51)$$

where we apply the equipartition condition, $\varepsilon = 1$, as well. The direction of the helicity flux due to the first contribution in (51), $\mathcal{F}^{\mathbf{W}} = \left(\left\langle u^{(0)2}\right\rangle/6 + 2\overline{B}^2/(15\mu\rho)\right)\mathbf{W}$, depends on distribution of the large-scale vorticity solely. The second term depends on details of the dynamo action. To estimate the direction of the helicity transport due to $\mathcal{F}^{\mathbf{W}}$ on the Sun we compute the vector field of the large-scale vorticity $\mathbf{W}$. In the spherical coordinate system we have $\mathbf{W} = \mathbf{e}^r(\sin\theta\partial\Omega/\partial\theta + \cos\theta\delta\Omega) - \sin\theta\mathbf{e}^\theta(r\partial\Omega/\partial r + 2\delta\Omega)$, where r, θ are the radial distance and the polar angle, respectively, and $\delta\Omega = \Omega - \Omega_0$, where Ω_0 is the equatorial angular velocity of the Sun at the surface. The distribution of the angular velocity is taken as an analytical fit given by Belvedere *et al.* (2000). It is shown at the left side figure 7. The computed vector field of the large-scale vorticity is shown at the right side figure 7.

The given figure shows the possibility of the outward helicity flux from the dynamo region due to shear. Note that one component of the helicity flux $\mathcal{F}^{\mathbf{W}}$ is due to the small-scale dynamo, $\left\langle u^{(0)2}\right\rangle\mathbf{W}/6$, and another is due to the LSMF, $2\overline{B}^2\mathbf{W}/(15\mu\rho)$. Among two, the contribution of the small-scale dynamo is likely to be dominated in the depth of convection zone. While the flux due to the LSMF may be important at near the surface level. The latter effect may produce the significant outward flux of helicity only with the open boundaries ((Brandenburg and Subramanian 2004, Subramanian and Brandenburg 2005). At the near surface level the amplitude of the large-scale vorticity, $|\mathbf{W}| \approx 4 \times 10^{-8}\mathrm{s}^{-1} \approx 1.5 \times 10^{-5}\mathrm{day}^{-1}$. The magnitude of the surface magnetic flux change during the solar cycle is about $10^{24} Mx$ (Schrijver and Harvey 1984). Then the magnitude of the helicity outflow from $2\overline{B}^2\mathbf{W}/(15\mu\rho)$ is about $10^{43} Mx^2\mathrm{day}^{-1}$. It is compatible with estimations given by (DeVore 2000).

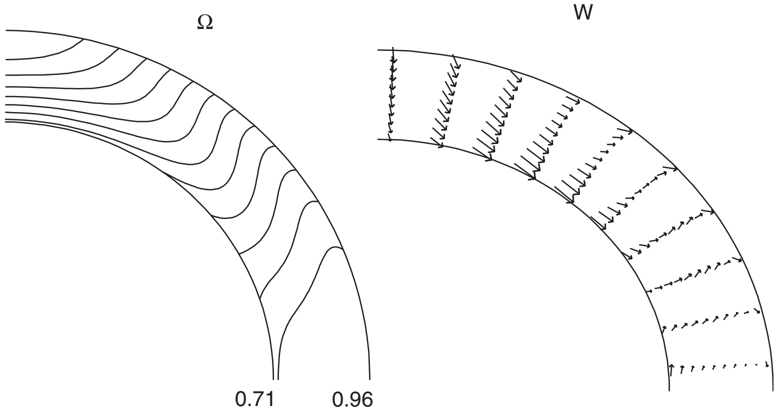

Figure 7. The isolines of the angular velocity distribution (left) and the corresponding vector field of the large-scale vorticity (right).

We have estimated only one part of the helicity flux. The numerical dynamo model based on the given results would help to get a more definite conclusions about this subject.

5. Summary

In this article, the mean electromotive force of turbulent flows and magnetic fields is computed analytically using the framework of mean-field dynamo theory and MTA (minimal τ approximation). There is an overlap in results obtained with SOCA and with MTA approximation. The two approximations give qualitatively the same results about nonlinear dependence of mean electromotive force on the strength of LSMF or on the Coriolis number. Also, there is a difference between predictions of SOCA and MTA for mean-electromotive force expressions if the shear is taken into account. This difference can be explained, in part, by the crudeness of the given version of tau approximation. The deficient accuracy of calculations of shear contribution is due to an assumption about the scale-independent τ. In whole, the accuracy of the theory presented in the article is comparable with the mixing-length approximation. This theory has no firm grounds and should be considered with caution.

Finally, I would like to focus on the new findings of the article. In this study it is shown that the new interesting component of transport of LSMF appears due to joint contribution of current helicity and shear. The effect does not disappear in the strong LSMF limit, $\beta \gg 1$. It may be important near the base of solar CZ where the influence of rotation and shear on the turbulence is quite strong. Furthermore, the analysis, which we carried out for the current helicity evolution, suggests that the shear and rotation may redistribute the helicity in solar CZ amplifying it (in amplitude) at the near equatorial regions in agreement with observations. Beside, the effect of rotation and stratification on the h_C evolution is calculated explicitly. Basically, the equation for current helicity is obtained using the same approach as for the mean electromotive force and on the base of quantities which are explicitly gauge invariants.

Therefore, we can expect that the dynamo model based on the above approach could be capable for meeting the requirements of both solar and stellar dynamo simulations.

Acknowledgments

This work is benefited from the research grants 05-02-16326, 07-02-00246, 4741.2006.2, the grant of Royal Society and grant of Trinity College of Cambridge. I thank for hospitality of the Department of Applied Mathematics and Theoretical Physics of University of Cambridge. I have benefited from stimulating discussion on the subject of the article with Michael Proctor. I thank sincerely K.-H. Rädler and A. Brandenburg for criticism and helpful suggestions. I acknowledge valuable discussion with Nathan Kleeorin and Igor Rogachevskii. Gratitude also goes to Kirill Kuzanyan for the critical reading the manuscript. The large portion of calculations presented in the article has been carried out with the use of Maxima computer algebra system (http://Maxima.sf.net). I owe sincere thanks to all developers of Maxima and personally to Victor Toth (Toth 2005) for maintaining the Maxima's package for tensor calculations.

References

Belvedere, G., Kuzanyan, K.M. and Sokoloff, D., A two-dimensional asymptotic solution for a dynamo wave in the light of the solar internal rotation. *Mon. Not. R. Astron. Soc.*, 2000, **315**, 778–790.

Blackman, E.G. and Brandenburg, A., Dynamic nonlinearity in large scale dynamos with shear. *Astrophys. J.*, 2002, **579**, 379–373.

Blackman, E.G. and Field, G.B., New dynamical mean-field dynamo theory and closure approach. *Phys. Rev. Lett.*, 2002, **89**, 265007, 1–4.

Brandenburg, A., The inverse cascade and nonlinear alpha effect in simulations of isotropic helical hydromagnetic turbulence. *Astrophys. J.*, 2001, **550**, 824–840.

Brandenburg, A. and Subramanian, K., Strong mean field dynamos require supercritical helicity fluxes. *Astron. Nachr.*, 2004, **325**, 400–408.

Brandenburg, A. and Subramanian, K., Astrophysical magnetic fields and nonlinear dynamo theory. *Phys. Rep.*, 2005, **417**, 1–209.

DeVore, C.R., Magnetic helicity generation by solar differential rotation. *Astrophys. J.*, 2000, **539**, 944–953.

Field, G.B. and Blackman, E.G., Dynamical quenching of the alpha^2 dynamo. *Astrophys. J.*, 2002, **572**, 685–692.

Frisch, U., Pouquet, A. and Léorat, J.A.M., Possibility of an inverse cascade of magnetic helicity in magnetohydrodynamic turbulence. *J. Fluid Mech.*, 1975, **68**, 769–778.

Kichatinov, L.L., Turbulent transport of magnetic fields in a highly conducting rotating fluid and the solar cycle. *Astron. Astrophys.*, 1991, **243**, 483–491.

Kichatinov, L.L., Turbulent transport in rotating and magnetized fluids. In: *Turbulence, Waves and Instabilities in the Solar Plasma*, edited by R. Erdélyi, K. Petroway, B. Roberts and M. Aschwanden, pp. 81–96, 2003 (Kluwer).

Kichatinov, L.L., Pipin, V. and Rüdiger, G., Turbulent viscosity, magnetic diffusivity, and heat conductivity under the influence of rotation and magnetic field. *Astron. Nachr.*, 1994, **315**, 157–170.

Kichatinov, L.L. and Rüdiger, G., Magnetic-field advection in inhomogeneous turbulence. *Astron. Astrophys.*, 1992, **260**, 494–498.

Kleeorin, N., Kuzanyan, K., Moss, D., Rogachevskii, I. and Sokoloff, D.H.Z., Magnetic helicity evolution during the solar activity cycle: observations and dynamo theory. *Astron. Astrophys.*, 2003, **409**, 1097–1105.

Kleeorin, N. and Rogachevskii, I., Magnetic helicity tensor for an anisotropic turbulence. *Phys. Rev. E.*, 1999, **59**, 6724–6729.

Kleeorin, N.I. and Ruzmaikin, A.A., Dynamics of the average turbulent helicity in a magnetic field. *Magnetohydrodynamics*, 1982, **18**, 116–122.

Krause, F. and Rädler, K.-H., *Mean-Field Magnetohydrodynamics and Dynamo Theory*, 1980 (Akedemie-Verlag, Berlin).

Moffatt, H.K., *Magnetic Field Generation in Electrically Conducting Fluids*, 1978 (Cambridge University Press, Cambridge, England).

Orszag, S., Analytical theories of turbulence. *J. Fluid Mech.*, 1970, **41**, 363–386.

Parker, E.N., *Cosmical Magnetic Fields: Their Origin and their Activity*, 1979 (Clarendon Press, Oxford).

Pipin, V.V., Generation of the solar magnetic fields in $\alpha\delta\Omega$ dynamo. *Astron. Rep.*, 2007, **51**, 5, 411–420.

Pouquet, A., Frisch, U. and Léorat, J., Strong MHD helical turbulence and the nonlinear dynamo effect. *J. Fluid Mech.*, 1978, **68**, 769–778.

Rädler, K.-H., On the electrodynamics of turbulent fields under the influence of corilois forces. *Monats. Dt. Akad. Wiss.*, 1969, **11**, 194–201.

Rädler, K.-H., Kleeorin, N. and Rogachevskii, I., The mean electromotive force for mhd turbulence: the case of a weak mean magnetic field and slow rotation. *Geophys. Astrophys. Fluid Dyn.*, 2003, **97**, 249–269.

Rädler, K.-H. and Rheinhardt, M., Mean-field electrodynamics: critical analysis of various analytical approaches to the mean electromotive force. *Geophys. Astrophys. Fluid Dyn.*, 2006, (astro– ph/0606267) (Accepted).

Rädler, K.-H. and Stepanov, R., The mean electromotive force due to turbulence of a conducting fuild in the presence of mean flow. *Phys. Rev.*, 2006, **73**(056311), 1–15.

Roberts, P. and Soward, A., A unified approach to mean field electrodynamics. *Astron. Nachr.*, 1975, **296**, 49–64.

Rogachevskii, I. and Kleeorin, N., Electromotive force and large-scale magnetic dynamo in a turbulent flow with a mean shear. *Phys. Rev. E*, 2003, **68**(036301), 1–12.

Rogachevskii, I. and Kleeorin, N., Effects of differential and uniform rotation on nonlinear electromotive force in a turbulent flow, 2004a, arXiv:astro-ph/0407375v2.

Rogachevskii, I. and Kleeorin, N., Nonlinear theory of a shear-current effect and mean-field magnetic dynamos. *Phys. Rev. E*, 2004b, **70**(046310), 1–15.

Rogachevskii, I. and Kleeorin, N., Shear-current effect in a turbulent convection with a large-scale shear. *Phys. Rev. E*, 2007, **75**, 046305/1–7.

Rüdiger, G. and Kichatinov, L.L., α-effect and alpha-quenching. *Astron. Astrophys.*, 1993, **269**, 581–588.

Rüdiger, G. and Kichatinov, L.L., Do mean-field dynamos in nonrotating turbulent shear-flows exist? *Astron. Nachr.*, 2006, **327**, 298–303.

Schrijver, C. and Harvey, K., The photospheric magnetic flux budget. *Sol. Phys.* 1984, **150**, 1–10.

Subramanian, K. and Brandenburg, A., Nonlinear current helicity fluxes in turbulent dynamos and alpha quenching. *Phys. Rev. Lett.*, 2004, **93**, 205001.

Subramanian, K. and Brandenburg, A., Magnetic helicity density and its flux in weakly inhomogeneous turbulence, *Astrophys. J.*, 2006, **648**, L71-L74, astro-ph/0509392.

Toth, V., Tensor manipulation in GPL Maxima, 2005, cs.SC/0503073.

Vainshtein, S.I., *Magnitnie polia v cosmose*. 1983 (Nauka, Moscow).

Vainshtein, S.I. and Kitchatinov, L.L., The macroscopic magnetohydrodynamics of inhomogeneously turbulent cosmic plasmas. Geophys. Astrophys. Fluid Dynam., 1983, **24**, 273–298.

Vishniac, E.T. and Cho, J., Magnetic helicity conservation and astrophysical dynamos. *Astrophys. J.*, 2001, **550**, 752–760.

Appendix

Appendix A

This part of appendix gives the functions of the Coriolis number defining the dependence of the turbulent transport generation and diffusivities on the angular velocity.

$$
\begin{aligned}
f_1^{(a)} &= \frac{1}{4\Omega^{*2}}\left(\left(\Omega^{*2}+3\right)\frac{\arctan\Omega^*}{\Omega^*}-3\right),\\
f_2^{(a)} &= \frac{1}{4\Omega^{*2}}\left(\left(\Omega^{*2}+1\right)\frac{\arctan\Omega^*}{\Omega^*}-1\right),\\
f_3^{(a)} &= \frac{1}{4\Omega^{*2}}\left(\left((\varepsilon-1)\Omega^{*2}+\varepsilon-3\right)\frac{\arctan\Omega^*}{\Omega^*}+3-\varepsilon\right),\\
f_4^{(a)} &= \frac{1}{6\Omega^{*3}}\left(3\left(\Omega^{*4}+6\varepsilon\Omega^{*2}+10\varepsilon-5\right)\frac{\arctan\Omega^*}{\Omega^*}-\left((8\varepsilon+5)\Omega^{*2}+30\varepsilon-15\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
f_5^{(a)} &= \frac{1}{3\Omega^{*3}}\left(3\left(\Omega^{*4}+3\varepsilon\Omega^{*2}+5(\varepsilon-1)\right)\frac{\arctan\Omega^*}{\Omega^*}-\left((4\varepsilon+5)\Omega^{*2}+15(\varepsilon-1)\right)\right),\\
f_6^{(a)} &= -\frac{1}{48\Omega^{*3}}\left(3\left((3\varepsilon-11)\Omega^{*2}+5\varepsilon-21\right)\frac{\arctan\Omega^*}{\Omega^*}-\left(4(\varepsilon-3)\Omega^{*2}+15\varepsilon-63\right)\right),\\
f_7^{(a)} &= \frac{1}{48\Omega^{*3}}\left(3\left((5\varepsilon+3)\Omega^{*2}+11\varepsilon+5\right)\frac{\arctan\Omega^*}{\Omega^*}-\left(4(\varepsilon+1)\Omega^{*2}+33\varepsilon+15\right)\right),\\
f_8^{(a)} &= -\frac{1}{12\Omega^{*3}}\left(3\left((3\varepsilon+1)\Omega^{*2}+4\varepsilon-2\right)\frac{\arctan\Omega^*}{\Omega^*}-\left(5(\varepsilon+1)\Omega^{*2}+12\varepsilon-6\right)\right),\\
f_9^{(a)} &= \frac{(\varepsilon+1)}{4\Omega^*}\left(\frac{\arctan\Omega^*}{\Omega^*}-1\right),\\
f_{10}^{(a)} &= -\frac{1}{3\Omega^{*3}}\left(3\left(\Omega^{*2}+1\right)\left(\Omega^{*2}+\varepsilon-1\right)\frac{\arctan\Omega^*}{\Omega^*}-\left((2\varepsilon+1)\Omega^{*2}+3\varepsilon-3\right)\right),\\
f_{11}^{(a)} &= -\frac{1}{6\Omega^{*3}}\left(3\left(\Omega^{*2}+1\right)\left(\Omega^{*2}+2\varepsilon-1\right)\frac{\arctan\Omega^*}{\Omega^*}-\left((4\varepsilon+1)\Omega^{*2}+6\varepsilon-3\right)\right).
\end{aligned}
$$

The dependence of turbulent diffusivities on the Coriolis number (equation (28)) is given by

$$
\begin{aligned}
f_1^{(d)} &= \frac{1}{2\Omega^{*3}}\left((\varepsilon+1)\Omega^{*2}+3\varepsilon-\left((2\varepsilon+1)\Omega^{*2}+3\varepsilon\right)\frac{\arctan(\Omega^*)}{\Omega^*}\right),\\
f_2^{(d)} &= \frac{1}{4\Omega^{*2}}\left(\left((\varepsilon-1)\Omega^{*2}+3\varepsilon+1\right)\frac{\arctan(\Omega^*)}{\Omega^*}-(3\varepsilon+1)\right),\\
f_3^{(d)} &= \frac{1}{2\Omega^{*3}}\left(3\left(3\Omega^{*2}+5\right)\frac{\arctan(\Omega^*)}{\Omega^*}-\left(4\Omega^{*2}+15\right)\right),\\
f_4^{(d)} &= \frac{1}{2\Omega^{*3}}\left(\left(2\Omega^{*2}+3\right)-3\left(\Omega^{*2}+1\right)\frac{\arctan(\Omega^*)}{\Omega^*}\right).
\end{aligned}
$$

The magnetic quenching functions of the generation and transport effects in equation (33) are

$$
\begin{aligned}
\varphi_1^{(a)} &= \frac{1}{64\beta^2}\left(\left(4(3\varepsilon+1)\beta^2-17\varepsilon+21\right)\frac{\arctan(2\beta)}{2\beta}+\frac{\left(4(11\varepsilon-15)\beta^2+17\varepsilon-21\right)}{(4\beta^2+1)}\right),\\
\varphi_2^{(a)} &= \frac{(1-\varepsilon)}{8\beta^2}\left(\frac{\arctan(2\beta)}{2\beta}-1\right),\\
\varphi_3^{(a)} &= \frac{1}{3072\beta^4}\left(\frac{8\beta^2\left(2\beta^2\left(16(45\varepsilon+107)\beta^2-1731\varepsilon+739\right)-2097\varepsilon+97\right)-2295\varepsilon-105}{(4\beta^2+1)^2}\right.\\
&\qquad\left.-3\left(12\beta^2\left(16(3\varepsilon+5)\beta^2-41\varepsilon+41\right)-765\varepsilon-35\right)\frac{\arctan(2\beta)}{2\beta}\right),\\
\varphi_4^{(a)} &= \frac{1}{3072\beta^4}\left(\frac{8\beta^2\left(2\beta^2\left(128(3\varepsilon+1)\beta^2+807\varepsilon-71\right)+555\varepsilon-11\right)+459\varepsilon+21}{(4\beta^2+1)^2}\right.\\
&\qquad\left.-3\left(4(115\varepsilon-19)\beta^2+153\varepsilon+7\right)\frac{\arctan(2\beta)}{2\beta}\right),
\end{aligned}
$$

$$
\begin{aligned}
\varphi_5^{(a)} &= \frac{1}{3072\beta^4}\left(3\big(4\beta^2\big(48(3\varepsilon+5)\beta^2 - 41\varepsilon + 41\big) - 153\varepsilon - 7\big)\frac{\arctan(2\beta)}{2\beta}\right.\\
&\quad \left. - \frac{4\beta^2\big(16(9\varepsilon+31)\beta^2 - 429\varepsilon + 109\big) - 459\varepsilon - 21}{(4\beta^2+1)}\right),\\
\varphi_6^{(a)} &= \frac{1}{3072\beta^4}\left(3\big(4(163\varepsilon-3)\beta^2 - 153\varepsilon - 7\big)\frac{\arctan(2\beta)}{2\beta}\right.\\
&\quad \left. - \frac{4\beta^2\big(64(21\varepsilon+19)\beta^2 + 183\varepsilon - 23\big) - 459\varepsilon - 21}{(4\beta^2+1)}\right),\\
\varphi_7^{(a)} &= -\frac{1}{48\beta^4}\left(3\big(2\beta^2\big(4(\varepsilon+1)\beta^2 - 3\varepsilon + 3\big) - 10\varepsilon + 5\big)\frac{\arctan(2\beta)}{2\beta}\right.\\
&\quad \left. - \frac{\big(2\beta^2\big(20(\varepsilon+1)\beta^2 - 49\varepsilon + 29\big) - 30\varepsilon + 15\big)}{(4\beta^2+1)}\right),
\end{aligned}
$$

$$
\begin{aligned}
\varphi_8^{(a)} &= \frac{1}{48\beta^4}\left(3\big(2\varepsilon\big(2\beta^2-1\big)+1\big)\frac{\arctan(2\beta)}{2\beta} - \frac{\big(4\beta^2\big(8(\varepsilon+1)\beta^2 - \varepsilon + 2\big) - 6\varepsilon + 3\big)}{(4\beta^2+1)}\right),\\
\varphi_9^{(a)} &= \frac{1}{48\beta^4}\left(3\big(2\beta^2\big(4(\varepsilon+1)\beta^2 - \varepsilon + 1\big) - 2\varepsilon + 1\big)\frac{\arctan(2\beta)}{2\beta} - \big(2(\varepsilon+1)\beta^2 - 6\varepsilon + 3\big)\right),\\
\varphi_{10}^{(a)} &= \frac{1}{48\beta^4}\left(\big(4(\varepsilon+1)\beta^2 + 6\varepsilon - 3\big) - 3\big(2\varepsilon\big(2\beta^2+1\big) - 1\big)\frac{\arctan(2\beta)}{2\beta}\right).
\end{aligned}
$$

The nonlinear turbulent diffusion of the LSMF in (37) is expressed with help of the following functions

$$
\begin{aligned}
\varphi_1 &= \frac{(\varepsilon-1)}{16\beta^2}\left(3\frac{\arctan(2\beta)}{\beta} - 2\frac{\big(8\beta^2+3\big)}{(4\beta^2+1)}\right),\\
\varphi_2 &= \frac{(\varepsilon+1)}{32\beta^2}\left(\big(4\beta^2+3\big)\frac{\arctan(2\beta)}{\beta} - 3\right),\\
\varphi_3 &= \frac{1}{8\beta^2}\left(\frac{\arctan(2\beta)}{\beta} - 2\right).
\end{aligned}
$$

The effect of slow rotation and nonuniform LSMF on the MEMF (equation (38)) is expressed with help of the following functions

$$
\begin{aligned}
\varphi_1^{(w)} &= -\frac{1}{6144\beta^4}\left(\frac{4\beta^2\big(4\beta^2\big(4(51\varepsilon-371)\beta^2 + 291\varepsilon + 29\big) - 219\varepsilon + 1819\big) - 315\varepsilon + 1275}{(4\beta^2+1)^2}\right.\\
&\quad \left. + 3\big(8\beta^2\big(2(17\varepsilon+47)\beta^2 - 51\varepsilon + 51\big) + 105\varepsilon - 425\big)\frac{\arctan(2\beta)}{2\beta}\right),\\
\varphi_2^{(w)} &= -\frac{1}{6144\beta^4}\left(3\big(8\beta^2\big(2(37\varepsilon+27)\beta^2 + 99\varepsilon - 99\big) + 105\varepsilon - 425\big)\frac{\arctan(2\beta)}{2\beta}\right.\\
&\quad \left. - \frac{4\beta^2\big(4\beta^2\big(4(273\varepsilon+47)\beta^2 + 1269\varepsilon - 1589\big) + 1119\varepsilon - 2719\big) + 315\varepsilon - 1275}{(4\beta^2+1)^2}\right),
\end{aligned}
$$

$$\varphi_3^{(w)} = \frac{1}{6144\beta^4}\left(3\big(24\beta^2\big(2(21\varepsilon+43)\beta^2+125\varepsilon-125\big)+735\varepsilon-2975\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left.-\frac{4\beta^2\big(4\beta^2\big(4(1347\varepsilon+125)\beta^2+5115\varepsilon-8123\big)+5925\varepsilon-17125\big)+2205\varepsilon-8925}{(4\beta^2+1)^2}\right),$$

$$\varphi_4^{(w)} = \frac{1}{6144\beta^4}\left(\frac{16\beta^2\big((321\varepsilon-1)\beta^2+165\varepsilon-325\big)+315\varepsilon-1275}{(4\beta^2+1)}\right.$$
$$\left.-3\big(8\beta^2\big(2(21\varepsilon+43)\beta^2+75\varepsilon-75\big)+105\varepsilon-425\big)\frac{\arctan(2\beta)}{2\beta}\right),$$

$$\varphi_6^{(w)} = \varphi_5^{(w)} = \varphi_4^{(w)},$$

$$\varphi_7^{(w)} = \frac{1}{6144\beta^4}\left(3\big(8\beta^2\big(2(37\varepsilon+27)\beta^2+33\varepsilon-33\big)+21\varepsilon-85\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left.-\frac{16\beta^2\big((81\varepsilon-17)\beta^2+60\varepsilon-92\big)+63\varepsilon-255}{(4\beta^2+1)}\right),$$

$$\varphi_8^{(w)} = \frac{1}{6144\beta^4}\left(3\big(8\beta^2\big(2(17\varepsilon+47)\beta^2-17\varepsilon+17\big)+21\varepsilon-85\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left.+\frac{16\beta^2\big((51\varepsilon-115)\beta^2+15\varepsilon+17\big)-63\varepsilon+255}{(4\beta^2+1)}\right).$$

$$\varphi_1^{(s)} = \frac{1}{192\beta^4}\left(3\big(-20\varepsilon+3\beta^2\big(-\varepsilon+4\beta^2(\varepsilon-1)-7\big)+10\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left.-\frac{\big(\beta^2\big(8\beta^2\big(2\beta^2(23\varepsilon-55)-67\varepsilon-25\big)-409\varepsilon+137\big)-60\varepsilon+30\big)}{(4\beta^2+1)^2}\right),$$

$$\varphi_2^{(s)} = \frac{1}{192\beta^4}\left(\frac{\big(\beta^2\big(4\beta^2(7\varepsilon-23)-35\varepsilon-5\big)-12\varepsilon+6\big)}{(4\beta^2+1)}\right.$$
$$\left.-3\big(2-4\varepsilon+\beta^2\big(-\varepsilon+12\beta^2(\varepsilon-1)-7\big)\big)\frac{\arctan(2\beta)}{2\beta}\right),$$

$$\varphi_3^{(s)} = \frac{(\varepsilon+1)}{16\beta^2}\left((4\beta^2+3)\frac{\arctan(2\beta)}{2\beta}-3\right),$$

$$\varphi_4^{(s)} = -\frac{(\varepsilon+1)}{8\beta^2}\left(\frac{\arctan(2\beta)}{2\beta}-1\right),$$

$$\varphi_5^{(s)} = \frac{1}{96\beta^4}\left(8\beta^2(\varepsilon+1)-6\varepsilon+3-3\big(4\beta^2+1-2\varepsilon\big)\frac{\arctan(2\beta)}{2\beta}\right).$$

The quenching functions of the current helicity effects obtained in the article are

$$\varphi_1^{(h)} = \frac{1}{4\beta^2}\left(1-\frac{\arctan(2\beta)}{2\beta}\right),$$

$$\varphi_2^{(h)} = \frac{1}{64\beta^2}\left(3\big(4\beta^2+5\big)\frac{\arctan(2\beta)}{2\beta}-\frac{5\big(4\beta^2+3\big)}{(4\beta^2+1)}\right),$$

$$\varphi_3^{(h)} = -\frac{1}{192\beta^4}\left(3\big(12\beta^4-\beta^2-8\big)\frac{\arctan(2\beta)}{2\beta}+\frac{\big(4\beta^4+67\beta^2+24\big)}{(4\beta^2+1)}\right),$$

$$\varphi_4^{(h)} = \frac{1}{96\beta^4}\left(3\big(12\beta^4 - 3\beta^2 - 20\big)\frac{\arctan(2\beta)}{2\beta} - \frac{\big(368\beta^6 - 536\beta^4 - 409\beta^2 - 60\big)}{(4\beta^2+1)^2}\right).$$

The magnetic quenching functions for the shear-current effects are

$$\varphi_1^{(V)} = \frac{1}{(8\beta)^4}\left(\big(16\beta^2\big(\beta^2(125\varepsilon+43) - 25\varepsilon - 3\big) - \varepsilon - 87\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. - \frac{\big(8\beta^2\big(2\beta^2\big(128\beta^2(3\varepsilon+13) - 19\varepsilon - 709\big) - 755\varepsilon - 525\big) - 15\varepsilon - 1305\big)}{15(4\beta^2+1)}\right),$$

$$\varphi_2^{(V)} = \frac{1}{2(8\beta)^4}\left(\big(8\beta^2\big(2\beta^2(151\varepsilon - 15) - 15\varepsilon - 57\big) - 125\varepsilon + 165\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. - \frac{\big(16\beta^2\big(\beta^2(443\varepsilon - 339) - 85\varepsilon - 3\big) - 375\varepsilon + 495\big)}{3(4\beta^2+1)}\right),$$

$$\varphi_3^{(V)} = -\frac{1}{2(8\beta)^4}\left(\big(8\beta^2\big(2\beta^2(247\varepsilon - 367) + 193\varepsilon - 393\big) - 125\varepsilon + 165\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. + \frac{\big(16\beta^2\big(\beta^2\big(512\beta^2(\varepsilon+3) - 1563\varepsilon + 2739\big) - 227\varepsilon + 507\big) + 375\varepsilon - 495\big)}{3(4\beta^2+1)}\right),$$

$$\varphi_4^{(V)} = \frac{1}{2(4\beta)^4}\left(\big(\beta^2\big(4\beta^2(17\varepsilon - 5) + 5\varepsilon + 51\big) + 10\varepsilon - 23\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. + \frac{\big(\beta^2\big(4\beta^2\big(64\beta^2(\varepsilon+15) - 609\varepsilon - 235\big) - 475\varepsilon + 155\big) - 150\varepsilon + 345\big)}{15(4\beta^2+1)}\right),$$

$$\varphi_5^{(V)} = -\frac{1}{2(4\beta)^4}\left(\big(8\beta^2\big((2\beta^2+5)\varepsilon - 6\big) - 7\varepsilon + 4\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. + \frac{\big(16\beta^2\big(\beta^2((8\beta-7)(8\beta+7)\varepsilon + 80) - 20\varepsilon + 35\big) + 105\varepsilon - 60\big)}{15(4\beta^2+1)}\right),$$

$$\varphi_6^{(V)} = -\frac{1}{2(4\beta)^4}\left(\big(8\beta^2\big(2\beta^2(3\varepsilon - 4) + 7\varepsilon + 10\big) + 35\varepsilon - 40\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. - \frac{\big(16\beta^2\big(\beta^2\big(64\beta^2(\varepsilon+1) + 23\varepsilon + 44\big) + 28\varepsilon - 5\big) + 105\varepsilon - 120\big)}{3(4\beta^2+1)}\right),$$

$$\varphi_7^{(V)} = \frac{1}{(4\beta)^2}\left(\big(4\beta^2(8\varepsilon - 7) - 4\varepsilon - 41\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. - \frac{\big(8\beta^2\big(2\beta^2\big(16\beta^2(3\varepsilon+1) + 48\varepsilon - 67\big) + 2\varepsilon - 113\big) - 12\varepsilon - 123\big)}{3(4\beta^2+1)^2}\right),$$

$$\varphi_8^{(V)} = -\frac{1}{128\beta^4}\left(\big(8\beta^2\big(2\beta^2(4\varepsilon+5) + 3\big) + 5\big)\frac{\arctan(2\beta)}{2\beta} - \frac{\big(16\beta^2\big(\beta^2(12\varepsilon+25) + 7\big) + 15\big)}{3(4\beta^2+1)}\right),$$

$$\varphi_9^{(V)} = -\frac{1}{(8\beta)^4}\left(\big(8\beta^2\big(2\beta^2(9\varepsilon+47) - 11\varepsilon + 19\big) + 25\varepsilon - 65\big)\frac{\arctan(2\beta)}{2\beta}\right.$$
$$\left. - \frac{1}{15}\big(4\beta^2\big(256\beta^2(\varepsilon+1) - 455\varepsilon + 895\big) + 375\varepsilon - 975\big)\right),$$

$$\varphi_{10}^{(V)} = -\frac{1}{128\beta^2}\left(\left(4\beta^2(3\varepsilon-11)+13\varepsilon-21\right)\frac{\arctan(2\beta)}{2\beta}+\frac{1}{3}\left(16\beta^2(\varepsilon+3)-39\varepsilon+63\right)\right),$$
$$\varphi_{11}^{(V)} = -\frac{(\varepsilon+1)}{48\beta^2}\left(3\left(4\beta^2+1\right)\frac{\arctan(2\beta)}{2\beta}-8\beta^2-3\right).$$

The magnetic quenching functions for the current helicity evolution equation:

$$\psi_1 = -\frac{\varphi_3^{(s)}}{(\varepsilon+1)},$$
$$\psi_2 = -\frac{1}{768\beta^2}\left(3\left(12(\varepsilon-1)\beta^2-21\varepsilon+5\right)\frac{\arctan(2\beta)}{2\beta} + \frac{\left(4\beta^2\left(32\beta^2(\varepsilon+1)+65\varepsilon-1\right)+63\varepsilon-15\right)}{(4\beta^2+1)}\right),$$
$$\psi_3 = \frac{1}{48\beta^2}\left(3(\varepsilon-1)\frac{\arctan(2\beta)}{2\beta}-\left(4\beta^2(\varepsilon+1)+3\varepsilon-3\right)\right),$$
$$\psi_4 = \frac{1}{192\beta^4}\left(\frac{\left(4\beta^2\left(15\varepsilon+32\beta^2+5\right)+9\varepsilon+3\right)}{(4\beta^2+1)}-3(3\varepsilon+1)\left(4\beta^2+1\right)\frac{\arctan(2\beta)}{2\beta}\right),$$
$$\psi_5 = \frac{1}{8(2\beta)^4}\left(\left(11\varepsilon+4\beta^2\left(-\varepsilon+8\beta^2(\varepsilon-1)+5\right)-7\right)\frac{\arctan(2\beta)}{2\beta} - \frac{1}{15}\left(8\beta^2\left(8\beta^2(\varepsilon+1)-35\varepsilon+55\right)+165\varepsilon-105\right)\right),$$
$$\psi_6 = \frac{\left(4\beta^2+4-3\varepsilon\right)}{48\beta^4}\left(3\frac{\arctan(2\beta)}{2\beta}-\frac{\left(8\beta^2+3\right)}{(4\beta^2+1)}\right).$$

Appendix B. Comparison with some of results given in the article by Rädler and Stepanov (2006)

This part of the article contains the comparison of some of our results with those from RS06. We apply the mixing-length (MLT) approximation to expressions obtained in RS06. In this procedure we replace the spectrum of turbulent fields by the single-scaled function of the form $\delta\left(k-\ell_c^{-1}\right)\delta(\omega)$, where ℓ_c is the correlation length of the turbulence and put $\eta k^2 = \nu k^2 = \tau_c^{-1}$ (Kichatinov 1991).

The effect of stratification and shear. The structure of the electromotive force obtained by RS06 can be reproduced if we decompose the gradient of the large-scale flow $\overline{V}_{i,j}$ into symmetric and antisymmetric parts *via*

$$\overline{V}_{i,j} = D_{ij} - \frac{1}{2}\varepsilon_{ijn}W_n, \tag{52}$$

where $W_i = \varepsilon_{inm}\overline{V}_{m,n}$ is the large-scale vorticity and $D_{ij} = \left(\overline{V}_{i,j} + \overline{V}_{j,i}\right)/2$ is the rate of strain tensor. After substitution (55) to (27) we obtain

$$\mathcal{E}_i^{(s)} = \left(\varepsilon_{inm}U_k\overline{B}_nD_{mk}A_4 - \frac{A_4}{2}\left(\mathbf{W}\cdot\overline{\mathbf{B}}\right)U_i + \left(\mathbf{U}\cdot\overline{\mathbf{B}}\right)\left(A_3 - \frac{A_2}{2} + \frac{A_1}{2} + \frac{A_4}{2}\right)W_i\right.$$
$$+ \varepsilon_{inm}\overline{B}_kD_{nk}U_m(A_2 + A_1) + (\mathbf{W}\cdot\mathbf{U})\overline{B}_i\left(\frac{A_2}{2} - \frac{A_1}{2}\right)\Bigg)\langle u^{(0)2}\rangle$$
$$\left. + \tau_c^2\frac{h_{\mathcal{C}}^{(0)}}{6}\left(\frac{5}{2}\left(\mathbf{W}\times\overline{\mathbf{B}}\right)_i - \frac{23}{5}D_{ik}\overline{B}_k\right)\right). \tag{53}$$

Using (53) we find

$$\tau_c^{-2}\tilde{\alpha}_1^{(W)} = \frac{1}{2}\tau_c^{-2}(A_1 - A_2) = 0, \tag{54}$$

$$\tau_c^{-2}\tilde{\alpha}_2^{(W)} = -\frac{1}{2}\tau_c^{-2}\left(A_3 + \frac{1}{2}(A_1 - A_2)\right) = -\frac{1}{12}, \tag{55}$$

$$\tau_c^{-2}\tilde{\gamma}^{(W)} = -\frac{1}{2}\tau_c^{-2}\left(A_3 + A_4 + \frac{1}{2}(A_1 - A_2)\right) = 0, \tag{56}$$

$$\tau_c^{-2}\tilde{\gamma}^{(D)} = \frac{1}{2}\tau_c^{-2}(3A_4 - A_1 - A_2) = -\frac{11}{60}, \tag{57}$$

$$\tau_c^{-2}\tilde{\alpha}^{(D)} = \frac{1}{2}\tau_c^{-2}(A_4 - A_1 - A_2) = -\frac{1}{60}, \tag{58}$$

where we put $\varepsilon = 0$. After applying the MLT to results obtained in RS06 we find $\tau_c^{-2}\tilde{\alpha}_1^{(W)} = 19/120$, $\tau_c^{-2}\tilde{\alpha}_2^{(W)} = -7/240$, $\tau_c^{-2}\tilde{\gamma}^{(W)} = -1/48$, $\tau_c^{-2}\tilde{\gamma}^{(D)} = -39/120$, $\tau_c^{-2}\tilde{\alpha}^{(D)} = -21/120$.

The effect of nonuniform LSMF and shear. For the shear-current effect, after substitution of (55) to (22) we arrive to the following representation of $\mathcal{E}_i^{(V)}$,

$$\mathcal{E}_i^{(V)} = \left\{\frac{C_3 - C_4}{2}(\mathbf{W}\cdot\nabla)\overline{B}_i + \frac{C_1 - C_2}{2}\nabla_i\left(\mathbf{W}\cdot\overline{\mathbf{B}}\right)\right\}\left\langle u^{(0)2}\right\rangle$$
$$+ \varepsilon_{inm}\left\{(C_1 + C_2)\overline{B}_{n,l} + (C_3 + C_4)\overline{B}_{l,l,n}\right\}D_{ml}\left\langle u^{(0)2}\right\rangle. \tag{60}$$

Using this formula we obtain

$$\tau_c^{-2}\tilde{\delta}^{(W)} = \frac{1}{4}\tau_c^{-2}(C_3 - C_4 - C_1 + C_2) = \frac{1}{12}, \tag{61}$$

$$\tau_c^{-2}\tilde{\kappa}^{(W)} = \frac{1}{2}\tau_c^{-2}(C_4 + C_2 - C_1 - C_3) = -\frac{4}{15}, \tag{62}$$

$$\tau_c^{-2}\tilde{\kappa}^{(D)} = -\frac{1}{2}\tau_c^{-2}(C_1 + C_2 + C_3 + C_4) = \frac{3}{10}, \tag{63}$$

$$\tau_c^{-2}\tilde{\beta}^{(D)} = -\frac{1}{2}\tau_c^{-2}(C_1 + C_2 - C_3 - C_4) = 0, \tag{64}$$

where we put $\varepsilon = 0$ in C_{1-4}. After applying the MLT to results in RS06 we find $\tau_c^{-2}\tilde{\delta}^{(W)} = 1/12$, $\tau_c^{-2}\tilde{\kappa}^{(W)} = -1/30$, $\tau_c^{-2}\tilde{\kappa}^{(D)} = 13/30$ and $\tau_c^{-2}\tilde{\beta}^{(D)} = 7/60$.