

Probing current and cross-helicity in the solar atmosphere: A challenge for theory

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Abstract

We review results of systematic data analysis of solar vector magnetograms and dopplergrams for revealing average values of current helicity and twist over active regions and their systematic interpretation in the framework of dynamo theory. In anticipation of opportunities for collection of data on cross-helicity and further improvement of dynamo models with respect to account of this quantity in future, we propose here a series of dynamo models which could fit available observational data. We have shown that the cross-helicity alternates in sign with the solar cycle (so it is zero in the long time average), and it changes from negative to positive following the toroidal field. We demonstrate how it is possible to tune such models with respect to account of different effects to reproduce particular features of the observable solar magnetic fields and its helical properties.

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1. Introduction

For about a decade observational data on current helicity of solar magnetic fields have been taken into account in analysis of properties of solar magnetic fields. Due to the links between these data that are believed to have some relationship with the magnetic helicity and a possible phenomenon called the α -effect, they are of higher importance for understanding of the basics dynamo theory. Namely,

how the poloidal magnetic fields can be amplified by toroidal fields and thus provide effective regeneration of the magnetic field from turbulent motions and rotation shear in the Solar convective zone.

A number of observational facts have been found. First of all, as a number of authors stressed (Pevtsov et al., 1994; Bao and Zhang, 1998), see further analysis of in Kuzanyan et al. (2000) and Zhang et al. (2002), the current helicity like the Coriolis force is antisymmetric over the solar equator. There is a serious argument that this quantity is not regularly changing with the phase of the solar cycle while some observations indicate its variations with time, although still under discussion (Hagino and Sakurai, 2004, 2005), cf. (Sokoloff et al., 2006).

By analysis of datasets of active regions with their relative rotation the effective depth, at which these magnetic structures are anchored, has been estimated in Kuzanyan et al. (2003). To carry out so, we use the data on solar internal rotation provided by helioseismic inversion technique. This enables us to conclude on possible change of the sign

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of helical quantities with depth in the solar convection zone Zhang et al. (2006).

Separate studies of evolution of the effective depth at which the main magnetic activity is located over the solar cycle may enable us in the future, with accumulation of a bigger dataset on current helicity of active regions, to track the effective depth at which solar magnetic field generation processes operate.

All these properties of spatial and temporal structure of helical properties of magnetic fields provide certain constrain to the dynamo theories in the Solar convective zone. In this paper, we consider several dynamo models with inclusion of the solar rotation and inhomogeneity of turbulence and compute both current and cross-helicities. We shall give some predictions on the distribution and behaviour of cross-helicity within the solar convection zone. We show the impact of different assumptions about mechanisms governing the generation of the large-scale poloidal field of the Sun to evolution of the current and cross-helicity in course of the solar cycle.

2. Basic equations and results

The evolution of the axisymmetric large-scale magnetic field (LSMF), $\mathbf{B} = \mathbf{e}_\phi \mathbf{B} + \text{curl} \left(\frac{A \mathbf{e}_\phi}{r \sin \theta} \right)$, (where r is radius, θ is colatitude, \mathbf{e}_ϕ is the unit azimuthal vector) in the turbulent media subjected to the differential rotation can be described with equations

$$\frac{\partial B}{\partial t} = \frac{1}{r} \frac{\partial(\Omega, A)}{\partial(r, \theta)} + \frac{1}{r} \left(\frac{\partial r \mathcal{E}^\theta}{\partial r} - \frac{\partial \mathcal{E}^r}{\partial \theta} \right), \quad (1)$$

$$\frac{\partial A}{\partial t} = r \sin \theta \mathcal{E}^\phi. \quad (2)$$

The contribution of turbulence in (1 and 2) is expressed through the components of the mean electromotive force $\mathcal{E} = \mathbf{u} \times \mathbf{b}$, where \mathbf{u} , \mathbf{b} are the small-scale fluctuated velocity and magnetic field, respectively. In the present paper, we adopt the expressions of mean electromotive force obtained in Pipin (2007a). The components of the mean electromotive force are given in attachment. One of the main purposes of the paper is to find out the effect of the large-scale magnetic fields and mean electromotive force on the evolution of the small-scale cross-helicity $h_\chi = \mathbf{u} \cdot \mathbf{b}$. As shown in Appendix A we can find that evolution of h_χ is governed by

$$\partial_t h_\chi = -\bar{\mathbf{V}} \cdot \nabla \times \mathcal{E} = -\Omega \sin \theta \left(\frac{\partial r \mathcal{E}^\theta}{\partial r} - \frac{\partial \mathcal{E}^r}{\partial \theta} \right).$$

In addition we need the equation for the current helicity $h_c = \mathbf{b} \cdot \nabla \times \mathbf{b} / (4\pi\rho)$,

$$\frac{\partial h_c}{\partial t} = -2 \frac{\mathcal{E} \cdot \mathbf{B}}{4\pi\rho\ell_c^2} + \eta_h \nabla^2 h_c - \frac{h_c}{\tau_h}. \quad (3)$$

It is in agreement with Kleorin et al. (2003) and Brandenburg and Subramanian (2005). Contributions h_c/τ_h and

$\eta_h \nabla^2 h_c$ serve to take into account the helicity loss in rough way. The parameter τ_h is normalized to the typical diffusion time $R_\odot^{-2} \eta_0$, and $\eta_h = \varepsilon_h \eta_0$, where $\varepsilon_h \ll 1$, $\eta_0 = (u_c \ell_c) / 3$ – typical turbulent diffusivity in convection zone. Boundary conditions, for helicity: vanishing of the radial derivatives both at the bottom and at the top

$$\left. \frac{\partial h_c}{\partial \theta} \right|_{\theta=0, \pi}, \quad \left. \frac{\partial h_c}{\partial r} \right|_{r=.71, .96R_\odot} = 0.$$

The dynamo problem is treated with the following types of the boundary conditions: superconductor at the bottom and external vacuum. One of the purposes of the paper is to consider various turbulent effects that may be responsible for generation of the large-scale poloidal magnetic fields. These effects are included in toroidal component of the mean-electromotive force (see Eq. (2)). In the paper, we consider two types of solar dynamos. One is the traditional $\alpha\Omega$ dynamo and another is $\alpha\delta\Omega$, where δ means that model includes the Rädler's $\Omega \times J$ -effect (Rädler, 1969), see also (Rädler et al., 2003; Rogachevskii and Kleorin, 2003; Pipin, 2007b). The expressions for mean electro-motive force in case of $\alpha\delta\Omega$ are given in Appendix A (see Eq. (15)). The construction of the pure $\alpha\Omega$ dynamo is very problematic in the solar case. The number of issues were discussed by Brandenburg (2005). Then to obtain the solar-like dynamo in this case we make several additional adjustments. First, we multiply the standard α -effect by factor $\sin^2\theta$ to make sure that the maximum generation of the poloidal large-scale magnetic fields (LSMF) is near equator. Second, we add the joint effects due to current helicity and shear to the mean electromotive force, see Pipin (2007b). This will ensure that activity of the toroidal LSMF drifts to the equator in course of the solar cycle. All these additions can be expressed in a formal way as follows:

$$\tilde{\mathcal{E}}^r = \mathcal{E}^r + \varphi_3^{(h)} \tau^2 h_c B r \sin \theta \frac{\partial \Omega}{\partial r}, \quad (4)$$

$$\tilde{\mathcal{E}}^\theta = \mathcal{E}^\theta + \varphi_3^{(h)} \tau^2 h_c B \sin \theta \frac{\partial \Omega}{\partial \theta}, \quad (5)$$

$$r \sin \theta \tilde{\mathcal{E}}^\phi = r \sin \theta \mathcal{E}^\phi - Br \tilde{\eta}_T C_\alpha G \sin \theta \times \left(f_{12}^{(a)} \varphi_6^{(a)} \cos^3 \theta - \varphi_2^{(s)} \tau \sin \theta \frac{\partial \Omega}{\partial \theta} \right), \quad (6)$$

where functions $f_{12}^{(a)}$, $\varphi_6^{(a)}$, $\varphi_3^{(h)}$, and $\varphi_2^{(s)}$ were defined in Pipin (2007b) (see further explanations in Appendix A). Thus, for the $\alpha\Omega$ we will use these components of mean electro-motive force. Note, that $\alpha\delta\Omega$ dynamo does not require such tuning. Further, the governing parameters of the model are $\tilde{\eta}_T = C_\eta u_c \ell_c$ with general parameter $C_\eta \leq 1$ controlling the power of turbulent sources; C_α , $C_{\omega j} \leq 1$ are parameters controlling the magnitude of the α - and $\Omega \times J$ -effects. Quantities \bar{u}^2 , ℓ_c , τ_c are taken from the Stix (2002) model. The distribution of the angular velocity is taken as an analytical fit given by Belvedere et al. (2000) who used the helioseismology data provided by Schou et al.

(1998), courtesy of Sasha Kosovichev. To match the period of solar cycle we have used $C_\eta = 1/3$, in case of $\alpha\delta\Omega$ dynamo $C_\eta = 1/20$, in case of $\alpha\Omega$ dynamo. Furthermore, our dynamo models include the dynamical constraints to the growth of the LSMF which rises due to helicity conservation. In preliminary runs it was found that the typical strength of the LSMF about 2 kGs in the SCZ can be found if the magnetic helicity loss from the dynamo region are more than $10^{28} \text{Wb}^2/\text{day}$ or $4 \times 10^{47} \text{Mx}^2/\text{cycle}$, where the period of the cycle is set to 11 y). Within the given formulation for the current helicity evolution (Eq. (3)) this can be achieved with given parameters $\eta_h = 0.01\tilde{\eta}_T$ and $\tau_h \geq 3T_D$, where $T_D = 3R_\odot^2/(u_c\ell_c)$ is a typical diffusion time of the system. We fix these parameters in all computations presented here. Further details about model can be found in Appendix A.

Let us give here two examples of possible dynamo models. As the first example we consider the results for $\alpha\delta\Omega$ dynamo. The governing parameters in these simulations are $C_\alpha = 0.1C_\eta$, $C_{\omega j} = 6.5C_\alpha$. The model is discussed in more details in the companion paper by Pipin (2007b). In Fig. 1 we give the Maunder's-like diagram for evolution of the radial LSMF field, toroidal LSMF, current and cross-helicities. The quantities are taken at the near surface level. This model demonstrates the basic properties of

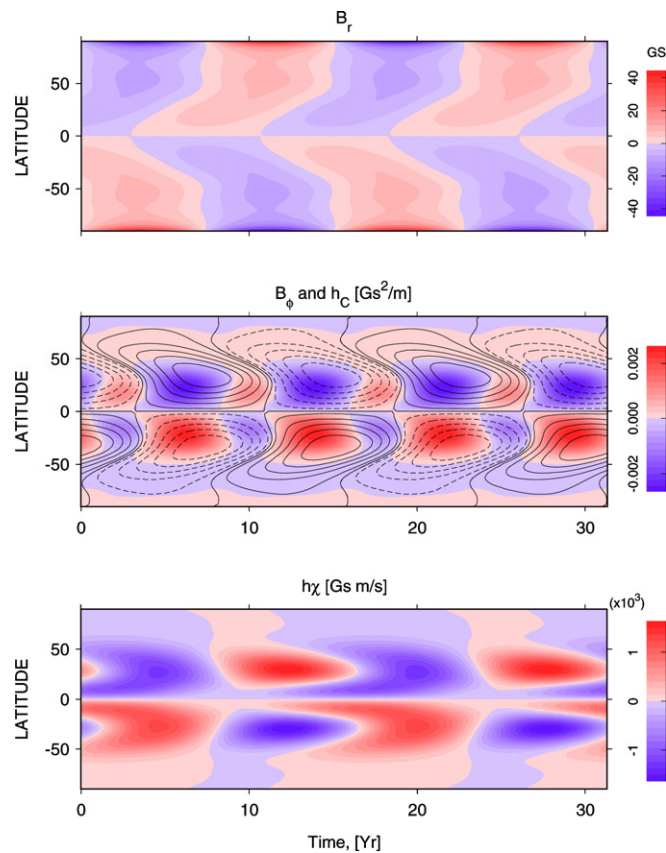


Fig. 1. $\alpha\delta\Omega$ dynamo. Top: radial magnetic field variations on the surface; middle: variations of current helicity with overlaid isocontours of the toroidal magnetic field; bottom: variations of the cross-helicity near the surface level.

magnetic activity that can be found from observations. Namely, the activity of the toroidal component of the LSMF drifts to equator as the cycle develops. The polar reversal of the radial LSMF takes place at the maximum of the cycle. The current helicity is concentrated near the equator having the preferable negative sign in the Northern hemisphere over the most of the cycle. The cross-helicity changes the sign after the maximum of the cycle.

Evolution of the LSMF and helicities is shown in radial section of convection zone is shown in Fig. 2.

A different behaviour can be demonstrated by $\alpha\Omega$ dynamo, which has the mean electro-motive force tuned as given by (4)–(6) and $C_\eta = 1/20$, $C_\alpha = .6C_\eta$. Here, we use the same boundary conditions and the same equations for the current- and cross-helicity evolution. The corresponding diagrams are shown in Fig. 3. In comparison with the previous model there is a difference in evolution of the radial LSMF near the equatorial regions. As in the previous model the current helicity has preferable negative sign in the Northern hemisphere over the most of the cycle. The cross-helicity changes the sign at the minimum of the cycle following to evolution of the toroidal component of the LSMF. Thus, these two models demonstrate a clear difference for the phase relations of the preferable sign of the cross-helicity in the hemisphere and the phase of the activity cycle.

3. Discussion

Here, we have considered an opportunity to develop a series of models with account of different effects and

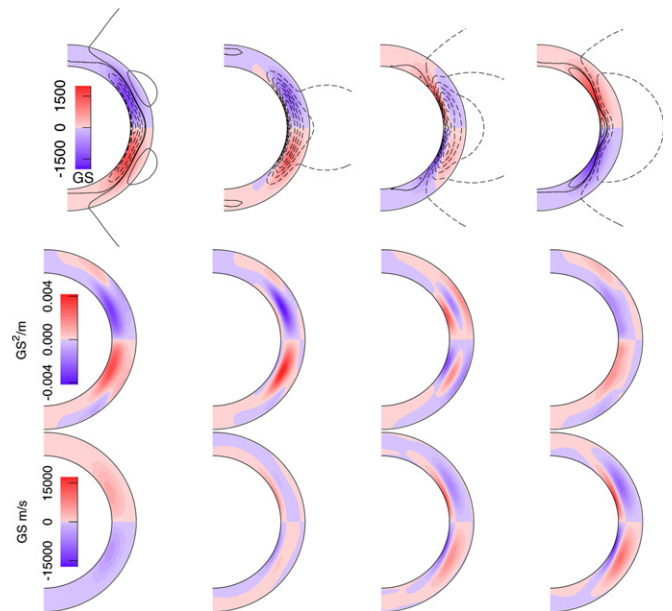


Fig. 2. Evolution of the large-scale magnetic fields and helicities is shown in radial section of the convection zone. Time grows from left to right, snapshots are taken at 0, 3, 5, and 8 y. The top: the poloidal field lines are overlaid by the gray-scale density plot of the toroidal LSMF; middle: current helicity, and bottom: h_x .

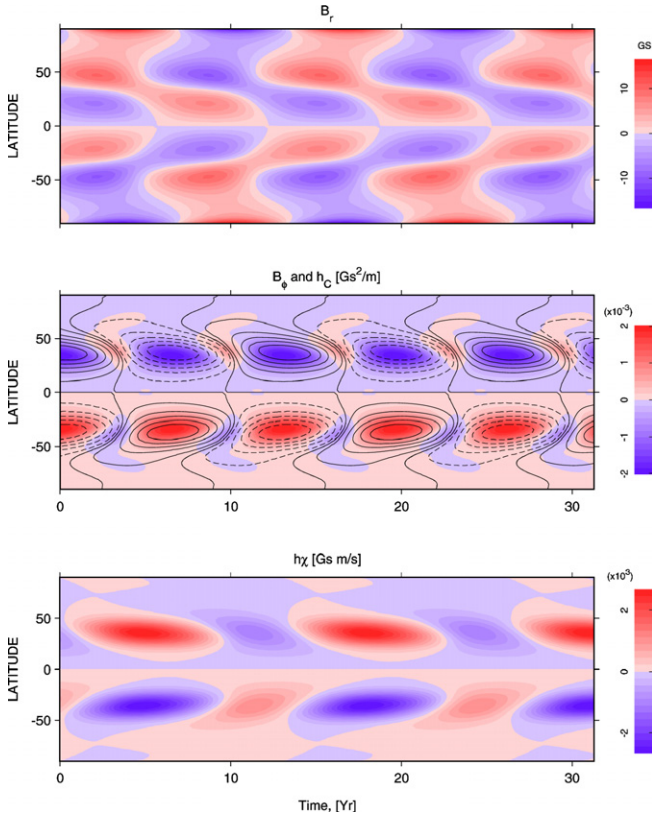


Fig. 3. The $\alpha\Omega$ dynamo, radial magnetic field variations on the surface, middle: variations of current helicity with overlaid isocontours of the toroidal magnetic fields, bottom: variations of the cross-helicity on the near surface level.

demonstrated how the choice of a particular model can provide reproduction of particular features of observable solar magnetic fields.

Cross-helicity is an inviscous invariant, and, therefore, our analysis within the Solar convective zone should well be relevant for the expected observational data. Precise knowledge on the properties of cross-helicity may help in selection of models which best fit the observational data.

Recently, a number of space missions has been launched and furthermore data on Solar magnetic fields and velocity fields are expected within a few forthcoming years. This will bring more observational data involved into the theoretical analysis and first of all, as never before, an opportunity to check the theory versus direct observations. We can predict the following properties to be seen upon systematic statistical studies of cross-helicity:

- Hemispheric rule: cross-helicity is anti-symmetric over the equator.
- Cyclic variation: cross-helicity changes sign with 11 y cycle.

However, at present times not sufficient amount of data has been processed yet, and more observational effort is

needed. Analysis of space and ground based observations opens a challenge for further theoretical studies.

Appendix A. Deriving equations for the small-scale cross-helicity evolution

The cross-helicity conservation law was established in the papers by Woltjer (1958) and Moffat (1969). Here, we adopt it for the case of the mean-field dynamo in the turbulent media. Generally, the conservation law can be formulated in integral form as follows (see Moffat (1969))

$$\int \mathbf{B} \cdot \mathbf{U} dV = \text{const}, \quad (7)$$

provided that $\mathbf{B} \cdot \mathbf{n} = 0$ on the surface comprising the volume V , where \mathbf{B} – induction vector and \mathbf{U} – velocity field. In the differential form the (7) can be expressed as

$$(\partial_t + (\mathbf{U} \cdot \nabla))(\mathbf{B} \cdot \mathbf{U}) = 0. \quad (8)$$

In the spirit of the mean-field magnetohydrodynamics, we split the physical quantities of the turbulent conducting fluid into the mean and randomly fluctuating part with the mean part defined as the ensemble average of the random fields. One assumes the validity of the Reynolds rules. The magnetic field \mathbf{B} and velocity of motions \mathbf{V} are decomposed as follows: $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$, $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$. Hereafter, everywhere, we use the small letters for the fluctuating part of the fields and capital letters with a bar above for the mean fields. From (8) we can arrive to

$$\partial_t(\overline{\mathbf{B}} \cdot \overline{\mathbf{U}}) + \partial_t(\overline{\mathbf{u}} \cdot \mathbf{b}) + (\overline{\mathbf{U}} \cdot \nabla)(\overline{\mathbf{B}} \cdot \overline{\mathbf{U}} + \overline{\mathbf{u}} \cdot \mathbf{b}) + \nabla \cdot \overline{\mathbf{u}(\mathbf{u} \cdot \mathbf{b})} = 0, \quad (9)$$

where we take into account $\nabla \cdot \mathbf{u} = 0$. Assuming the high-conductive and inviscid ideal plasma and averaging the induction and NavieStokes equations we arrive to

$$\partial_t \overline{\mathbf{B}} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathbf{u}} \times \mathbf{b}), \quad (10)$$

$$\overline{\rho}(\partial_t \overline{U}_i + (\overline{U}_j \nabla_j \overline{U}_i)) = \nabla_j \left(\overline{T}_{ij} + \frac{1}{4\pi} \overline{B}_i \overline{B}_j \right) - \nabla_i \left(\overline{P} + \frac{1}{8\pi} \overline{B}^2 - \Psi \right), \quad (11)$$

where $\overline{\rho}$, \overline{P} , Ψ is the mean density, pressure and the gravity. The effect of the turbulent velocity and magnetic fields on the mean-field evolution is described by the mean electromotive force $\mathcal{E} = \overline{\mathbf{u}} \times \mathbf{b}$ and the mean turbulent stresses $\overline{T}_{ij} = \overline{\rho u_i u_j} + (\overline{b_i b_j} - \delta_{ij} \overline{b^2}/2)/(4\pi)$. Excluding $\partial_t(\overline{\mathbf{B}} \cdot \overline{\mathbf{U}})$ from (9) by combining (10) and (11) we arrive to

$$\begin{aligned} \partial_t(\overline{\mathbf{u}} \cdot \mathbf{b}) = & \frac{\overline{\mathbf{B}}}{\rho} \cdot \nabla \left(\overline{P} + \frac{\overline{B}^2}{8\pi} - \Psi \right) \\ & - \frac{\overline{B}_i}{\rho} \nabla_j \left(\overline{T}_{ij} + \frac{1}{4\pi} \overline{B}_i \overline{B}_j \right) - (\overline{\mathbf{B}} \cdot \nabla) \frac{\overline{U}^2}{2} \\ & - \overline{\mathbf{U}} \cdot \nabla \times \mathcal{E} - (\overline{\mathbf{U}} \cdot \nabla) \overline{\mathbf{u}} \cdot \mathbf{b} + \nabla \cdot \overline{\mathbf{u}(\mathbf{u} \cdot \mathbf{b})}. \end{aligned} \quad (12)$$

For the sake of simplicity we consider the axisymmetric mean magnetic field and mean flows. Moreover, we assume that the azimuthal components of the large-scale magnetic fields (LSMF) and flows dominate their poloidal counterpart. Next we neglect the deviations of the angular momentum balance which are resulted due to LSMF. Further we assume that LSMF influence on the hydrostatic equilibrium in solar convection zone is also negligible then (12) is reduced to

$$\partial_t(\overline{\mathbf{u} \cdot \mathbf{b}}) = -\overline{\mathbf{U}} \cdot \nabla \times \mathcal{E},$$

where $\overline{\mathbf{U}}$ – is the differential rotation and \mathcal{E} is the mean electromotive force of turbulent flows and magnetic fields.

In the numerical model discussed in the paper we have used the following representation for the components of the mean electro-motive force

$$\mathcal{E}^r = -\tilde{\eta}_T \psi_\eta \left(\frac{(f_2^{(d)} + (1 + \varepsilon)f_1^{(a)} \sin^2 \theta)}{r \sin \theta} \frac{\partial \sin \theta B}{\partial \theta} - \frac{(1 + \varepsilon)f_1^{(a)} \sin 2\theta}{2r} \frac{\partial r B}{\partial r} \right) - G \tilde{\eta}_T \frac{\sin 2\theta}{2} f_5^{(a)} \psi_\eta B, \quad (13)$$

$$\mathcal{E}^\theta = \tilde{\eta}_T \psi_\eta \left(\frac{(f_2^{(d)} + (1 + \varepsilon)f_1^{(a)} \cos^2 \theta)}{r} \frac{\partial r B}{\partial r} - \frac{(1 + \varepsilon)f_1^{(a)} \cos \theta}{r} \frac{\partial \sin \theta B}{\partial \theta} \right) - \tilde{\eta}_T G \left(f_3^{(a)} \phi_1^{(a)}(\beta) + \cos^2 \theta f_5^{(a)} \psi_\eta(\beta) \right) B, \quad (14)$$

$$\begin{aligned} r \sin \theta \mathcal{E}^\phi = & \tilde{\eta}_T \psi_\eta(\beta) \left((f_2^{(d)} + (1 + \varepsilon)f_1^{(a)} \cos^2 \theta) \frac{\partial^2 A}{\partial r^2} + \frac{(f_2^{(d)} + (1 + \varepsilon)f_1^{(a)} \sin^2 \theta) \sin \theta}{r^2} \frac{\partial}{\partial \theta} \frac{1}{\sin \theta} \frac{\partial A}{\partial \theta} \right) \\ & + \tilde{\eta}_T \psi_\eta(\beta) \frac{(1 + \varepsilon)f_1^{(a)}}{r} \\ & \times \left(\sin^2 \theta \frac{\partial A}{\partial r} + \frac{3 \sin 2\theta}{2r} \frac{\partial A}{\partial \theta} - \sin 2\theta \frac{\partial^2 A}{\partial \theta \partial r} \right) \\ & + \tilde{\eta}_T G (\psi_\eta(\beta) (f_4^{(a)} \sin^2 \theta - f_5^{(a)} \cos^2 \theta) \\ & - f_3^{(a)} \phi_1^{(a)}(\beta)) \frac{\partial A}{\partial r} + \tilde{\eta}_T G \psi_\eta(\beta) \\ & \times \left(f_4^{(a)} + f_5^{(a)} \right) \frac{\sin 2\theta}{2r} \frac{\partial A}{\partial \theta} \\ & + Br \sin \theta \left(\tilde{\eta}_T C_x G \cos \theta f_{12}^{(a)} \phi_6^{(a)} + \tau \theta c f_1^{(h)} \phi_1^{(h)} \right) \\ & + \tilde{\eta}_T C_{\omega j} f_4^{(d)} \phi_{2+7}^{(w)}(\beta) \left(\frac{r \sin 2\theta}{2} \frac{\partial B}{\partial r} - \sin^2 \theta \frac{\partial B}{\partial \theta} \right), \end{aligned} \quad (15)$$

where, $\tilde{\eta}_T = C_\eta u_c \ell_c$ with $C_\eta \leq 1$ to control the power of turbulent sources. Functions of the Coriolis number $f_{\{n\}}^{(a)}$, $f_{\{n\}}^{(d)}$, as well as functions of β describing the LSMFs influence

ψ_η , $\phi_{\{n\}}^{(a)}$, $\phi_{\{n\}}^{(h)}$, $\phi_{\{n\}}^{(w)}$ are given in the papers Pipin (2007a,b), $\varepsilon = \sqrt{b^2} / \left(\sqrt{4\pi\rho u^2} \right) = 1$, $\overline{b^2}$ is the energy of the small-scale magnetic fields which stem from the small-scale dynamo. In expressions of the mean electromotive force (13)–(15) we take into account the anisotropy of magnetic diffusivity and turbulent pumping of LSMF with regards for the influence of rotation and LSMF on the turbulent motions. Quantities $\overline{u^2}$, ℓ_c , τ_c are taken from the model of Stix (2002). The distribution of the angular velocity is taken as an analytical fit given by Belvedere et al. (2000) who used the helioseismology data provided by Schou et al. (1998), see, e.g., Fig. 2 of Kuzanyan et al. (2006). The contribution of the $\Omega \times J$ -effect to the mean electromotive force is defined by the last line in (15), while the α -effect is defined by the line above.

Both effects are computed on the base of the solar interior model. Additionally, the power of the α - and $\Omega \times J$ -effects are controlled by parameters C_α , $C_{\omega j} \leq 1$. The distribution of the α -effect which is used in the model is similar to what is shown on Fig. 4 of Kuzanyan et al. (2006). The only difference is that in our model the α -effect is positive everywhere, and so it does not change the sign at the bottom of convection zone because when computing the α -effect we did not take into account the variation of the rms convective velocity with the radius. In the present model the effect of meridional circulation on the mean field dynamo is neglected.

References

- Bao, S., Zhang, H. Patterns of current helicity for the twenty-second solar. *ApJ* 496, L43–L46, 1998.
- Belvedere, G., Kuzanyan, K.M., Sokoloff, D. A two-dimensional asymptotic solution for a dynamo wave in the light of the solar internal rotation. *Mon. Not. R. Astron. Soc.* 315, 778–790, 2000.
- Brandenburg, A. The case for a distributed solar dynamo shaped by near-surface shear. *Astrophys. J.* 625, 539–547, 2005.
- Brandenburg, A., Subramanian, K. Strong mean field dynamos require supercritical helicity fluxes. *Astron. Nachr.* 325, 400–408, 2005.
- Hagino, M., Sakurai, T. Latitude variation of helicity in solar. *Publ. Astron. Soc. Jpn.* 56, 831–843, 2004.
- Hagino, M., Sakurai, T. Solar-cycle variation of magnetic helicity in active regions. *Publ. Astron. Soc. Jpn.* 57, 481–485, 2005.
- Kleeorin, N., Kuzanyan, K., Moss, D., Rogachevskii, I., Sokoloff, D.H.Z. Magnetic helicity evolution during the solar activity cycle: observations and dynamo theory. *Astron. Astrophys.* 409, 1097–1105, 2003.
- Kuzanyan, K., Zhang, H., Bao, S. Probing signatures of the alpha-effect in the sun. *Solar Phys.* 191 (2), 231–246, 2000.
- Kuzanyan, K., Lamburt, V., Zhang, H., Bao, S. Studies of helical properties of solar magnetic fields. *Chin. J. Astron. Astrophys.* 3 (3), 257–265, 2003.
- Kuzanyan, K., Pipin, V., Seehafer, N. The alpha effect and the observed twist and current helicity of the solar magnetic fields. *Solar Phys.* 233, 185–204, 2006.
- Moffat, H. The degree of knottedness of tangled vortex lines. *J. Fluid Mech.* 35 (1), 117–129, 1969.
- Pevtsov, A.A., Canfield, R.C., Metchalf, T.R. Patterns of helicity in solar active regions. *ApJ* 425 (2), 117–119, 1994.
- Pipin, V.V. The mean electro-motive force and current helicity under the influence of rotation, magnetic field and shear. *Geophys. Astrophys. Fluid Dynam.* 101, in press, astro-ph/0606265, 2007a.

- Pipin, V.V. Solar magnetic fields generation by $\alpha\delta\Omega$ dynamo. *Astron. Rep.* No. 5, in press, 2007b.
- Rädler, K.-H. On the electrodynamics of turbulent fields under the influence of coriolis forces. *Monatsh. Dt. Akad. Wiss.* 11, 194–201, 1969.
- Rädler, K.-H., Kleeorin, N., Rogachevskii, I. The mean electromotive force for mhd turbulence: the case of a weak mean magnetic field and slow rotation. *Geophys. Astrophys. Fluid Dyn.* 97, 249–269, 2003.
- Rogachevskii, I., Kleeorin, N. Electromotive force and large-scale magnetic dynamo in a turbulent flow with a mean shear. *Phys. Rev. E* 68 (036301), 1–12, 2003.
- Schou, J., Antia, H.M., Basu, S., Bogart, R.S., Bush, R.I., Chitre, S.M., Christensen-Dalsgaard, J., Di Mauro, M.P., Dziembowski, W.A., Eff-Darwich, A., Gough, D.O., Haber, D.A., Hoeksema, J.T., Howe, R., Korzenik, S.G., Kosovichev, A.G., Larsen, R.M., Pijpers, F.P., Scherrer, P.H., Sekii, T., Tarbell, T.D., Title, A.M., Thompson, M.J., Toomre, J. Helioseismic studies of differential rotation in the solar envelope by the Solar Oscillations Investigation using the Michelson Doppler Imager. *Astrophys. J.* 505, 390–417, 1998.
- Sokoloff, D., Bao, S.D., Kleeorin, N., Kuzanyan, K., Moss, D., Rogachevskii, I., Tomin, D., Zhang, H. The distribution of current helicity at the solar surface at the beginning of the solar cycle. *Astron. Nachr.* 327 (9), 876–883, 2006.
- Stix, M. *The Sun. An Introduction.* Springer, Berlin, 2002.
- Woltjer, L. On hydromagnetic equilibrium. *Proc. Natl. Acad. Sci. USA* 44, 833–841, 1958.
- Zhang, H., Bao, S., Kuzanyan, K. Twist of magnetic fields in solar active regions. *Astron. Rep.* 46 (5), 424–434, 2002.
- Zhang, H., Sokoloff, D., Rogachevskii, I.D.M., Lamburt, V., Kuzanyan, K., Kleeorin, N. The radial distribution of magnetic helicity in the solar convective zone: observations and dynamo theory. *Mon. Not. R. Astron. Soc.* 365 (1), 276–286, 2006.