Available online at www.sciencedirect.com
ScienceDirect
Planetary and Space Science 56 (2008) 1273–1279
Planetary and Space Science
www.elsevier.com/locate/pss

On the spatial structure and dispersion of slow magnetosonic modes coupled with Alfvén modes in planetary magnetospheres due to field line curvature

Dmitri Yu. Klimushkin*, Pavel N. Mager

Institute of Solar-Terrestrial Physics, P.O. Box 291, 664033 Irkutsk, Russia

Received 31 October 2007; received in revised form 18 March 2008; accepted 19 March 2008
Available online 26 March 2008

Abstract

The structure of the slow mode coupled with Alfvén mode in the axially symmetric magnetosphere is studied in the paper. Due to the coupling, the slow magnetosonic wave gets dispersion across magnetic shells and becomes not strictly guided. The slow mode is found to be captured between the resonant and cutoff surfaces, where the wave vector radial component goes to infinity and to zero, accordingly. The resonant surface is farther from the Earth than the cutoff surface. The slow mode resonance frequency is much lower than the Alfvén resonance frequency due to small value of the sound velocity near the equator. The maximum of the slow mode amplitude expressed in terms of the parallel magnetic field is concentrated near the equator, but expressed in hydromagnetic terms is concentrated near the ionospheres.

PACS: 94.30.cq; 94.30.Ms; 52.35.Bj; 47.35.Tv

Keywords: MHD waves; Mode coupling; Spatial structure

1. Introduction

Space plasmas is a natural medium for the propagation of a vast number of various kinds of waves. In the ULF range (wave frequency is much lower than ion cyclotron frequency), a prevalent mode is the Alfvén mode, because it demands only plasma and magnetic field. This is a guided wave, since its group velocity is directed along magnetic field lines. In finite pressure plasma, another guided wave can propagate under some conditions: the slow magnetosonic mode. With this mode some Pi2 (Saka et al., 1999) and Pc5 (Nishida et al., 1997) pulsations have been identified in the terrestrial magnetosphere and several wave events in the magnetosheath and the boundary layer (Gleaves and Southwood, 1991; Stasiewicz, 2004).

The field line curvature causes the coupling of the Alfvén and slow modes (Southwood, 1977; Walker, 1987; Cheng and Chance, 1986; Klimushkin, 1998; Erkaev et al., 2005). The physical mechanism of this coupling was elucidated by Southwood and Saunders (1985) and Ohtani et al. (1989a). By now much attention has been given to spectra and field-aligned structure of the slow modes (Taylor and Walker, 1987; Ohtani et al., 1989b; Cheng et al., 1993; Lui and Cheng, 2001; Cheng, 2003; Cheremnykh et al., 2004; Parnowski, 2007). The structure across magnetic shells has almost escaped the attention of the researchers, with only few exceptions (Yumoto, 1985; Klimushkin, 1998; Leonovich et al., 2006). For example, Klimushkin (1998) (hereinafter, Paper 1) showed that due to the coupling the slow mode becomes not strictly guided any more, since the dispersion across magnetic shells appears; the slow mode analogy of the poloidal surface had been found.

These studies revealed some contradictions. For example, according to Leonovich et al. (2006) there is only one kind of slow mode transverse dispersion, the one caused by the coupling with the fast mode. The field-aligned mode structure is also a subject of some controversy: it is not

*Corresponding author. Tel.: +7 3952 564547; fax: +7 3952 511675.
E-mail address: klimush@iszf.irk.ru (D.Yu. Klimushkin).

0032-0633/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.pss.2008.03.002

clear whether the amplitude maximum is situated near the ionospheres (Cheremnykh et al., 2004; Parnowski, 2007) or near the equator (Leonovich et al., 2006). Then, no other paper has found a slow mode analogy of the poloidal surface. So, the spatial structure of the slow modes in the space plasmas is not clear yet. However, at the moment the problem of the mode spatial structure becomes rather urgent in the context of the Cluster mission with its possibility to separate in the spacecraft reference system the spatial structure from the temporal evolution of the wave. Thus, it is the time to study these issues anew, in as simple terms as possible.

Similar to a number of previous studies (e.g. Klimushkin, 1998; Klimushkin et al., 2004; Erkaev et al., 2005; Cheremnykh et al., 2004; Parnowski, 2007), the present paper is concerned with transversally small-scale waves. In the Earth's magnetosphere, such are the waves with high azimuthal wave numbers ($m \gg 1$) often observed with Cluster (Eriksson et al., 2006; Schäfer et al., 2007). In the magnetosheath plasmas, these are the pulsations with the wave vector approximately perpendicular to an ambient magnetic field (Denton, 2000; Shevyrev et al., 2006). The convenience of the transversally small-scale approximation is caused by the possibility of using the WKB approximation and also by the fact that only two MHD modes can propagate in plasma in this case, the Alfvén mode and the slow magnetosonic mode; the third MHD mode, the fast magnetosonic wave, cannot propagate in this limit (e.g. Leonovich and Mazur, 2001).

The current study is based on the system of equation for the coupled modes obtained in Paper 1. This system will be presented in Section 1. Section 2 is devoted to the study of the wave structure across magnetic shells by application of the two-dimensional version of the WKB-approximation developed by Leonovich and Mazur (1993) and the perturbation theory; the small parameter is a ratio of the eigenfrequencies of the slow and Alfvén modes. The field-aligned structure of the slow modes will be studied in Section 3.

2. Basic equations

A curvilinear orthogonal coordinate system $\{x^1, x^2, x^3\}$ is used, where the field lines play the role of the coordinate lines x^3, the stream lines are the coordinate lines x^2, and the surfaces of constant pressure (magnetic shells) are coordinate surfaces $x^1 = const$. The coordinates x^1 and x^2 have the role of the radial and azimuthal coordinates. The components of the metric tensor are designated as g_i. The determinant of the metric tensor is $g = g_1 g_2 g_3$. The equilibrium magnetic field B, pressure P, and current J are related by the equilibrium condition $\nabla P = (4\pi)^{-1} \vec{J} \times \vec{B}$ (here current is defined as $\vec{J} = \vec{\nabla} \times \vec{B}$). The field line curvature radius is denoted as R.

In the axisymmetric model all perturbed quantities can be specified in the form $e^{-i\omega t + ik_2 x^2}$, where k_2 is the azimuthal component of the wave vector. If the azimuthal angle φ is used as the coordinate x^2, then $k_2 = m$, where m is the azimuthal wave number.

Within the approximation of infinite plasma conductivity, the longitudinal component of the wave's electric field $\vec{E}$ is zero. A two-dimensional field $\vec{E}$ can be split into the sum of the potential and vortical components:

$$\vec{E} = -\nabla_\perp \Phi + \nabla_\perp \times \vec{e}_\parallel \Psi, \tag{1}$$

where $\vec{e}_\parallel = \vec{B}/B$. As the third field variable, the quantity

$$\Theta = \frac{\sqrt{4\pi\gamma P}}{ck_2} \nabla \cdot \vec{\xi} \tag{2}$$

is used, where $\vec{\xi}$ is the plasma displacement. Large value of the azimuthal wave number m allows using the WKB approximation with respect to the radial coordinate, when the functions Φ and Θ can be written as

$$\begin{pmatrix} \Phi \\ \Theta \end{pmatrix} = \begin{pmatrix} F(x^3, x^1) \\ H(x^3, x^1) \end{pmatrix} \cdot \exp i \int k_1(x^1)\, dx^1,$$

where k_1 is the radial component of the wave vector.

Let us designate $s = \sqrt{\gamma P/\rho}$, $A = B/\sqrt{4\pi\rho}$, and $v_s = sA/\sqrt{s^2 + A^2}$ as the sound speed, Alfvén speed, and slow magnetosonic speed, accordingly. The operators

$$\hat{L}_T(\omega) = \partial_3 \frac{g_2}{\sqrt{g}} \partial_3 + \frac{\sqrt{g}}{g_1} \frac{\omega^2}{A^2}$$

and

$$\hat{L}_P(\omega) = \partial_3 \frac{g_1}{\sqrt{g}} \partial_3 + \frac{\sqrt{g}}{g_2} \frac{\omega^2}{A^2} - \frac{\sqrt{g}}{g_2} \frac{2J}{BR}$$

are referred to, respectively, as the toroidal and poloidal ones. The operator

$$\hat{L}_S(\omega) = \sqrt{g}\, s^2 \frac{\omega^2}{v_s^2} + \partial_3\, s^2 \frac{\sqrt{g}}{g_3}\, \partial_3$$

is called the slow mode operator. The parameter

$$\alpha = 2\sqrt{4\pi\gamma P} \frac{\sqrt{g_1 g_3}}{BR}$$

is responsible for coupling of the Alfvén and slow modes (here R is the curvature radius of a field line).

As was shown in the Paper 1, in the $m \gg 1$ limit, the functions F and H are related by the equations

$$\hat{L}_S(\omega) H = \alpha\omega F \tag{3}$$

and

$$k_1^2 \hat{L}_T(\omega) F + k_2^2 \hat{L}_P(\omega) F = k_2^2 \alpha\omega H. \tag{4}$$

These equations are the principal equations of this paper. In the homogeneous plasma, system (3), (4) yields the dispersion equations of the slow mode $s^2(\omega^2 - k_\parallel^2 v_s^2) = 0$ and Alfvén mode $k_\perp^2(\omega^2 - k_\parallel^2 A^2) = 0$. Therefore, we will call (3) and (4) as the slow and Alfvén mode equations, respectively, and the system describes slow and Alfvén modes coupled due to the field line curvature.

It should be noted that the slow mode dispersion relation in the transversally small-scale limit, $\omega^2 = k_\parallel^2 v_s^2$, contains

no wave vector transverse component, that is, the transverse dispersion of the wave is absent. The same situation is valid also for the Alfvén mode in a homogeneous plasma, but the transverse dispersion appears due to the field line curvature (Leonovich and Mazur, 1993; Klimushkin et al., 2004). Klimushkin (1998) came to the same conclusion with respect to the slow mode. We are to prove this conclusion in the present paper.

We will use the so-called 'hard' boundary conditions, where the ionosphere is supposed as an ideally conducting fixed surface, where all perturbed quantities go to zero:

$$F(x^3,x^1), H(x^3,x^1)|_{x^3=x^3_\pm} = 0. \tag{5}$$

Here $x^3_\pm$ stands for the intersection points of a field line with the upper ionospheric boundary.

As for the variable Ψ, it yields the longitudinal component of the wave magnetic field as

$$b_3 = \frac{\mathrm{i}c}{\omega}\frac{g_3}{\sqrt{g}}\left(\partial_1\frac{g_2}{\sqrt{g}}\partial_1 + \partial_2\frac{g_1}{\sqrt{g}}\partial_2\right)\Psi,$$

which is expressed through Φ and Θ as

$$b_3 = \frac{ck_2}{\omega}\frac{g_3}{\sqrt{g}}\left[\sqrt{\frac{g}{g_3}}\frac{s^2B}{A^2}\frac{\omega}{\sqrt{4\pi\gamma P}}\Theta - \sqrt{g_1}\frac{J}{B}\Phi\right]. \tag{6}$$

The perturbation of the plasma pressure $\delta P = -\vec{\xi}\cdot\nabla P - \gamma P\nabla\cdot\vec{\xi}$ is expressed in terms of Φ and Θ as

$$\delta P = -\frac{ck_2}{\omega}\left(\gamma P\frac{\omega}{\sqrt{4\pi\gamma P}}\Theta - \frac{J}{4\pi\sqrt{g_2}}\Phi\right). \tag{7}$$

Comparing it with the previous equation we get

$$\delta P + \frac{Bb_3}{4\pi\sqrt{g_3}} = 0, \tag{8}$$

that is, the perturbation of the total pressure (plasma plus magnetic pressure) is zero.

3. Slow mode localization region

As system (3), (4) involves the large parameter $m \gg 1$, to solve it we can use the WKB approximation in the radial coordinate x^1, a two-dimensional analogue of which was developed by Leonovich and Mazur (1993).

When the frequency ω is fixed, the solution of the problem (3), (4), (5) yields the eigenfunctions $H_N(x^3,x^1)$ and $F_N(x^3,x^1)$, describing the field-aligned structure of the oscillation (N is the harmonic wave number); these functions depend also on the radial coordinate x^1 as on parameter. The eigenvalue is $k_1 = k_{1N}(x^1,\omega)$. It describes the main features of the radial structure of the wave field. The most important are surfaces where k_1 goes to infinity (resonance surfaces) and to zero (cutoff surfaces).

In order to compensate the large value of the k_1^2 term in Eq. (4), the factor at this term must go to zero. There are two possibilities to arrange it. The first of them takes place when the function F is nonvanishing. Then, the product $\hat{L}_T F$ must go to zero. Thus, Alfvén or toroidal resonance is familiar, defined by the solution of the problem

$$\hat{L}_T(\omega)T(x^3,x^1) = 0, \quad T(x^3_\pm) = 0. \tag{9}$$

The solution (toroidal eigenfunction) is designated as T_N and the eigenfrequency as $\Omega_{TN}(x^1)$. The toroidal eigenfunctions are conveniently normalized in the following manner:

$$\left\langle \frac{\sqrt{g}}{g_1}\frac{T_N T_{N'}}{A^2}\right\rangle = \delta_{NN'} \tag{10}$$

(here the angle brackets designate integration along a field line between the ionospheres, $\langle\ldots\rangle = \int_{x^3_-}^{x^3_+}(\ldots)\,\mathrm{d}x^3$). The resonance (which is called the Alfvén, or toroidal resonance) takes place on the surface x^1_{TN} where the equality $\omega = \Omega_{TN}(x^1)$ is satisfied. As itself, it is beyond the scope of our present work, but the functions T_N and eigenfrequencies Ω_{TN} will be useful for other needs.

The other possibility for the resonance takes place when the function F in Eq. (4) is going to zero itself. Then, from Eq. (3) it is seen that it is possible when the product $\hat{L}_S H$ goes to zero. Let us call this case the slow mode resonance. Thus, this resonance is defined as a solution of the problem

$$\hat{L}_S(\omega)S(x^3,x^1) = 0, \quad S(x^3_\pm) = 0. \tag{11}$$

The eigenfunction is designated as S_N. Its amplitude is fixed by the normalization condition

$$\left\langle \frac{s^2}{v_s^2}\sqrt{g}S_N S_{N'}\right\rangle = \delta_{NN'}. \tag{12}$$

The slow mode eigenfrequency is designated as $\Omega_{SN}(x^1)$. This value was shown to be much lower than the toroidal eigenfrequency (Cheng et al., 1993; Lui and Cheng, 2001; Cheng, 2003; Cheremnykh et al., 2004; Parnowski, 2007). The resonance (which is called the slow mode resonance) takes place on the surface x^1_{SN} where the equality $\omega = \Omega_{SN}(x^1)$ is satisfied.

We are going to find the $k^2_{1N}(x^1,\omega)$ dependence near the slow resonance surface. To do it, let us use the perturbation method putting

$$H = S_N + h, \tag{13}$$

where h is a small correction. Notice that near the resonance, where $k_1^2 \gg k_2^2$, Eq. (4) comes to the form

$$k_1^2\hat{L}_T(\omega)F = k_2^2\alpha\omega S_N, \tag{14}$$

where we used S_N for H. We find the solution in the form

$$F = \frac{1}{k^2_{1N}}\sum_{N'} a_{N'}T_{N'}, \tag{15}$$

where $a_{N'}$ are the coefficients yet to be defined. Then we substitute F into (14), multiply by $T_{N'}$ and integrate along a field line. Using the Hermitian character of the operator $\hat{L}_T$ and the normalization (10) we obtain after some algebra an expression defining $a_{N'}$:

$$a_{N'}(\Omega^2_{SN} - \Omega^2_{TN'}) = k_2^2\Omega_{SN}\langle\alpha S_N T_{N'}\rangle.$$

Hence, if $\Omega_{SN} \ll \Omega_{TN'}$ for any N', N, we find

$$F = -\frac{k_2^2}{k_{1N}^2} \sum_{N'} \frac{\Omega_{SN}}{\Omega_{TN'}^2} \langle \alpha S_N T_{N'} \rangle T_{N'}. \tag{16}$$

This expression is valid for low slow mode harmonics.

Then we come back to Eq. (3). Substituting it in (13) and (16), multiplying by S_N and integrating along a field line, taking into account the Hermitian character of the operator $\hat{L}_S$, equality (11) and normalization (12), we obtain after some algebra:

$$k_{1N}^2(\omega, x^1) = -\frac{k_2^2}{\omega^2 - \Omega_{SN}^2(x^1)} \sum_{N'} \left(\frac{\Omega_{SN}}{\Omega_{TN'}}\right)^2 \langle \alpha S_N T_{N'} \rangle^2. \tag{17}$$

It is important to notice that positive signs of k_1^2 correspond to frequencies $\omega < \Omega_{SN}$. It means that if Ω_{SN} decreases with distance from the Earth, then the propagating region (where $k_1^2 > 0$) is situated inside the resonant surface. Using Eqs. (16) and (17) we can check our premise: the function F goes in the vicinity of the slow resonance to zero as $F \sim [\omega^2 - \Omega_{SN}^2(x^1)]$.

Now we consider cutoff surfaces. Let us designate a frequency corresponding to $k_1 = 0$ as Ω_{CN}. This frequency appears as an eigenfunction of the problem

$$\hat{L}_S(\Omega_{CN}) H = \alpha \Omega_{CN} F \tag{18}$$

and

$$\hat{L}_P(\Omega_{CN}) F = \alpha \Omega_{CN} H, \tag{19}$$

with the boundary condition (5). Since this system is equivalent to fourth-order equation with respect to the longitudinal coordinate, any N-number corresponds to two cutoff frequencies, one of which is close to the toroidal frequency and the other to the resonant SMS frequency. The first of them will be called the poloidal frequency and designated as Ω_{PN}. The emphasis is on the second one, which will be called simply the cutoff frequency. The coordinate of the cutoff surface is determined as a solution of the equation $\omega = \Omega_{CN}(x^1)$.

To determine $\Omega_{CN}(x^1)$, we express F through H by means of (14). To do it, let us introduce the "poloidal" eigenfunction determined as a solution of the problem

$$\hat{L}_P(\omega) P(x^3, x^1) = 0, \quad P(x_\pm^3, x^1) = 0. \tag{20}$$

As a normalization condition, the expression

$$\left\langle \frac{\sqrt{g}}{g_2} \frac{P_N P_{N'}}{A^2} \right\rangle = \delta_{NN'} \tag{21}$$

will be used.

Let us decompose the function F in terms of the poloidal eigenfunctions P_N:

$$F = \sum_{N'} b_{N'} P_{N'}, \tag{22}$$

where $b_{N'}$ are the coefficients yet to be defined. Proceeding in the same manner as in the derivation of (16), we get:

$$F = \sum_{N'} \frac{\Omega_{CN}}{\Omega_{CN}^2 - \Omega_{PN'}^2} \langle \alpha H_N P_{N'} \rangle P_{N'}. \tag{23}$$

Substituting this expression into (18) yields the integro-differential equation for the function H:

$$\hat{L}_S(\Omega_{CN}) H = \alpha \sum_{N'} \frac{\Omega_{CN}}{\Omega_{CN}^2 - \Omega_{PN'}^2} \langle \alpha H P_{N'} \rangle P_{N'}. \tag{24}$$

Let us suppose that the values Ω_{CN} and Ω_{SN} are close to each other. Then, the perturbation theory can be applied for the solution of Eq. (24). To do it, the function H and the frequency Ω_{CN} can be represented as

$$H = S_N + s, \tag{25}$$

$$\Omega_{CN} = \Omega_{SN} + \varepsilon, \tag{26}$$

where s and ε are small additions. The right-hand side of (24) is supposed to be small in view of the assumed inequality $\Omega_{PN}^2 / \Omega_{CN}^2 \ll 1$. Then the further equation is obtained for these additions:

$$\hat{L}_S(\Omega_{CN}) s + \sqrt{g} \frac{s^2}{v_s^2} \varepsilon S_N = -\sum_{N'} \frac{\Omega_{SN}^2}{\Omega_{PN'}^2} \langle \alpha S_N P_{N'} \rangle \alpha P_{N'}.$$

Then we multiply this expression by S_N and integrate along the field line. We get

$$\Omega_{CN}^2 = \Omega_{SN}^2 - \sum_{N'} \frac{\Omega_{SN}^2}{\Omega_{PN'}^2} \langle \alpha S_N P_{N'} \rangle^2. \tag{27}$$

Hence, it follows that the cutoff frequency is always less than the slow mode resonant eigenfrequency. Consequently, if the resonant eigenfrequency grows with the radial coordinate, then the cutoff surfaces is closer to the Earth than the resonant surface. It corresponds to the above-mentioned property: $k_1^2 > 0$ at $x^1 < x_{SN}^1$.

4. Slow mode field-aligned structure and eigenfrequencies in longitudinal WKB approximation

In order to clarify the field-aligned structure and eigenfrequencies of the slow magnetosonic waves, let us apply the WKB approximation with respect to the longitudinal coordinate, which is valid for relatively high N numbers. In this approximation, the required function can be presented as $S = e^{i\psi_0 + i\psi_1}$, where ψ_0 and ψ_1 describe the leading and the first term, accordingly. The covariant and 'physical' components of the longitudinal wave vector are determined as $\partial_3 \psi_0 = k_3$ and $k_\parallel = k_3 / \sqrt{g_3}$. Then, the leading order of system (3), (4) yields the algebraic equation

$$\omega^2 - k_\parallel^2 v_s^2 = \frac{k_y^2}{k_\perp^2} \frac{4\omega^2 v_s^2}{R^2} \left[(\omega^2 - k_\parallel^2 A^2) - \frac{k_y^2}{k_\perp^2} \frac{2P'}{\rho R} \right]^{-1}. \tag{28}$$

Here, $k_y = k_2 / \sqrt{g_2}$, the prime means differentiating with respect to the radial coordinate, and the transverse wave

vector is defined as $k_\perp^2 = (k_1^2/g_1) + (k_2^2/g_2)$. This equation can be considered as a local dispersion relation for the coupled Alfvén and slow modes. It is the same expression as derived by Mikhailovskii and Skovoroda (2002).

First, it is clearly seen that the transverse wave vector approaches infinity ($k_\perp \to \infty$) under either of two conditions: (i) $\omega^2 - k_\parallel^2 A^2 = 0$ or (ii) $\omega^2 - k_\parallel^2 v_s^2 = 0$. It confirms the existence of two resonances: (i) the Alfvén resonance and (ii) the slow resonance.

The field-aligned structure of the slow resonance is defined by Eq. (11). In the leading order of the WKB approximation, the parallel wave number is

$$k_3 = \frac{\omega}{v_s}\sqrt{g_3}. \tag{29}$$

The next order of the WKB approximation (function ψ_1) defines the wave amplitude. It is determined from the equation

$$\left(s^2 \frac{\sqrt{g}}{g_3}\right)' + s^2 \frac{\sqrt{g}}{g_3}(\mathrm{i}k_3' - 2k_3\psi_1) = 0.$$

Here, prime means the differentiation with respect to the x^3 coordinate. Thus, we get the solution of the equation $\hat{L}_S S = 0$:

$$S = Q \sin \int_{x_-^3}^{x^3} k_3(x^{3\prime})\,\mathrm{d}x^{3\prime}, \tag{30}$$

where Q is the wave amplitude and

$$t_s = \int_{x_-^3}^{x_+^3} \mathrm{d}x^3 \frac{\sqrt{g_3}}{v_s}$$

is the travel time along a field line with the local slow magnetosonic speed. The eigenfrequency becomes

$$\Omega_{SN} = \frac{\pi N}{t_s}. \tag{31}$$

It is clearly seen that the small value of the slow magnetosonic resonant frequency is caused by the travel time t_s of large value due to small value of the speed velocity near the equator.

Using the normalization condition (12), we find the amplitude:

$$Q = \frac{\omega}{t_s}\left(s^2 \frac{\sqrt{g}}{g_3} k_3\right)^{-1/2}. \tag{32}$$

Taking into account that due to condition $\nabla \cdot \vec{B} = 0$ the term $B\sqrt{g_\perp}$ does not depend on the longitudinal coordinate and Eq. (29), we can also find the dependence of the wave amplitude on the length along a field line:

$$Q \propto \left(B^2 \frac{\rho}{P}\right)^{1/4}. \tag{33}$$

The magnetic field B and density ρ increases away from the equator, and the pressure P is constant along a field line. Hence, if the slow magnetosonic function is determined in the hydromagnetic terms (through the divergence of the plasma displacement according to Eq. (2)), the maximum of the wave amplitude must be located near the ionosphere. This result is in agreement with previous studies (Cheremnykh et al., 2004; Parnowski, 2007).

It is easy also to find the behavior of the longitudinal magnetic field of the wave determined by Eq. (6). In this expression, the function Φ can be neglected because near the slow mode resonance surface it behaves as $\Phi \propto (\omega^2 - \Omega_{SN}^2(x^1))$ (see expressions (16) and (17)) and it is much smaller than Θ. Then it yields

$$|b_\parallel| \propto A^{-1/2}.$$

But Alfvén speed A increases away from the equator. Hence, the wave parallel magnetic field is concentrated near the equator, as was found by Leonovich et al. (2006). Thus, the contradiction with regard to the field-aligned wave structure disappears: different authors simply used different definitions of the slow mode: hydromagnetic (Cheremnykh et al., 2004; Parnowski, 2007) or electrodynamic (Leonovich et al., 2006).

Now we turn to the surfaces where the perpendicular wave vector equals zero, $k_\perp = 0$. Since Eq. (28) is of the fourth order with respect to the frequency, there are two solutions for the frequency. The first solution is more close to the toroidal eigenfrequency. It is the familiar Alfvén poloidal eigenfrequency. The second one is more close to the slow mode resonance frequency. It is the slow mode cutoff frequency, which we are going to study in more detail.

The slow mode cutoff frequency Ω_{CN} can be found in terms of the perturbation theory (in addition to the WKB approximation). The local value of the parallel wave vector can be searched in the form $k_\parallel^2 = (\omega^2 + \delta)/v_s^2$, where δ is the small addition yet to be determined. Let us suppose the right-hand side of Eq. (28) to be small, which is possible provided that the condition $\beta/(k_\parallel R)^2 \ll 1$ is satisfied. By means of the perturbation method we find

$$k_\parallel^2 = \frac{\omega^2}{v_s^2} + \frac{4}{R^2}\frac{v_s^2}{A^2}.$$

The frequency can be found from the quantization condition $\oint k_\parallel \,\mathrm{d}l_\parallel = 2\pi N$ (the integration is performed along the field line between ionospheres 'there and back'). Using the perturbation method one more time, we get

$$\Omega_{CN} = \frac{\pi N}{t_s} - \frac{1}{\pi N}\oint \frac{\mathrm{d}l_\parallel}{R^2}\frac{v_s^3}{A^2}. \tag{34}$$

In accordance with Eq. (27) from the previous section, the cutoff frequency is smaller than the resonance frequency, $\Omega_{CN} < \Omega_{SN}$.

5. Conclusions

Thus, we get the following conclusion: due to the coupling with the Alfvén mode, the slow magnetosonic wave gets dispersion across magnetic shells. The wave vector radial component goes to infinity when the wave frequency equals slow magnetosonic resonance frequency

$\Omega_{SN}(x^1)$, and it goes to zero when the frequency equals the cutoff eigenfrequency $\Omega_{CN}(x^1)$ (Fig. 1). Since both eigenfrequencies depend on the radial coordinate, two selected magnetic surfaces can be introduced, resonant and cutoff surfaces, defined as solutions of the equations $\omega = \Omega_{SN}(x^1)$ and $\omega = \Omega_{CN}(x^1)$, accordingly. The mode is enclosed between these two magnetic surfaces. This resembles the structure of the Alfvén wave (Leonovich and Mazur, 1993; Klimushkin et al., 2004). As the inequality $\Omega_{CN} < \Omega_{SN}$ holds and these functions typically decrease with the radial coordinate, the resonant surface is farther from the Earth than the cutoff surface. The slow mode resonance frequency Ω_{SN} is much lower than the Alfvén resonance frequency due to small value of the sound velocity near the equator.

The reasons for the appearance of the slow mode cutoff surface (the second cutoff surface in addition to the poloidal surface, which is a cutoff surface for the Alfvén mode) is that the system of two second-order equations (3), (4) is equivalent to one fourth-order equation, which has two solutions for the frequency squared, corresponded to any chosen $k_\perp$ value. Thus, the choice $k_\perp = 0$ leads to a couple of cutoff frequencies, one is the poloidal Alfvén frequency and the other is the slow mode cutoff frequency. This is clearly confirmed by the study performed in the longitudinal WKB approximation. Moreover, the dispersion relation for the coupled Alfvén and slow modes coincides with that found in the independent study (Mikhailovskii and Skovoroda, 2002).

These results confirm the conclusions of Paper 1. The disagreement between the present study and the work of Leonovich et al. (2006) is fictitious since it is caused by the difference in the assumptions made in these studies: Leonovich et al. (2006) considered such low wave frequencies that the coupling between the Alfvén and slow modes can be neglected, but they did not limit the m numbers, so the fast mode must be taken into account; while we did not limit the frequency, the coupling between the Alfvén and slow modes is essential, but considered only high m numbers, which allowed us to neglect the fast mode. So, the assumptions are quite opposite.

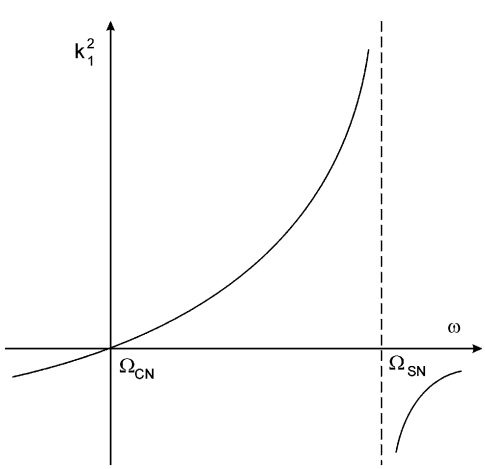

Fig. 1. Dispersion low of the slow magnetosonic wave coupled with the Alfvén mode.

The disagreement between the longitudinal mode structure revealed in different studies is also found to be fictitious. Near the magnetosonic resonance surface, the parallel magnetic field is concentrated near the magnetic equator, but the plasma displacement is concentrated near the ionospheres.

Acknowledgments

The work by D.K. and P.M. is supported by INTAS Grant 05-1000008-7978, RFBR Grant 07-05-00185, Program of presidium of Russian Academy of Sciences no. 16, and OFN RAS no. 16. The work of P.M. is also supported by Russian Science Support Foundation.

References

Cheng, C.Z., 2003. MHD field line resonances and global modes in three-dimensional magnetic fields. J. Geophys. Res. 108 (A1), 1002.

Cheng, C.Z., Chance, M.S., 1986. Low-n shear Alfvén spectra in axisymmetric toroidal plasmas. Phys. Fluids 29, 3695–3701.

Cheng, C.Z., Chang, T.C., Lin, C.A., Tsai, W.H., 1993. Magnetohydrodynamic theory of field line resonances in the magnetosphere. J. Geophys. Res. 98, 11339–11347.

Cheremnykh, O.K., Parnowski, A.S., Burdo, O.S., 2004. Ballooning modes in the inner magnetosphere of the Earth. Planet. Space Sci. 52, 1217–1229.

Denton, R.E., 2000. ULF waves in the magnetosheath. Int. J. Geomagn. Aeron. 2, 45–55.

Eriksson, P.T.I., Blomberg, L.G., Glassmeier, K.-H., 2006. Cluster satellite observations of mHz pulsations in the dayside magnetosphere. Adv. Space Res. 38, 1730–1737.

Erkaev, N.V., Shaidurov, V.A., Semenov, V.S., Langmayr, D., Biernat, H.K., 2005. Peculiarities of Alfvén wave propagation along a nonuniform magnetic flux tube. Phys. Plasmas 12, 012905-12.

Gleaves, D.G., Southwood, D.J., 1991. Magnetohydrodynamic fluctuations in the Earth's agnetosheath at 1500 LT: ISEE 1 and ISEE 2. J. Geophys. Res. 96 (A1), 129–142.

Klimushkin, D.Yu., 1998. Theory of azimuthally small-scale hydromagnetic waves in the axisymmetric magnetosphere with finite plasma pressure. Ann. Geophys. 16, 303–321.

Klimushkin, D.Yu., Mager, P.N., Glassmeier, K.-H., 2004. Toroidal and poloidal Alfvén waves with arbitrary azimuthal wave numbers in a finite pressure plasma in the Earth's magnetosphere. Ann. Geophys. 22, 267–288.

Leonovich, A.S., Mazur, V.A., 1993. A theory of transverse small-scale standing Alfvén waves in an axially symmetric magnetosphere. Planet. Space Sci. 41, 697–717.

Leonovich, A.S., Mazur, V.A., 2001. On the spectrum of magnetosonic eigenoscillations of an axisymmetric magnetosphere. J. Geophys. Res. 106, 3919–3928.

Leonovich, A.S., Kozlov, D.A., Pilipenko, V.A., 2006. Magnetosonic resonance in a dipole-like magnetosphere. Ann. Geophys. 24, 2277–2289.

Lui, A.T.Y., Cheng, C.Z., 2001. Resonance frequency of stretched magnetic field lines based on a self-consistent equilibrium magnetosphere model. J. Geophys. Res. 106 (A11), 25793–25802.

Mikhailovskii, A.B., Skovoroda, A.A., 2002. Stability of alternative toroidal systems with respect to ideal local modes. Plasma Phys. Control. Fusion 44, 2033–2050.

Nishida, A., Mukai, T., Yamamoto, T., Saito, Y., Kokubun, S., Lepping, R.P., 1997. Traversal of the nightside magnetosphere at 10 to 15 Re during northward IMF. Geophys. Res. Lett. 24 (8), 939–942.

Ohtani, S., Miura, A., Tamao, T., 1989a. Coupling between Alfvén and slow magnetosonic waves in an inhomogeneous finite-β plasma—I. Coupled equations and physical mechanism. Planet. Space Sci. 37, 567–577.

Ohtani, S., Miura, A., Tamao, T., 1989b. Coupling between Alfvén and slow magnetosonic waves in an inhomogeneous finite-β plasma—II. Eigenmode analysis of localized ballooning-interchange instability. Planet. Space Sci. 37, 579–588.

Parnowski, A.S., 2007. Eigenmode analysis of ballooning perturbations in the inner magnetosphere of the Earth. Ann. Geophys. 25, 1391–1403.

Saka, O., Watanabe, O., Okada, K., Baker, D.N., 1999. A slow mode wave as a possible source of Pi 2 and associated particle precipitation: a case study. Ann. Geophys. 17, 674–681.

Schäfer, S., Glassmeier, K.-H., Eriksson, P.T.I., Pierrard, V., Fornasson, K.-H., Blomberg, L.G., 2007. Spatial and temporal characteristics of poloidal waves in the terrestrial plasmasphere: a CLUSTER case study. Ann. Geophys. 25, 1011–1024.

Shevyrev, N.N., Zastenker, G.N., Eiges, P.E., Richardson, J.D., 2006. Low frequency waves observed by Interball-1 in foreshock and magnetosheath. Adv. Space Res. 37, 1516–1521.

Southwood, D.J., 1977. Localised compressional hydromagnetic waves in the magnetospheric ring current. Planet. Space Sci. 25, 549–554.

Southwood, D.J., Saunders, M.A., 1985. Curvature coupling of slow and Alfvén MHD waves in a magnetotail field configuration. Planet. Space Sci. 33, 127–134.

Stasiewicz, K., 2004. Theory and observations of slow-mode solitons in space plasmas. Phys. Rev. Lett. 93.

Taylor, J.P.N., Walker, A.D.M., 1987. Theory of magnetospheric standing hydromagnetic waves with large azimuthal wave number: 2. Eigenmodes of the magnetosonic and Alfvén oscillations. J. Geophys. Res. 92, 10046–10052.

Walker, A.D.M., 1987. Theory of magnetospheric standing hydromagnetic waves with large azimuthal wave number: 1. Coupled magnetosonic and Alfvén waves. J. Geophys. Res. 92, 10039–10045.

Yumoto, K., 1985. Characteristics of localized resonance coupling oscillations of the slow magnetosonic wave in a non-uniform plasma. Planet. Space Sci. 33, 1029–1036.