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DEFINITION OF THE ALFVÉN MODE IN INHOMOGENEOUS MAGNETIC FIELD

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Abstract. The article is methodological and defines the concept of the linear Alfvén mode. There are two definitions — electrodynamic and hydrodynamic. In the former, the Alfvén mode is considered a wave with a potential transverse electric field. In the latter, waves are more often identified with the Alfvén mode, plasma motion in which is purely vortex. While these definitions are equivalent for homogeneous plasma, they are incompatible if the field line curvature is taken into account: if the transverse electric field is purely potential, the plasma speed has not only a vortex component, but also a potential one, and vice versa. The electrodynamic and hydrodynamic definitions are equivalent only if the wave electric field completely lacks a component along the binormal to the external magnetic field. However, such waves do not exist in nature.

Keywords: Alfvén waves, magnetosphere, solar corona, field line curvature.

INTRODUCTION

Magnetohydrodynamic (MHD) waves are often observed in space plasma (planetary magnetospheres, solar chromosphere and corona, interplanetary medium) [Nakariakov et al., 2016]. As is known, there are three MHD modes: Alfvén, fast and slow magnetosonic (FMS and SMS) [Kadomtsev, 1988]. Only the Alfvén mode is able to propagate over considerable distances without noticeable attenuation. Indeed, its group velocity is directed along the field line so that the wave energy density remains virtually unchanged. On the contrary, the FMS wave propagates isotropically, thereby leading to a decrease in the energy density when moving away from the source. On the other hand, the SMS wave experiences strong collisionless attenuation when interacting with particles. A considerable amount of MHD oscillations in Earth's magnetosphere [Leonovich, Mazur, 2016; Alperovich, Fedorov, 2007] and the solar wind [Belcher, Davis, 1971; Tu, Marsch, 1995] are usually identified with Alfvén waves. In the solar atmosphere, these modes are technically more difficult to observe, but even here there is evidence of their existence [Jess et al., 2009; Kobanov et al., 2017; Chelpanov et al., 2018]. Recent observations of Alfvén waves in the solar atmosphere have been reviewed in [Ruderman, Petrukhin, 2022].

In order to understand the mechanisms of their generation and figure out what information MHD modes contain about the medium, it is necessary to learn how to correctly identify the oscillation mode. There is considerable disagreement in the literature about what should be called Alfvén modes. For instance, in magnetospheric physics this term is generally applied to waves with a potential electric field (we are talking about the electric field component perpendicular to the external magnetic field; recall that in single-fluid MHD the field-aligned electric field is equal to zero). In solar physics, waves that do not change the density of surrounding plasma are more often identified with the Alfvén mode. The same definition is often adopted by researchers dealing with "pure" magnetic hydrodynamics. There are no density perturbations only if plasma velocity fluctuations in the wave are vortex. In homogeneous plasma, these definitions are equivalent, but in inhomogeneous plasma, especially in a curved magnetic field, the situation is different. This fact is not generally known, which sometimes leads to mutual misunderstanding among researchers studying MHD waves in various space plasma regions. The article gives an insight into this issue. For simplicity, we limit ourselves to the case of cold plasma, where there is no SMS wave, but only Alfvén and FMS waves. The waves are considered in ideal single-fluid MHD in a linear approximation.

ELECTRODYNAMIC AND HYDRODYNAMIC DEFINITIONS OF ALFVÉN MODES

As is known, in ideal single-fluid MHD it is assumed that the electric field is concentrated on a surface orthogonal to field lines: $\vec{E}=\vec{E}_\perp$, where the index $\perp$ indicates the direction perpendicular to the magnetic field. According to the Helmholtz theorem, any vector field can be expanded into the sum of potential and vortex components. In application to the MHD-wave electric field

$$\vec{E}_\perp = -\nabla_\perp \Phi + \nabla_\perp \times \vec{e}_\parallel \Psi. \qquad (1)$$

Here $\vec{e}_\parallel$ is a unit vector along the external magnetic field $\vec{B}_0$. It can be shown that the potentials Φ and Ψ correspond to Alfvén and FMS waves respectively [Tamao, 1986; Klimushkin, 1994]. Thus, the Alfvén-wave electric field is characterized by the condition

$$\nabla_\perp \times \vec{E}_\perp = 0, \tag{2}$$

which can be called the electrodynamic definition of the Alfvén mode.

The transverse plasma velocity in MHD is equal to the electric drift velocity

$$\vec{\upsilon}_\perp = c\frac{\vec{E}_\perp \times \vec{B}_0}{B_0^2}. \tag{3}$$

If a magnetic field is homogeneous, $\vec{B}_0 = const$, it follows from (2) and (3) that plasma motion in the Alfvén wave is vortex:

$$\nabla_\perp \cdot \vec{\upsilon}_\perp = 0. \tag{4}$$

This condition can be called the hydrodynamic definition of the Alfvén mode.

Now consider the curved magnetic field. For simplicity, assume that the equilibrium magnetic field is potential: $\nabla \times \vec{B}_0 = 0$. In cold plasma, the equilibrium condition is written as

$$\nabla_\perp B_0 = \vec{n}\frac{B_0}{R}, \tag{5}$$

where $\vec{n}$ is the unit vector along the normal to the field line; R is the radius of field line curvature.

Calculate the divergence of a velocity vector

$$\begin{aligned}\nabla_\perp \cdot \vec{\upsilon}_\perp &= c\left(\nabla_\perp B_0^{-2}\right)\cdot\left[\vec{E}_\perp \times \vec{B}_0\right] - \\ &- cB_0^{-2}\left\{\vec{E}_\perp \cdot \left[\nabla_\perp \times \vec{B}_0\right] - \vec{B}_0 \cdot \left[\nabla_\perp \times \vec{E}_\perp\right]\right\}.\end{aligned} \tag{6}$$

As a result of calculations in terms of (5), this value is found to be proportional to the field line curvature and, in general, is not equal to zero:

$$\nabla_\perp \cdot \vec{\upsilon}_\perp = \frac{2c}{B_0^2 R}\vec{B}_0 \cdot \left[\vec{n} \times \vec{E}_\perp\right] = -\frac{2c}{B_0 R}E_b \neq 0. \tag{7}$$

Here $E_b = \vec{E}_\perp \cdot \left[\vec{n} \times \vec{e}_\parallel\right]$ is the electric field component along the binormal to the field line (Figure 1). Thus, if condition (2) is valid in a curved magnetic field, condition (4) is ruled out: the plasma velocity has not only a vortex component, but also a potential one. It is easy to verify the opposite: if the plasma motion is purely vortex (condition (4) holds), the wave electric field cannot be purely potential.

The effects of field line curvature leading to inequality (7) can be ignored only if two conditions are fulfilled simultaneously:

$$\lambda_\parallel \ll R \tag{8}$$

and

$$\lambda_n \ll \lambda_b, \tag{9}$$

where $\lambda_\parallel, \lambda_n, \lambda_b$ is the wavelength in the projection on the magnetic field direction, normal, and binormal to the magnetic field respectively. In the magnetosphere for Pc4-5 MHD waves, condition (8) is not obviously met. Indeed, such waves are low harmonics of waves standing along the field line, whose longitudinal length is equal

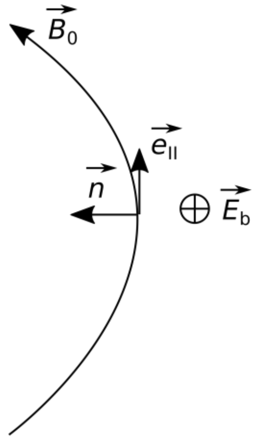

Figure. Binormal component of wave electric field

to the length L of the field line between ionospheres of magnetoconjugated hemispheres or 2–3 times less. Yet, the radius of field line curvature and the length of the field line have the same order of magnitude: L~R [Leonovich, Mazur, 2016]. Note that it is the parallel rather than transverse wavelength enters in consition (8) since the latter is not contained in the dispersion relation for Alfvén wave. Condition (9) means that the Alfvén wave has a toroidal polarization such that the field line oscillates mainly in the azimuthal direction. In this case, the binormal wave electric field component E_b is much smaller than the normal one E_n. The vortex-free motion in the Alfvén wave is especially important if there is finite plasma pressure since it contributes to the coupling of the poloidal Alfvén mode and the slow magnetic sound due to field line curvature [Southwood, Saunders, 1985], which may cause ballooning instability of magnetospheric plasma [Cheremnykh et al., 2004; Leonovich, Kozlov, 2014; Rubtsov et al., 2018].

The perturbation of the plasma density ρ deserves separate consideration. Its absence is often considered a characteristic feature of the Alfvén wave. In homogeneous plasma (ρ_0=*const*), vortex motion (4) does not in fact lead to plasma condensation or rarefaction (in the linear approximation):

$$\frac{\partial\rho}{\partial t} = -\nabla_\perp \cdot \rho_0\vec{\upsilon}_\perp = -\rho_0\nabla_\perp \cdot \vec{\upsilon}_\perp = 0. \tag{10}$$

However, if plasma is inhomogeneous, $\rho_0 \neq const$, a density perturbation occurs even in a straight magnetic field with a purely vortex motion:

$$\frac{\partial\rho}{\partial t} = -\nabla_\perp \cdot \rho_0\vec{\upsilon}_\perp = -\vec{\upsilon}_\perp \cdot \nabla_\perp\rho_0 - \rho_0\nabla_\perp \cdot \vec{\upsilon}_\perp \neq 0. \tag{11}$$

There is no density perturbation only if the velocity component along the density gradient $\nabla_\perp\rho_0$ is zero. It is often assumed that this gradient is directed along the normal to field lines. Then, in view of (3), the binormal electric field component E_b is also zero. In this case, the density perturbation does occur.

Note that the mode with $E_b \neq 0$ is sometimes called ballooning and is distinguished from the Alfvén one. We think this definition is improper since in the general case the wave has both normal E_n and binormal E_b electric field components. There are no waves with E_b=0 in nature. Note that in magnetospheric physics the Alfvén waves with $E_n \gg E_b$ are referred to as toroidal; with $E_n \ll E_b$, poloidal. Both toroidal [Shi et al., 2020; Yamamoto et al., 2022] and poloidal [Mager, 2021; Mikhailova et al., 2022] Alfvén waves are regularly observed in the magnetosphere.

CONCLUSION

Thus, the two definitions of the Alfvén mode, electrodynamic $\nabla_\perp \times \vec{E}_\perp = 0$ and hydrodynamic $\nabla_\perp \cdot \vec{\upsilon}_\perp = 0$, in a curved magnetic field are incompatible with each other. Which to accept is a matter of researcher's opinion.

We will, however, give an additional argument for the electrodynamic definition. The velocity $\vec{\upsilon}$ is usually used for the hydrodynamic description of plasma. In kinetics, the main variables describing the wave are electromagnetic field components, whereas the macroscopic plasma velocity $\vec{\upsilon}$ appears only as a result of calculation of the distribution function moment and is usually omitted. Thus, in the context of joining of MHD and kinetics, the electrodynamic definition of Alfvén wave $\nabla_\perp \times \vec{E}_\perp = 0$ seems to be more preferable.

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