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Rotation-induced lithium depletion of solar-type stars in young stellar clusters

G. Rüdiger and V.V. Pipin[1]

Astrophysikalisches Institut Potsdam,
An der Sternwarte 16, D-14482 Potsdam, Germany

Abstract. The eddy-diffusion tensor for rotating anisotropic turbulence fields is considered in order to explain the lithium decay during the spin-down process of solar-type stars. Rotation proves to be effective in the transfer of passive tracers through the solar/stellar tachoclines, located beneath the outer convection zones and assumed to possess only horizontal turbulence fields. The transport efficacy is so high that the tachocline turbulence must not exceed a limit of (say) $10^{-(3...4)}$ of the rms velocity of the convection zone in order to leave the lithium surviving after Gigayears. A correlation is predicted between rotation rate and lithium abundance for (fast rotating) stars of one and the same age. It can thus be tested with the solar-type stars of young clusters such as IC 2391 and others. The public database of observational parameters confirms the existence of such a correlation, which becomes weaker for older clusters like Pleiades and Hyades.

1. Introduction

There must be a drift process for the chemicals from the bottom of the convection zone through the solar 'tachocline' to the burning domain. The effect must be small, however, in order to allow the existence of lithium in the solar atmosphere even after 4.6 Gyr. The lithium decay time is about 10^7 times the convection zone diffusion time of ~100 yrs. We are thus looking for a rather small effect which, however, cannot simply be microscopic diffusion (Zahn 1989). In the following the consequences are presented of a quasilinear mean-field approximation of an anisotropic turbulence field which might be located in the solar 'tachocline', which is stably stratified in opposition to the unstably-stratified solar convection zone. However, to form reduction factors of $10^{7...8}$ for any eddy diffusivity is nontrivial.

The transport of a passive scalar is governed by the diffusion equation

$$\frac{\partial \rho C}{\partial t} + \mathrm{div}\,(\rho C \boldsymbol{u} - \rho D \nabla C) = 0 \qquad (1)$$

with D as the microscopic diffusion coefficient.

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[1] On leave from the Institute for Solar-Terrestrial Physics Irkutsk.

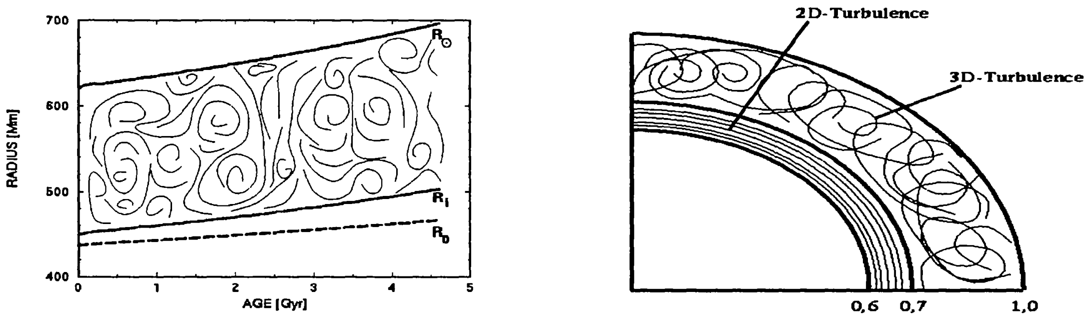

Figure 1. LEFT: The outer structure of the Sun during its MS life. R_i limits the outer convection zone, R_0 displays the lithium burning zone hotter than 2.6 Mio K. RIGHT: The turbulence model. Below the convection zone with isotropic turbulence is a stable 'tachocline' layer with horizontal turbulence.

Averaging (1) we get the well-known diffusion equation in the presence of turbulence (see Eq. (5) below). For the fluctuations of the chemical concentration in a quasilinear approximation follows

$$\frac{\partial \rho C'}{\partial t} + \mathrm{div}\left(\rho \boldsymbol{u}' \bar{C} - \rho D \nabla C'\right) = 0. \tag{2}$$

In a corotating frame of reference the equation for the fluctuating part of the velocity field $\boldsymbol{u}'$ is

$$\frac{\partial \boldsymbol{u}'}{\partial t} + 2\boldsymbol{\Omega} \times \boldsymbol{u}' + \frac{1}{\rho}\nabla p' - \nu_{\mathrm{t}} \Delta \boldsymbol{u}' = \boldsymbol{f}' \tag{3}$$

with $\boldsymbol{f}'$ as the random turbulence driver and ν_{t} as some background eddy viscosity. The desired correlation between concentration fluctuations and velocity fluctuations, $\langle C'\boldsymbol{u}'\rangle$, can be found by a Fourier transform of (3). The radial turbulence intensity may be denoted by w^2 while the azimuthal turbulence intensity may be v^2. For convenience, an anisotropy parameter s is defined by $v^2 = sw^2$, so that large s denote horizontal-type turbulences.
The results for the turbulent concentration-flux vector may be written as anisotropic diffusion down the mean concentration gradient, i.e.

$$\langle C' u_i' \rangle = -D_{ij}\, \frac{\partial \bar{C}}{\partial x_j}. \tag{4}$$

For horizontal turbulence the diffusion tensor *with rotation* becomes highly anisotropic and is given in the τ-approximation in a paper by Rüdiger et al. (1999). The main parameter is the Coriolis number $\Omega^* = 2\tau_{\mathrm{corr}}\Omega$ with τ correlation highly radius-dependent. We observe the appearance of the off-diagonal components $D_{r\theta} = D_{\theta r}$, which will play a key role role in the mean-field equation for the large-scale concentration for rotating turbulence fields. The mean-field

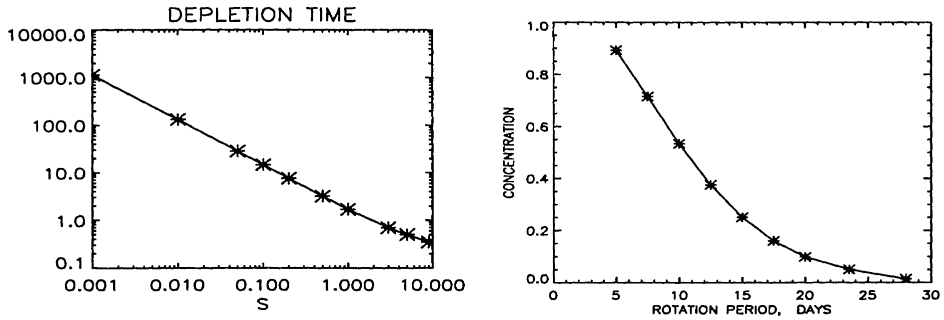

Figure 2. LEFT: Depletion time (in units of the diffusion time) for weak horizontal turbulence varies with $1/s$. $\Omega_0^* = 6$ at the base of the convection zone and below. RIGHT: Situation for young stellar clusters: One and the same turbulence model with $s = 0.01$ but angular velocity is varied. The surface concentration at the equator is shown after 100 diffusion times.

diffusion equation

$$r^2 \rho \frac{\partial \bar{C}}{\partial t} = \frac{\partial}{\partial r} \left\{ \rho r^2 \left(D_{rr} \frac{\partial \bar{C}}{\partial r} + \frac{D_{r\theta}}{r} \frac{\partial \bar{C}}{\partial \theta} \right) \right\} + \frac{\rho}{\sin\theta} \frac{\partial}{\partial \theta} \sin\theta \left(D_{\theta\theta} \frac{\partial \bar{C}}{\partial \theta} + r D_{r\theta} \frac{\partial \bar{C}}{\partial r} \right) \tag{5}$$

must be solved with $\bar{C} = 0$ at the lower boundary and an no-flux condition at the surface.

2. The lithium problem

Following Spiegel & Zahn (1992) we shall refer to the region $x_0 \leq x \leq x_i$ as the solar/stellar 'tachocline'. We model the turbulence in the convection zone as isotropic turbulence also under the influence of rotation. Below the convection zone the turbulence may be so anisotropic that vertical motions do not exist. Thus we have 2 free parameters for the tachocline turbulence: intensity s of the horizontal motion and correlation time $\tau_{\rm corr}$ of the eddies. For $s \to 0$ the turbulence disappears and for $\tau_{\rm corr} \to 0$ the rotational influence disappears. In both cases the decay time of the chemical at the solar surface *must* become infinite. In the Fig. 2 the decay time of an initially uniform concentration is plotted for various horizontal turbulence intensities in the tachocline. For $s = 0$, of course, there is no depletion, and it is strikingly short for nearly homogeneous turbulence fields with (say) $s = 1$. The correlation time profile and the basic rotation are taken from a solar model by Stix & Skaley with $\Omega^* = 6$ at the base of the convection zone. We find *that rotation is highly effective to transport the chemicals* to the burning zone at x_0. An approximate relation

$$\tau_{\rm dec} \simeq \tau_{\rm diff}/s \tag{6}$$

is found. The factor 10^7 between both the characteristic times provided, the horizontal rms velocity of the tachocline turbulence must not exceed $10^{-(3\ldots4)}$ of the convection zone turbulence. That there is still lithium at the solar surface is here only compatible with *very low horizontal turbulence intensities.*

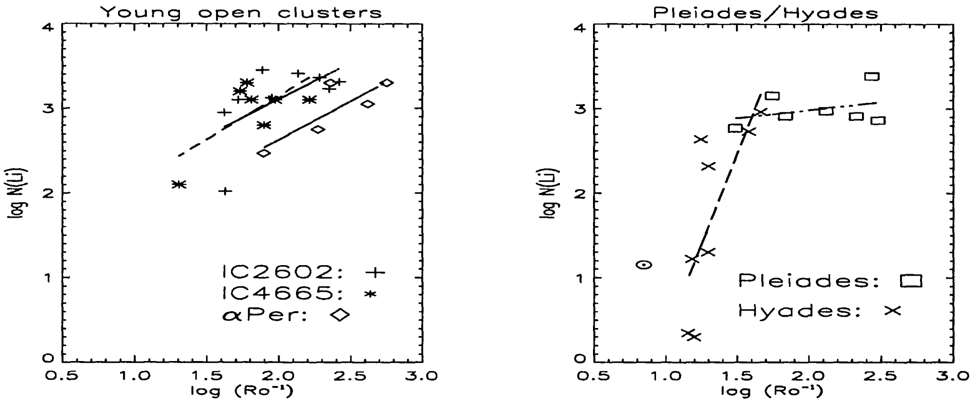

Figure 3. Solar-type stars in open clusters with lithium abundance versus inverse Rossby number Ro^{-1} after Tschäpe & Rüdiger (1999). LEFT: young clusters (30...50 Myr). RIGHT: older cluster (100...600 Myr).

3. Rotation rate variations

One can vary the basic rotation rate such as it varies in a young stellar cluster for all stars with the same age. The question is how the lithium varies with the prescribed rotation rate for fixed turbulence model. We take $s = 0.01$ and $\tau_{\mathrm{corr}} = 12$ days. The right panel of Fig. 2 presents the results. The faster the rotation the more lithium remains at the surface. The rotational quenching of the eddy diffusivity dominates the effect of the large latitudinal gradients of the concentration. The *slower rotators are more effective in mixing* the chemical downwards. The plot, however, does not display that for even slower rotation, in the limit for $\Omega^* < 1$, the mixing again becomes slower and slower and also the decay of the primordial lithium.

The observation of the lithium-rotation correlation in young stellar clusters seems to confirm the prediction. In Fig. 3 the data of the Prosser-Stauffer Internet archive are presented for various open clusters. At the left panel young clusters are considered (30...50 Myr) and on the right panel older clusters are considered (100...600 Myr). The results are well related to our computations: The faster the rotation the higher the lithium surface value. If the theory is correct the data are representing the rotational quenching of the eddy diffusivity. More computations and more observations of rotation periods for stars with known Li-abundance are still necessary in order to confirm the given turbulence-founded concept of the lithium problem.

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