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OSCILLATIONS AND WAVES IN PLASMA

Propagation of MHD Waves in a Plasma in a Sheared Magnetic Field with Straight Field Lines

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Abstract—The propagation of MHD plasma waves in a sheared magnetic field is investigated. The problem is solved using a simplified model: a cold plasma is inhomogeneous in one direction, and the magnetic field lines are straight. The waves are assumed to travel in the plane perpendicular to the radial coordinate (i.e., the coordinate along which the plasma and magnetic field are inhomogeneous). It is shown that the character of the singularity at the resonance surface is the same as that in a homogeneous magnetic field. It is found that the shear gives rise to the transverse dispersion of Alfvén waves, i.e., the dependence of the radial component of the wave vector on the wave frequency. In the presence of shear, Alfvén waves are found to propagate across magnetic surfaces. In this case, the transparent region is bounded by two turning points, at one of which, the radial component of the wave vector approaches infinity and, at the other one, it vanishes. At the turning point for magnetosonic waves, the electric and magnetic fields are finite; however, the radial component of the wave vector approaches infinity, rather than vanishes as in the case with a homogeneous field. *© 2002 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

In this paper, we study the resonant excitation of Alfvén waves in an inhomogeneous plasma. The phenomenon of Alfvén resonance is of fundamental importance in the physics of wave processes in the magnetosphere [1, 2]. At present, there are numerous observations indicating the resonance excitation of Alfvén modes by fast magnetosonic (FMS) waves arriving from the outer layers of the magnetosphere or from interplanetary space [3]. This involves the excitation of Alfvén waves in which the magnetic field lines execute azimuthal oscillations (these waves are usually referred to as toroidal). At the same time, there are Alfvén waves that are characterized by radial oscillations of the magnetic field lines (poloidal oscillations). The origin of these waves is still poorly understood [4]. When studying hydromagnetic waves in an inhomogeneous plasma, we will focus mainly on magnetospheric issues. We note, however, that Alfvén resonance is a more universal phenomenon. For example, it was invoked to explain solar corona heating [5] and to develop new methods for plasma heating in fusion devices [6–9]. The character of the models used in this study makes it possible to easily extend our results to Alfvén resonances in these and other branches of physics.

In the simplest model in which the plasma is assumed to be inhomogeneous in one direction and the magnetic field lines are straight and mutually parallel, the Alfvén resonance implies that FMS waves arriving from the outer layers of the magnetosphere excite an Alfvén mode near the surface at which the wave frequency ω is equal to the local Alfvén frequency $\Omega_A = k_\parallel A(x)$, where A is the Alfvén velocity [1, 2]. A large number of studies were devoted to the development of the theory of Alfvén resonance with the use of more complicated (but, at the same time, more realistic) models. Thus, it was shown that Alfvén resonance also occurs in a two-dimensional model that takes into account the curvature of the magnetic field lines and the inhomogeneity of the background plasma in the direction of the magnetic field [10–16]. Furthermore, the field line curvature gives rise to a specific transverse dispersion of Alfvén waves, i.e., the dependence of the radial component of the wave vector on the Alfvén wave frequency [16]. In this case, the waves propagate across the magnetic shells. Taking into account the finite plasma pressure and equilibrium current in a magnetic field with curved field lines leads to an even more drastic change of the transverse dispersion law for hydromagnetic waves [17].

When the problem is treated in a two-dimensional model, some of the factors related to inhomogeneity are often ignored. This naturally brings up the question as to whether these factors can significantly contribute to the overall picture of the process. In recent paper [18], attention was drawn to one of the factors that was previously disregarded—the magnetic field shear caused by the current flowing along the magnetic field. Since field-aligned currents are a rather common phenomenon in the magnetosphere [19], it is worthwhile to take magnetic shear into account in order to gain a comprehensive insight into the physics of MHD waves in the magnetosphere. Up to the present time, much attention has been given to the effect of shear on the plasma stability (see, e.g., [20]). In our study, the problem is formulated in a different way: we investigate the structure of the wave field at a fixed wave frequency ω. Note that many authors treated the wave structure near the resonance surface without allowance for shear (see, e.g., [1, 2, 11]). In this paper, we investigate the wave structure not only in the vicinity of the resonance, but also in the entire plasma volume.

The paper is organized as follows. In Section 2, we introduce the coordinate system and specify the equilibrium plasma parameters. In Sections 3 and 4, we derive an equation describing the structure of a wave traveling in the plane parallel to the magnetic field lines and study the character of the wave field at the singular points. Section 5 is devoted to the study of the wave structure. The results obtained are summarized in Section 6.

2. EQUILIBRIUM STATE AND COORDINATE SYSTEM

To ascertain how shear can influence the structure of the wave field, we investigate a relatively simple model in which all the equilibrium parameters depend only on one coordinate x, imitating the radial coordinate in the magnetosphere. The magnetic field lines are straight and lie in the (y, z) plane. At a given coordinate x, the magnetic field lines are parallel to each other; however, the angle between the field lines and the z-axis depends on the x coordinate (the field lines rotate about the x-axis). One-dimensional models similar to that employed in this study are widely used to investigate MHD waves in the Earth's magnetosphere and the resonance heating of space and laboratory plasmas, whereas the plasma stability is usually examined in cylindrical or toroidal geometry. As a shear parameter, we will use the tangent of the angle between the field lines and the z-axis,

$$\tan\alpha = \frac{B_{0y}}{B_{0z}} = \tau(x),$$

where B_{0y} and B_{0z} are the components of the ambient magnetic field $\mathbf{B}_0$. The nonzero derivative $d\tau/dx$ implies the presence of magnetic shear.

The plasma is assumed to be cold; therefore, the equilibrium current with the density $\mathbf{J}_0 = \nabla \times \mathbf{B}_0$ can only flow along the magnetic field lines; i.e. $\mathbf{J}_0 \times \mathbf{B}_0 = 0$. It is easy to see that the current density and the shear parameter τ are related by

$$J_0 = B_0 \frac{\tau'}{1+\tau^2}.$$

In this paper, we focus on the study of the wave processes in the magnetosphere. The x and y coordinates imitate, respectively, the radial and azimuthal coordinates in the magnetosphere. The parameter $\tau(x)$ and the Alfvén velocity $A(x)$ are assumed to be monotonic functions varying on the same scale length L, as is shown in Fig. 1. It is also assumed that, these functions tend to constant values as $x \longrightarrow \pm\infty$.

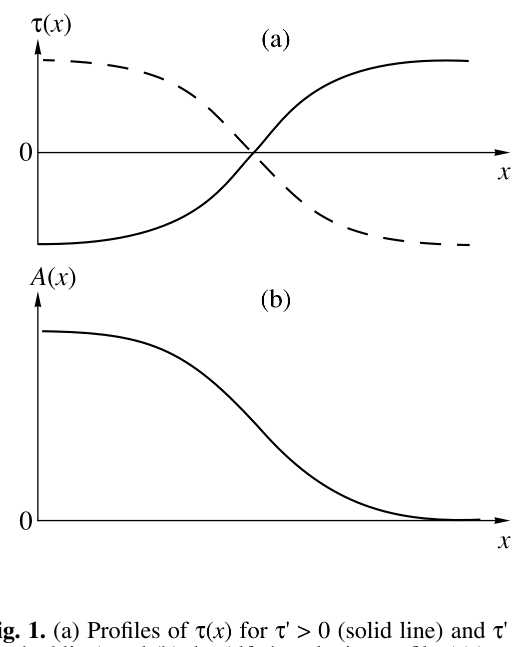

Fig. 1. (a) Profiles of $\tau(x)$ for $\tau' > 0$ (solid line) and $\tau' < 0$ (dashed line) and (b) the Alfvén velocity profile $A(x)$.

3. DERIVATION OF AN EQUATION DESCRIBING THE WAVE STRUCTURE ALONG THE x COORDINATE

A linear monochromatic MHD wave propagating through a cold plasma is described by the equation

$$-4\pi\rho_0\omega^2\boldsymbol{\xi} = \mathbf{J}_0 \times \delta\mathbf{B} + \delta\mathbf{J} \times \mathbf{B}_0, \qquad (1)$$

where $\boldsymbol{\xi}(x, y, z)$ is the vector of the plasma displacement from the equilibrium position; ω is the wave frequency; ρ_0 is the plasma mass density; and $\delta\mathbf{B}$ and $\delta\mathbf{J}$ are small deviations of the magnetic field and current density from their equilibrium values $\mathbf{B}_0$ and $\mathbf{J}_0$, respectively. The magnetic field perturbation $\delta\mathbf{B}$ can be expressed in terms of the wave electric field $\mathbf{E}$:

$$\delta\mathbf{B} = -\frac{ic}{\omega}\nabla \times \mathbf{E}, \qquad (2)$$

where c is the speed of light. The displacement $\boldsymbol{\xi}$ can be found from the frozen-in condition by assuming the plasma to be perfectly conducting:

$$\boldsymbol{\xi} = -\frac{ic}{\omega}B_0^{-2}\mathbf{B}_0 \times \mathbf{E}. \qquad (3)$$

The infinite plasma conductivity also implies that the wave electric field is perpendicular to field lines,

$$E_y B_{0y} + E_z B_{0z} = 0. \qquad (4)$$

Substituting expressions (2) and (3) into Eq. (1), we obtain:

$$-\frac{\omega^2}{A^2}\mathbf{B}_0 \times \mathbf{E} = \mathbf{J}_0 \times (\nabla \times \mathbf{E}) - \mathbf{B}_0 \times (\nabla \times \nabla \times \mathbf{E}),$$

where $A = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén velocity. We take the vector product of the above equation with $\mathbf{B}_0$ and, in view of Eq. (4), retain only the transverse components. As a result, we arrive at the equation

$$\frac{\omega^2}{A^2}\mathbf{E} = (\nabla \times \nabla \times \mathbf{E})_\perp - \kappa(\nabla \times \mathbf{E})_\perp, \tag{5}$$

where $\kappa = \mathbf{J}_0 \cdot \mathbf{B}_0/B_0^2 = \tau'/(1+\tau^2)$ is the shear-related quantity. Thus, we have obtained an equation describing an MHD wave in a cold plasma in a sheared magnetic field. A particular model of the medium described in the previous section has not yet been used; hence, Eq. (5) describes an MHD wave in a cold plasma with an arbitrary magnetic field configuration. Below, we will apply the above model to examine the propagation of MHD waves.

We seek the solution to Eq. (5) in the form

$$E_i(x, y, z, t) = E_i(x)\exp(-i\omega + ik_y y + ik_z z)$$

which implies that the wave is a traveling wave in the (y, z) plane. Then, after simple but laborious manipulations, we obtain from Eq. (5) the equation for the E_y component,

$$E_y'' + a(x)E_y' + b(x)E_y = 0, \tag{6}$$

where the prime denotes differentiation with respect to x. Here, the following notation is introduced:

$$a(x) = \frac{K_A'}{K_A} - \frac{K_F'}{K_F} + 2\tau\kappa, \tag{7}$$

$$b(x) = K_F + \kappa\tau\left(\frac{K_A'}{K_A} - \frac{K_F'}{K_F}\right) + \left(\frac{\tau\tau''}{1+\tau^2} + \kappa^2\right), \tag{8}$$

where $K_A = \omega^2/A^2 - k_\parallel^2$, $K_F = \omega^2/A^2 - k_y^2 - k_z^2$, and $k_\perp = (k_y - \tau k_z)/\sqrt{1+\tau^2}$ and $k_\parallel = (k_z + \tau k_y)/\sqrt{1+\tau^2}$ are the transverse and longitudinal components of the wave vector, respectively. If $\tau' = 0$, Eq. (6) reduces to a one-dimensional equation describing the wave structure in a cold plasma in a magnetic field with straight parallel field lines:

$$E_y'' - k_\perp^2\left(\frac{\omega^2}{A^2}\right)'\frac{E_y'}{K_A K_F} + K_F E_y = 0, \tag{9}$$

which was derived in [1, 2].

4. SINGULAR POINTS

Equation (6) involves singular points x_A and x_F, at which the functions $a(x)$ and $b(x)$ become infinite. It is seen from expressions (7) and (8) that these points satisfy the equations $K_A(x_A) = 0$ and $K_F(x_F) = 0$; i.e.,

$$\omega^2 = \Omega_A^2(x_A),$$
$$\omega^2 = \Omega_F^2(x_F),$$

where

$$\Omega_A^2(x) = A^2(x)k_\parallel^2(x),$$
$$\Omega_F^2(x) = A^2(x)(k_y^2 + k_z^2).$$

To analyze the solution to Eq. (6), we expand K_A and K_F in the vicinities of the points x_A and x_F:

$$K_A = K_A'(x - x_A), \tag{10}$$

$$K_F = K_F'(x - x_F). \tag{11}$$

In view of expansion (11), Eq. (6) in the limit $x \longrightarrow x_F$ reduces to the equation

$$E_y'' - \frac{E_y'}{x - x_F} + \gamma\frac{E_y}{x - x_F} = 0, \tag{12}$$

where $\gamma = -\tau\kappa$ is a quantity related to shear. The solution to this equation is the function

$$E_y = C_1(x - x_F)J_2(Z) + C_2(x - x_F)Y_2(Z);$$

where $Y_2(Z)$ and $J_2(Z)$ are the linearly independent solutions to the Bessel equation and $Z = \sqrt{4\gamma(x - x_F)}$. This solution contains a singularity of the form $(x - x_F)^2\ln(x - x_F)$; i.e., x_F is a branch point. Nevertheless, the function $E_y(x)$ is finite near the point x_F. In this regard, the situation is similar to that in the absence of shear, when Eq. (9) near the point x_F takes the form

$$E_y'' - \frac{E_y'}{x - x_F} + K_F'(x - x_F)E_y = 0$$

(see, e.g., [1]). This equation has the same singular point x_F, but its solution has no singularity. In both cases (with and without shear), the electromagnetic field is finite near this point. An important difference between these cases arises when the WKB approximation is applied (see the next section).

In the vicinity of the point x_A, expansion (10) is valid and Eq. (6) reduces to

$$E_y'' + \frac{E_y'}{x - x_A} + \beta\frac{E_y}{x - x_A} = 0, \tag{13}$$

where $\beta = \tau\kappa$. The solution to this equation is

$$E_y = C_1 J_0(Z) + C_2 Y_0(Z),$$

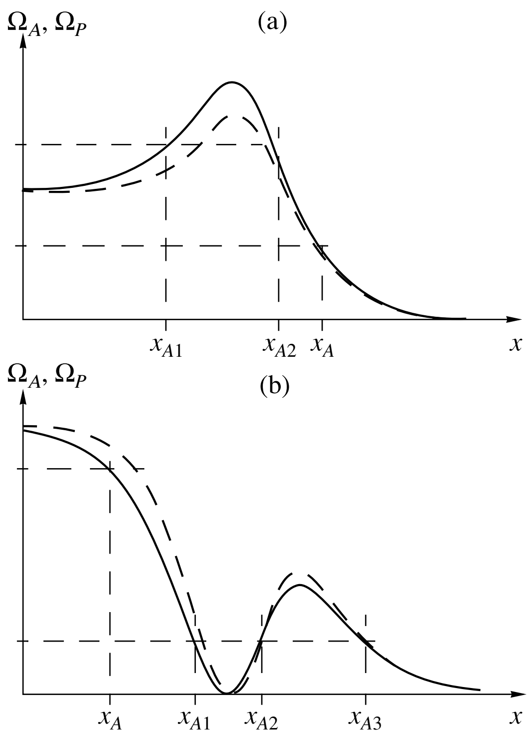

Fig. 2. Functions $\Omega_A^2(x)$ (solid line) and $\Omega_P^2(x)$ (dashed line) for (a) $k_z \gg k_y$ and (b) $k_y \gg k_z$. Indices 1, 2, and 3 stand for different Alfvén resonances at the same frequency ω.

where $J_0(Z)$ and $Y_0(Z)$ are the linearly independent solutions to the Bessel equation and $Z = \sqrt{4\beta(x - x_F)}$. As $x \approx x_A$, this solution has the following asymptotic representation:

$$E_y \approx C_1 \ln(x - x_F) + C_2,$$

i.e. the wave field has a singularity of the same type as in the absence of shear [1, 2]. Hence, at the point x_A, Alfvén resonance occurs. In view of this, the surface $x = x_A$ will be referred to as the resonance surface. The functions $\Omega_A(x)$ for $k_z \gg k_y$ and $k_y \gg k_z$ are plotted in Fig. 2. It is seen in the figure that, even with a monotonic $A(x)$ profile, there can be several Alfvén resonances, whose number depends on the $\tau(x)$ profile and the relation between the wave vector components k_y and k_z. Note that resonance exists even if $A(x)$ = const. Hence, magnetic shear is an additional factor that, along with the plasma inhomogeneity, gives rise to Alfvén resonance. We note that the inequalities $E_x \gg E_y$ and $\delta B_x \ll \delta B_y$ hold near the point x_A; i.e., the field lines oscillate in the (y, z) plane. Such oscillations are often referred to as toroidal pulsations (especially, in publications on magnetosphere physics).

5. RADIAL STRUCTURE OF AN MHD WAVE IN THE WKB APPROXIMATION

When studying the excitation of Alfvén waves in the magnetosphere, the following scenario is usually considered: FMS waves arrive from the outer layers of the magnetosphere, reach the boundary of the transparent region, and are reflected back; however, their field partially penetrates deep into the magnetosphere and excites oscillations in the Alfvén resonance region. With this scenario in mind, we will use, as the boundary condition, the boundedness of the function E_y as $x \longrightarrow -\infty$. Furthermore, in the case of an inhomogeneous plasma, we should speak of a single MHD mode, because the separation of the solution into an Alfvén mode and an FMS wave is, strictly speaking, rather arbitrary. Nevertheless, by tradition, we will use these terms, trying to more strictly define them.

To solve Eq. (6) with the boundary condition $|E_y(x \longrightarrow -\infty)| < \infty$, we will use the WKB approach, assuming that the inequalities $k_y \gg L^{-1}$ and $k_z \gg L^{-1}$ are satisfied, where L is the typical scale length on which the equilibrium parameters of the medium vary. The main WKB order gives the radial component of the wave vector. In the case at hand, we have

$$k_x^2(x, \omega) = K_F + \kappa\tau\left(\frac{K'_A}{K_A} - \frac{K'_F}{K_F}\right). \tag{14}$$

In the next WKB order, we can determine the wave amplitude as a function of the radial coordinate. In our analysis, the terms with the first derivative with respect to the radial coordinate refer just to this order because they contain a large parameter to the first power.

In the transparent region, we have $k_x^2 > 0$. One of the transparent regions exists even in the absence of shear: if $\tau' = 0$ and $\kappa = 0$, then, from Eq. (14), we obtain a well-known FMS dispersion relation, $k_x^2 = K_F$. In the presence of shear, the FMS transparent region consists of two separate (but close to each other) regions (Fig. 3b). One of these regions (to the left of the point x_F) is similar to the transparent region in the absence of shear (Fig. 3a). The second region is bounded by the point where $k_x^2 = 0$ and the point x_F, where $k_x^2 = \infty$; i.e., it resembles the Alfvén transparent region described below. Recall that the wave amplitude is finite near the point x_F.

The other transparent region is adjacent to the resonance surface $x_A(\omega)$. In this transparent region, the waves have, to a high degree of accuracy, the Alfvén mode polarization ($E_x/E_y = k_x/k_y$). Hence, this region will be referred to as the Alfvén transparent region. It is seen from relation (14) that, when $\omega = \Omega_A(x)$, we have $k_x^2 \longrightarrow \infty$. Let us introduce the poloidal frequency $\Omega_P(x)$, such that, when $\omega = \Omega_P(x)$, the equality $k_x = 0$ is satisfied. Accordingly, the surface $x_P(\omega)$, on which the equality $\omega = \Omega_P(x)$ is satisfied, will be referred to as poloidal, because, near the point x_P, the inequality $k_x \ll k_y$ holds and, consequently, we have $E_x \ll E_y$ and $\delta B_x \gg \delta B_y$. The second of the last two inequalities suggests that the field lines oscillate in the radial direction; i.e., the oscillations are poloidal in character. At large values of k_y and k_z, it is easy to obtain from relation (14) the difference between the poloidal and resonance frequencies,

$$\Omega_P^2(x) - \Omega_A^2(x) = qA^2k^{-2},$$

where $k^2 = k_y^2 + k_z^2$ and $q = \tau\kappa K_A'$. The distance between the poloidal and resonance surfaces is determined by the expression

$$\Delta = x_P - x_A = \frac{q}{k^2 K_A'}.$$

The function $\Omega_P(x)$ is plotted in Fig. 2. It is seen in the figure that $\Delta \ll L$. In the vicinities of the resonance and poloidal surfaces, relation (14) takes a simpler form:

$$k_x^2 = k^2\frac{\Omega_P^2(x) - \omega^2}{\omega^2 - \Omega_A^2(x)}. \tag{15}$$

This formula can be regarded as a dispersion relation for Alfvén waves in the presence of magnetic shear in a plasma whose parameters vary along one coordinate. The dependence $k_x(\omega)$ indicates the emergence of the transverse dispersion of Alfvén waves. As is known, the dispersion relation for the Alfvén mode in a homogeneous plasma is $\omega^2 = k_\parallel^2 A^2$. The dependence of the transverse component of the wave vector on the frequency appears when nonideal MHD effects (such as the electron inertia and the effects related to the finite ion Larmor radius) are taken into account. We can see that, even in an ideal one-fluid magnetohydrodynamics, the transverse dispersion arises if magnetic shear is taken into account. It should be noted that the transverse dispersion similar to that described by Eq. (15) was found for waves in a two-dimensional plasma in a magnetic field with curved field lines [16], whereas, in our model, the magnetic field lines are straight and the plasma is inhomogeneous along one direction.

The profiles of $k_x^2(x)$ outside the FMS transparent region (i.e., at $x < x_F$) are plotted in Fig. 4. The plots illustrate the arrangement of the transparent regions for Alfvén waves under different assumptions about the ambient medium. It is seen in the figure that there are generally several such regions. Let us consider Fig. 4a in more detail, where only one Alfvén transparent

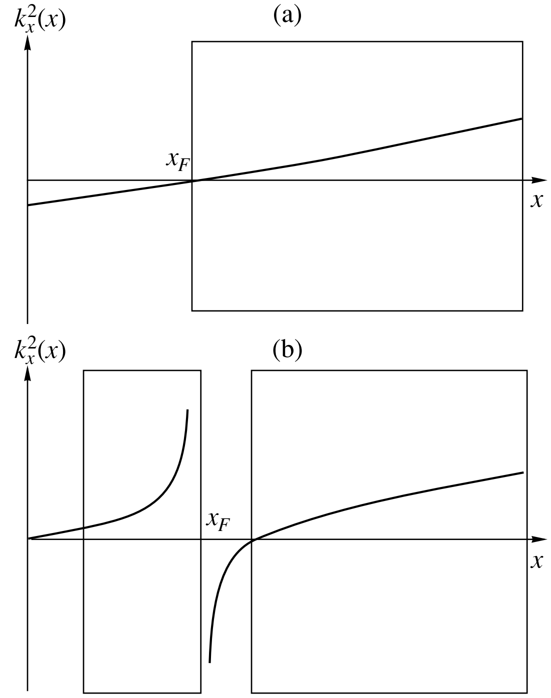

Fig. 3. Profiles of $k_x^2(x)$ near the x_F point (a) without and (b) with shear.

region is present. If $k_y, k_z \gg L^{-1}$, then, in this region and around it, Eq. (6) can be brought to the form

$$E_y'' + \frac{1}{x - x_A}E_y' + k^2\frac{x_P - x}{x - x_A}E_y = 0. \tag{16}$$

Near the Alfvén resonance, this equation can be even more simplified [see Eq. (13)]. Near the poloidal point, it reduces to the Airy equation. To solve Eq. (6), the WKB solutions must be matched with the solutions near the points x_A, x_P, and x_F. Omitting intermediate manipulations, we give the final solution. In accordance with the boundary condition, in the opaque region (at $x \ll x_A$), we have

$$E_y = c_1\exp(-kx)$$

(here and below, we will not give the expressions for the constants because they are rather unwieldy). In the Alfvén transparent region (i.e., at $x_A < x < x_P$), the wave is described by the expression

$$E_y = c_2[(x_P - x)(x - x_A)]^{-1/4}\exp ik\int_{x_P}^{x}\sqrt{\frac{x_P - x'}{x' - x_A}}dx';$$

i.e., it is a traveling wave propagating across magnetic shells. In this case, the wave phase velocity is directed along the x-axis ($v_{\rm ph} > 0$) and the group velocity, as is seen from relation (15), is directed from the poloidal surface toward the resonance surface. Note that this is a

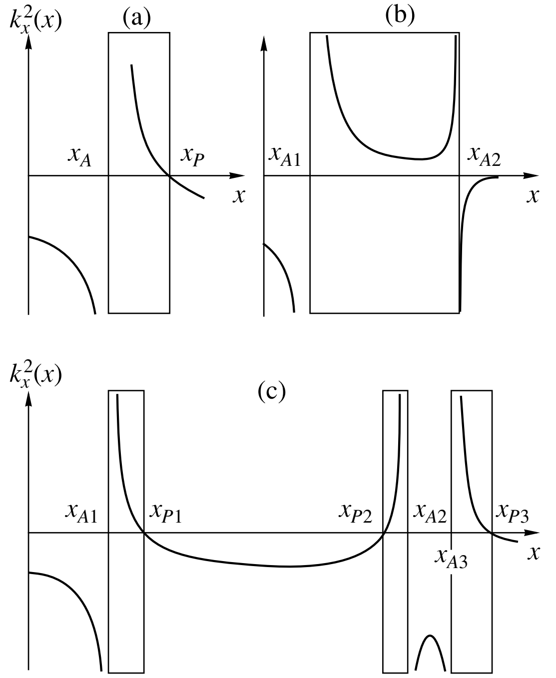

Fig. 4. Profiles of $k_x^2(x)$ at $x < x_F$ for the cases of (a) one, (b) two, and (c) three resonances. Indices 1, 2, and 3 stand for the Alfvén transparent regions adjacent to the different Alfvén resonances shown in Fig. 2.

general result, irrespective of the relative positions of x_P and x_A; i.e., we always have $v_{ph} > 0$, and the group velocity is always directed toward x_A. As the wave propagates, its radial wavelength decreases and the character of the mode polarization also changes. At $x \approx x_P$, the wave is poloidally polarized ($E_y \gg E_x$ and $\delta B_y \ll \delta B_x$), whereas at $x \approx x_A$, it is toroidally polarized ($E_y \ll E_x$ and $\delta B_y \gg \delta B_x$). Near the points x_P and x_A, the applicability conditions of the WKB approximation fail to satisfy and the structure of the wave is described in terms of the functions obtained by solving Eq. (16) near these points:

$$E_y = \begin{cases} c_3 K_0(\sqrt{4k^2\Delta(x_A - x)}) & \text{for } x \le x_A \\ c_4 H_0^{(1)}(\sqrt{4k^2\Delta(x - x_A)}) & \text{for } x_A \le x \ll x_P \\ c_5 \mathrm{Ai}((k^2/\Delta)^{1/3}(x - x_p)) & \text{for } x \approx x_P, \end{cases}$$

where K_0 is the modified Bessel function, $H_0^{(1)}$ is the Hankel function, and Ai is the Airy function. The constants are obtained by matching these solutions with the WKB solutions. Finally, in the FMS transparent region, we have a standing wave; thus, at $x \gg x_F$, we obtain

$$E_y = c_6 \cos\left(kx - \frac{\pi}{4}\right).$$

Note that the penetration of an FMS wave beyond the transparent region slightly affects the structure of this mode (see also [21]).

Such a structure of an MHD wave generally corresponds to the conventional propagation scheme of low-frequency waves in the magnetosphere, as was described at the beginning of this section. However, the situation with the Alfvén transparent region differs radically from that in the absence of shear [1, 2] (in the latter case, the wave is a solitary resonance and there is no poloidal surface). On the other hand, it resembles a situation considered in [16, 17]. In that case, the waves also propagate across the magnetic shells; however, this is related to the curvature of magnetic field lines.

6. CONCLUSION

(i) An equation describing the electric field of a wave propagating in a cold plasma in a sheared magnetic field has been derived. The equation holds for any magnetic field configuration. A particular case of a wave traveling in the plane parallel to the magnetic field lines has been examined assuming that the plasma is inhomogeneous along one direction and the magnetic field lines are straight.

(ii) It has been shown that, on the magnetic surfaces where the condition $\omega^2 = A^2 k_\parallel^2$ is satisfied, there are logarithmic singularities similar to that in the absence of shear. Thus, we can state that the Alfvén resonances occur just on these surfaces. In a sheared magnetic field (unlike the case without magnetic shear), there can be several Alfvén resonances at a fixed frequency even if the Alfvén velocity $A(x)$ has no local extrema. The number of resonances depends on the $\tau = \tau(x)$ profile and the relation between the wave vector components k_y and k_z. It has been shown that the Alfvén resonance exists even if $A(x) = \text{const}$. This suggests that shear is one of the factors (along with the plasma inhomogeneity) that gives rise to Alfvén resonance.

(iii) It has been shown that, at the turning point for magnetosonic waves, the equation possesses a singularity. At this point, the solution is finite, but has a branching singularity.

(iv) It has been established that the presence of shear gives rise to the transverse dispersion of Alfvén waves, i.e., the dependence of the radial component of the wave vector on the wave frequency ω. This phenomenon has no analogue in the case of a magnetic field with straight parallel field lines and a plasma that is inhomogeneous along one direction. However, transverse dispersion can also arise due to the field line curvature [16] or the finite plasma pressure [17]. The presence of shear also slightly changes the FMS dispersion law; however, this change does not play an important role because this mode has a significant transverse dispersion ($k_x^2 = \omega^2/A^2(x) - k_y^2 - k_z^2$) even in the absence of shear.

(v) The wave structure has been studied in the model in question. It is shown that the MHD mode has two transparent regions. The first region, corresponding to small values of the Alfvén velocity, refers to FMS waves. In this region, the mode is a standing wave occurring between magnetic shells. The second transparent region is adjacent to the Alfvén resonance surface and, thus, can be called the Alfvén region. This region is bounded by the Alfvén resonance surface on the one side and the poloidal surface ($k_x = 0$) on the other side. Within this transparent region, the mode is a traveling wave and the energy of the wave is transported from the poloidal surface to the resonance surface. This situation differs radically from that with straight parallel field lines [1, 2], in which case an Alfvén wave is a solitary resonance and there is no poloidal surface. However, it resembles a situation in which a similar phenomenon is caused by the curvature of magnetic field lines [16, 17].

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REFERENCES

1. D. J. Southwood, Planet. Space Sci. **22**, 483 (1974).
2. L. Chen and A. Hasegawa, J. Geophys. Res. **79**, 1024 (1974).
3. A. S. Potapov and V. A. Mazur, in *Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves*, Ed. by M. J. Engebretson, K. Takahashi, and M. Scholer (American Geophysical Union, Washington, 1994), Geophysical Monograph, Vol. 81, p. 135.
4. H. J. Singer, W. J. Hughes, and C. T. Russel, J. Geophys. Res. **87**, 3519 (1982).
5. J. A. Ionson, Astrophys. J. **226**, 650 (1978).
6. A. V. Timofeev and K. Yu. Kharitonov, Fiz. Plazmy **15**, 674 (1989) [Sov. J. Plasma Phys. **15**, 389 (1989)].
7. P. E. Moroz, Fiz. Plazmy **13**, 558 (1987) [Sov. J. Plasma Phys. **13**, 317 (1987)].
8. W. Grossmann and J. A. Tataronis, J. Phys. **261**, 217 (1973).
9. L. Chen and A. Hasegawa, Phys. Fluids **17**, 1399 (1974).
10. A. E. Lifshits and E. N. Fedorov, Dokl. Akad. Nauk SSSR **287**, 90 (1986).
11. A. S. Leonovich and V. A. Mazur, Fiz. Plazmy **15**, 660 (1989) [Sov. J. Plasma Phys. **15**, 381 (1989)].
12. E. N. Fedorov, N. G. Mazur, and V. A. Pilipenko, Fiz. Plazmy **21**, 333 (1995) [Plasma Phys. Rep. **21**, 311 (1995)].
13. A. L. Krylov and A. F. Lifshitz, Planet. Space Sci. **32**, 481 (1984).
14. A. N. Wright, J. Geophys. Res. **97**, 6429 (1992).
15. A. N. Wright and M. J. Thompson, Phys. Plasmas **1**, 691 (1994).
16. A. S. Leonovich and V. A. Mazur, Planet. Space Sci. **41**, 697 (1993).
17. D. Yu. Klimushkin, Fiz. Plazmy **23**, 931 (1997) [Plasma Phys. Rep. **23**, 858 (1997)].
18. A. Salat and J. A. Tataronis, J. Geophys. Res. **105**, 13055 (2000).
19. D. P. Stern, J. Geophys. Res. **99**, 17169 (1994).
20. G. Bateman, *MHD Instabilities* (MIT Press, Cambridge, 1978; Énergoatomizdat, Moscow, 1982).
21. A. S. Leonovich and V. A. Mazur, J. Geophys. Res. **106**, 3919 (2001).

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