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Concerning ULF Pulsations in Mercury's Magnetosphere

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Abstract. Upcoming missions such as Messenger or BepiColombo will revolutionize our knowledge about planet Mercury and its magnetosphere. To contribute to a comparison between the terrestrial and the Hermean magnetosphere. The system of partial differential equations describing coupled directional and isotropic eigenmodes of a magnetosphere filled with a multi-component plasma is derived and investigated to study eigenoscillations of Mercury's magnetosphere. As anticipated eigenfrequencies are smaller, but comparable to the ion gyrofrequencies of the plasma assumed, an MHD treatment as used by Dungey [1954] is not appropriate. Instead, a cold plasma approximation is used keeping the off-diagonal components of the dielectric tensor thus allowing higher frequencies to be tackled. Unlike in the terrestrial magnetosphere, the directional and the isotropic modes in the Hermean magnetosphere remain coupled even for axisymmetric perturbations with azimuthal waves numbers $m = 0$.

1. The problem

NASA's Messenger mission and the joint European Japanese BepiColombo project will provide us with new and outstanding possibilities to study the magnetosphere of planet Mercury. Comparing the very different magnetospheres of Earth and Mercury will greatly enhance our understanding of magnetospheric systems in general. Even before arriving in the Hermean system it is worthwhile and most timely to investigate which particular physical conditions we can expect. In this brief article we address the problem of coupled and decoupled global magnetospheric eigenoscillations and concentrate on the question of axisymmetric perturbations.

Analyzing Mariner 10 magnetic field observations during the first encounter with planet Mercury Russell [1989] detected an about 2 second narrow band pulsation event in the night side magnetosphere. Though this oscillation exhibits a clear compressional component, it is mainly transverse to the ambient magnetic field with a an almost linear polarization in the meridian plane. The narrow bandwidth suggests a resonant excitation mechanism, and Russell [1989] proposes it to be a standing Alfvén wave along a Hermean magnetic field line. Using an electron density of 3 cm^3 and assuming the plasma ions to be protons, he derives an Alfvén velocity of about 1000 km/s. With a field line length of about 4000 km, Russell [1989] estimates an eigenperiod of the fundamental oscillation of the resonant field line of 8 seconds. Accordingly, the observed 2-s wave could be the fourth harmonic of the fundamental.

This interpretation renders the question whether an MHD approach is anyhow reasonable for the description of this kind of ULF pulsation in Mercury's magnetosphere. Proton cyclotron periods of the Hermean plasma are of the order of a few seconds. Thus, the observed wave period is very close to typical cyclotron periods, and a plasma description using MHD is prohibited. This has already been noted by e.g. Othmer et al. [1999], when discussing resonant mode couplings in the Hermean plasma. Profound differences are found by these authors considering field line resonances in a multi-component, cold plasma. For example, the resonance point is determined by the cross-over frequency rather than by the local resonance frequency of a standing field line oscillation.

That the plasma at Mercury must be treated as a multi-component plasma is evident from ground-based observations of sodium ions in the Hermean environment [e.g. Killen and Ip, 1999; Cheng et al., 1987; Ip, 1986].

Here, we like to investigate in particular in which way the classical picture of coupled directional and isotropic modes or toroidal and poloidal eigenoscillations as first discussed by Dungey [1954] for a terrestrial dipole magnetosphere must be changed when taking into account a multi-component, cold plasma and eigenperiods smaller, but close to the ion gyroperiods. Global axisymmetric perturbations, that is perturbations with azimuthal wave number $m = 0$, are of interest as its Mercury's magnetosphere is rather small compared to that one of the Earth and global, large scale oscillations are probably easier to be realized. The question to be studied is: Are toroidal and poloidal axisymmetric eigenoscillations still decoupled at Mercury if $m = 0$.

2. The system of coupled wave equations

The treatment used here is different from Dungey's [1954] original approach, but adapted to the physical problem sketched above. We formulate the problem in general curvilinear and orthogonal coordinates [e.g. Stratton, 1941], and we follow an approach used by e.g. Klimushkin [1994]. First, we introduce a contravariant coordinate system organized by the background magnetic field. In this system (x^1, x^2, x^3) the coordinate surfaces $x^1 = const.$ coincide with magnetic

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shells defined as constant magnetic pressure surfaces, x^2 determines a given field line on this magnetic surface, and the coordinate x^3 refers to a point on the field line. The coordinates x^1 and x^2 correspond to radial and azimuthal coordinates, respectively. The physical length of the field line is expressed by the increment of the corresponding coordinate as $dl_3 = \sqrt{g_3}dx^3$. Similarly, $dl_1 = \sqrt{g_1}dx^1$ and $dl_2 = \sqrt{g_2}dx^2$, where the g_i denote the diagonal components of the metric tensor. The nondiagonal components vanish as the coordinate system used is an orthogonal one. The metric tensor determinant is $g = g_1g_2g_3$. Here the superscripts and subscripts denote contravariant and covariant vector components, respectively. They are related by $a^j = a_j/g_j$ (no summation with respect to the recurring index is assumed here). As co- and contravariant basis vectors are not of unit length, an appropriate set of unit vector components can be introduced according to the relation $\breve{a}_j = a_j/\sqrt{g_j}$. We call these components "physical" vector components.

We now use Maxwells equations to derive a relation between the electric $\vec{E}$ and magnetic $\vec{B}$ fields of the wave propagating in a cold plasma with frequency ω:

$$\nabla \times \vec{E} = i\omega\vec{B}, \tag{1}$$

$$\nabla \times \vec{B} = -i\frac{\omega}{c^2}\hat{\varepsilon}\vec{E}. \tag{2}$$

Here

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & -i\eta & 0 \\ i\eta & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \tag{3}$$

is the dielectric tensor. In the Hermean magnetosphere, the ULF oscillation frequencies are only slightly inferior to the ion cyclotron frequency Ω_{ci}, but significantly lower than the electron cyclotron frequency Ω_{ce}. Furthermore, consideration should be given to the significant content of sodium ions Na^+ in the Hermean magnetospheric plasma. The dielectric tensor elements in the ULF range in the Hermean magnetosphere can thus be written as

$$\varepsilon_\parallel = -\frac{\Omega_{pe}^2}{\omega^2},$$

$$\varepsilon_\perp = \frac{\Omega_{pH}^2}{\Omega_{cH}^2-\omega^2} + \frac{\Omega_{pNa}^2}{\Omega_{cNa}^2-\omega^2},$$

$$\eta = \frac{\Omega_{pe}^2}{\omega\Omega_{ce}} - \frac{\Omega_{cH}}{\omega}\frac{\Omega_{pH}^2}{\Omega_{cH}^2-\omega^2} - \frac{\Omega_{cNa}}{\omega}\frac{\Omega_{pNa}^2}{\Omega_{cNa}^2-\omega^2},$$

where Ω_p and Ω_c denote plasma and cyclotron frequencies, respectively, and the second index on these frequencies denotes the corresponding particle, that is electrons (e), protons (H), and sodium ions (Na).

From equations (1) and (2) we obtain the equation for the wave electric field:

$$\nabla \times \nabla \times \vec{E} = \frac{\omega^2}{c^2}\hat{\varepsilon}\vec{E}. \tag{4}$$

Equations (4) represent a system of partial differential equations which we need to evaluate for the three covariant electric field components $\vec{E} = (E_1, E_2, E_3)$ with E_1 and E_2 denoting the components transverse to the background equilibrium magnetic field, and E_3 being the longitudinal component. In principal we need to evaluate the dielectric tensor by formulating the magneto-ionic theory using curvilinear coordinates. However, here we like to approximate the dielectric tensor by its Cartesian coordinate representation. This is locally justified as long as one of the curvilinear coordinates is aligned with the ambient magnetic field which is this case here. By taking the divergence of equation (2), we obtain $\nabla\cdot\hat{\varepsilon}\vec{E} = 0$, which is equivalent to the current continuity condition $\nabla\cdot\vec{j} = 0$. Upon developing this equation component-wise, we obtain

$$\frac{1}{\sqrt{g}}\,\partial_1\frac{\sqrt{g}}{g_1}(\varepsilon_\perp E_1 - i\eta E_2) + \frac{1}{\sqrt{g}}\,\partial_2\frac{\sqrt{g}}{g_2}(\varepsilon_\perp E_2 + i\eta E_1) +$$

$$\frac{1}{\sqrt{g}}\,\partial_3\frac{\sqrt{g}}{g_3}\varepsilon_{||}E_3 = 0, \tag{5}$$

where $\partial_i = \partial/\partial x^i$. Note, that in the axisymmetric magnetosphere the derivative with respect to the azimuthal coordinate is proportional to the azimuthal wave number m, $\partial_2 = i\,m$.

In the frequency range considered here, $\omega \sim \Omega_{cH}, \Omega_{cNa}$, the relation $\varepsilon_\parallel \gg \varepsilon_\perp, \eta$ holds true for the dielectric tensor $\hat{\varepsilon}$ components. Thus, it follows from (5) that the longitudinal component E_3 of the electric field of the wave will be significantly smaller than the transverse components E_1 and E_2. This justifies to consider the electric field of the wave as a two-dimensional one, $\vec{E} = (E_1, E_2, 0)$. Within this approximation, from (4) we obtain a system of two equations for E_1 and E_2:

$$(L^{ij} + P^{ij} + Q^{ij})E_j = 0, \tag{6}$$

where

$$L^{ij} = \begin{pmatrix} \hat{L}_T & 0 \\ 0 & \hat{L}_P \end{pmatrix}, P^{ij} = \begin{pmatrix} \partial_2\frac{g_3}{\sqrt{g}}\partial_2 & -\partial_2\frac{g_3}{\sqrt{g}}\partial_1 \\ -\partial_1\frac{g_3}{\sqrt{g}}\partial_2 & \partial_1\frac{g_3}{\sqrt{g}}\partial_1 \end{pmatrix}$$

$$Q^{ij} = \begin{pmatrix} 0 & -i\sqrt{g_3}\frac{\omega^2}{c^2}\eta \\ i\sqrt{g_3}\frac{\omega^2}{c^2}\eta & 0 \end{pmatrix},$$

and

$$\hat{L}_T = \partial_3\frac{g_2}{\sqrt{g}}\partial_3 + \frac{\sqrt{g}}{g_1}\varepsilon_\perp\frac{\omega^2}{c^2},$$

$$\hat{L}_P = \partial_3\frac{g_1}{\sqrt{g}}\partial_3 + \frac{\sqrt{g}}{g_2}\varepsilon_\perp\frac{\omega^2}{c^2}.$$

In relation (6) and below, indices i and j can take the values 1 and 2, and summation with respect to indices recurring twice is implied.

According to the Helmholtz theorem [e.g. Duschek and Hochrainer, 1961], an arbitrary vector field can be represented as the sum of its potential and vortex components. Applying this theorem to a two-dimensional field $\vec{E}$, we use

$$\vec{E} = -\nabla_\perp\Phi + \nabla_\perp \times \vec{e}_{||}\Psi, \tag{7}$$

where $\vec{e}_{||} = \vec{B}/B$, and $\nabla_\perp$ is the two-dimensional nabla operator in the plane (x^1, x^2). In a homogeneous plasma within the MHD approximation ($\omega \ll \Omega_{c;i,e}$), the "potentials" Φ and Ψ describe the electric fields of the Alfvén wave and of the fast magnetosound (FMS), respectively [e.g. Klimushkin, 1994; Glassmeier, 1995].

We now transform (6) into a system of equations for Φ and Ψ. By letting the operator ∂_i act on (6), we obtain the first equation

$$\left[\partial_1\hat{L}_T\partial_1 + \partial_2\hat{L}_P\partial_2\right]\Phi - i\frac{\omega^2}{c^2}\left[\partial_1\sqrt{g_3}\,\eta\,\partial_2 - \partial_2\sqrt{g_3}\,\eta\,\partial_1\right]\Phi =$$

$$i\frac{\omega^2}{c^2}\left[\partial_1\sqrt{\frac{g_1}{g_2}}\,\eta\,\partial_1+\partial_2\sqrt{\frac{g_2}{g_1}}\,\eta\,\partial_2\right]\Psi+$$

$$\left[\partial_1\hat{L}_T\frac{g_1}{\sqrt{g}}\partial_2-\partial_2\hat{L}_P\frac{g_2}{\sqrt{g}}\partial_1\right]\Psi. \quad (8)$$

To obtain the second equation, we treat (6) by the operator $-\epsilon^{kj}\partial_k\, g_{ij}/\sqrt{g}$, where ϵ^{kj} is an antisymmetric unit pseudotensor, $\epsilon^{12}=-\epsilon^{21}=1$, and $\epsilon^{11}=\epsilon^{22}=0$. As a result, we have

$$\left[\partial_1\frac{g_2}{\sqrt{g}}\hat{L}_T\frac{g_2}{\sqrt{g}}\partial_1+\partial_2\frac{g_1}{\sqrt{g}}\hat{L}_P\frac{g_1}{\sqrt{g}}\partial_2\right]\Psi$$

$$-i\frac{\omega^2}{c^2}\left[\partial_1\frac{\eta}{\sqrt{g_3}}\partial_2-\partial_2\frac{\eta}{\sqrt{g_3}}\partial_1\right]\Psi+\Delta_\perp\frac{g_3}{\sqrt{g}}\Delta_\perp\Psi\;=$$

$$-i\frac{\omega^2}{c^2}\left[\partial_1\sqrt{\frac{g_2}{g_1}}\,\eta\,\partial_1+\partial_2\sqrt{\frac{g_1}{g_2}}\,\eta\,\partial_2\right]\Phi+$$

$$\left[\partial_2\frac{g_1}{\sqrt{g}}\,\hat{L}_T\partial_1-\partial_1\frac{g_2}{\sqrt{g}}\,\hat{L}_P\partial_2\right]\Phi. \quad (9)$$

Here $\Delta_\perp\equiv\partial_1(g_2/\sqrt{g})\partial_1+\partial_2(g_1/\sqrt{g})\partial_2$ is defined as the "transverse Laplacian". Equations (9) and (10) are coupled equations describing small-amplitude wave perturbations in an arbitrary curvilinear and orthogonal coordinate system.

3. Discussion of wave equations

Within the MHD approximation, that is for $\omega\ll\Omega_{cH},\Omega_{cNa}$, the dielectric tensor components reduce to the following values:

$$\varepsilon_\perp\rightarrow c^2/v_A^2,\varepsilon_\parallel\rightarrow-\infty,\eta\rightarrow 0, \quad (10)$$

where $v_A=B_0/\sqrt{\mu_0(\rho_H+\rho_{Na})}$ is the Alfvén velocity, B_0 the background equilibrium magnetic field, and ρ_H and ρ_{Na} are the hydrogen and sodium mass densities, respectively. Thus, within this approximation, we obtain a system of equations from (8) and (9) describing coupled Alfvén and magnetosonic modes in the terrestrial magnetosphere [Klimushkin, 1994]. They are the generalized form of Dungey's [1954] famous equations.

As can be seen, the system of equations describing ULF waves in Mercury's magnetosphere has a similar structure, that is two ULF modes exist, one of which, Φ, is a guided mode, propagating predominantly along magnetic field lines, and the other one, Ψ, is an isotropic mode. Both modes are the analogues of the Alfvén and magnetosonic modes of MHD theory, respectively.

Comparing terrestrial and Hermean plasma conditions we may note the following differences:

1) The dielectric tensor in the Hermean case does not reduce to the form (10) as the ratio ω/Ω_{cH} is not small, and the ratio ω/Ω_{cNa} is in general very large. Thus, ULF waves in Mercury's magnetosphere cannot be called hydromagnetic waves.

2) Several new terms appear for the inhomogeneous plasma case and are proportional to the nondiagonal components of the dielectric tensor η. These new terms are the second terms on the left-hand sides of equations (8) and (9). They are proportional to the derivative with respect to the azimuthal coordinate, i.e. to the azimuthal wave number m. Hence it follows that the solution of the wave equation depends on the sign of m and waves with different signs propagate asymmetrically.

3) The most important difference, however, is caused by the first terms on the right-hand sides of equations (8) and (9), which make an additional contribution to the coupling of the guided and isotropic modes. These terms are also proportional to η and do not disappear when $m=0$, which radically distinguishes ULF waves in Mercury's magnetosphere from the ones in our terrestrial magnetosphere.

For the special case $m=0$ equations (8) and (9) read

$$\left[\partial_1\hat{L}_T\partial_1\right]\Phi=i\frac{\omega^2}{c^2}\left[\partial_1\sqrt{\frac{g_1}{g_2}}\,\eta\,\partial_1\right]\Psi. \quad (11)$$

$$\left[\partial_1\frac{g_2}{\sqrt{g}}\hat{L}_T\frac{g_2}{\sqrt{g}}\partial_1\right]\Psi+\Delta_\perp\frac{g_3}{\sqrt{g}}\Delta_\perp\Psi=-i\frac{\omega^2}{c^2}\left[\partial_1\sqrt{\frac{g_2}{g_1}}\,\eta\,\partial_1\right]\Phi. \quad (12)$$

It is easily seen that the two modes do not decouple due to the finite value of η. This coupling does also not disappear when assuming a homogeneous plasma, in which case the system (8) and (9) reduces to the dispersion equation

$$\left(\frac{\omega^2}{c^2}\varepsilon_\perp-k_\parallel^2\right)\left(\frac{\omega^2}{c^2}\varepsilon_\perp-k_\parallel^2-k_\perp^2\right)=\frac{\omega^4}{c^4}\eta^2. \quad (13)$$

The fact that a finite ratio ω/Ω_c introduces linear coupling of the guided and isotropic modes is well known in plasma physics (see, for example, Swanson [1989]). Dmitrienko and Mazur [1985] and Leonovich et al. [1983] studied this linear coupling in detail within the context of the physics of MHD waves in the Earth's magnetosphere. But this coupling in that application can be important only for Pc1 pulsations, that is for higher frequency wave packets travelling along field lines, while standing waves (Pc 3-5) probably have too low frequencies for this coupling to have a perceptible effect on them.

Unlike the terrestrial magnetosphere, in the Hermean magnetosphere the ω/Ω_{cH} ratio is only slightly smaller than unity even for low harmonic numbers of standing oscillations along field lines. Furthermore, a noticeable admixture of sodium ions, for which the cyclotron frequency can be even lower than the wave frequency, makes the linear coupling between the guided and isotropic modes perhaps even more important than coupling caused by plasma inhomogeneities.

For further study we numerically solved the set of coupled equations (11) and (12) assuming a box-model magnetosphere with the plasma density varying linearly in the x-direction and assuming a background magnetic field $\vec{B}=(0,0,B)$. *For this special case equations (11) and (12) read*

$$\frac{d}{dx}\hat{L}_T(\omega,x)\frac{d\Phi}{dx}=i\frac{\omega^2}{c^2}\frac{d}{dx}\eta\frac{d\Psi}{dx} \quad (14)$$

$$\frac{d}{dx}\hat{L}_T(\omega,x)\frac{d\Psi}{dx}+\frac{d^4}{dx^4}\Psi=-i\frac{\omega^2}{c^2}\frac{d}{dx}\eta\frac{d\Phi}{dx} \quad (15)$$

with $\hat{L}_T(\omega,x)=\omega^2/c^2\,\epsilon(\omega,x)-k_\parallel^2$. *The point* $\mathrm{x_s}$ *where* $\hat{L}_T(\omega,x_s)=0$ *is a singular point of the equations. To* *check whether this point is also a singular point of the solution we solved the above set of equations numerically in the vicinity of this point. For* $\mathrm{x} >> \mathrm{x_s}$ *we assumed that the electric field components vanish and propagate the solution across the point* $\mathrm{x_s}$ *with the result displayed in Fig. 1. Parameters used in our numerical solution are:* $B = 40$ *nT,* $\rho_H = 4.7 \cdot 10^{-21} kg\, m^{-3}$*,* $\rho_{Na} = 3.3 \cdot 10^{-22} kg\, m^{-3}$*, and* $\omega = 3.2\,\mathrm{Hz}$*, which gives one* $k_\parallel = 1.1 \cdot \mathrm{km}^{-1}$*. The scale length for the density variation was assumed at 1/10 of the planetary radius.*

The spatial variation of the electric field components exhibits a clear singular behavior of the solution around $\mathrm{x_s}$*. Furthermore, the numerical solution indicates oscillatory behavior near the singular point, a feature known for the MHD case if field line curvature is taken into account and* $m \gg 1$ *[e.g. Klimushkin, 1998]. Our numerical solution indicates that in the Hermean magnetosphere resonant mode coupling may occur even in the case* $m = 0$*.*

4. Summary and Outlook

The system of coupled eigenoscillations in an inhomogeneous magnetized plasma resembling that of the magnetosphere of Mercury has been formulated and discussed. Significant differences as compared to the corresponding eigenmodes in the terrestrial situation are found, the most pronounced one being the coupling of the guided and isotropic mode even for azimuthal symmetric perturbations. This coupling is introduced by the off-diagonal terms of the dielectric tensor. *Assuming a box-model magnetosphere numerical solution indicates the existence of a singular point similar to the resonance point in an MHD approximation [e.g. Tamao, 1965; Southwood et al., 1974; Chen and Hasegawa, 1974]. The singular solutions exhibit oscillatory behavior around the singular point. Further study is required to study this singular solution in detail and to understand its relation to a singular solution discussed by Othmer et al. [1999].*

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Figure 1. Variation of the electric field components across the singular point $\mathrm{x_s}$.