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Stability of magnetohydrodynamic shear flows with and without bounding walls

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We have solved the shear flow stability problem of compressible electrically conducting fluid (plasma) in a magnetic field. A comparative analysis is made of the influence of different boundary conditions on shear flow stability. We consider the problems for flows in an unbounded medium, a flow between fixed walls, as well as in the presence of such a wall on one side of the shear flow only. Shear flow velocity profiles are treated both in the form of a tangential discontinuity and as being described by a hyperbolic tangent function. For the case of an unbounded flow with a velocity vortex sheet, analytical solutions are found; for all other cases, the solutions are found numerically. Problems are solved for two different positions of the tangential wave vector of oscillations and magnetic field $\mathbf{k_t} \perp \mathbf{B_0}$ and $\mathbf{k_t} \parallel \mathbf{B_0}$. For shear flows bounded by a fixed wall we found an unstable mode of oscillations produced by the wave, reflected from the wall and transmitted through the shear layer.

1. Introduction

The study of the instability of shear flows of fluids and gases (Kelvin-Helmholtz instability) has been and is the subject of scrutiny for many investigators. Such flows are of widespread occurrence in many regions, both on the Earth and in space plasmas (Dungey 1955 , Parker 1964). Analytical treatments devoted to this topic are usually based on models of the medium in which the shear layer is regarded as a vortex sheet of the shear flow velocity profile (Landau 1944 , Syrovatsky 1954 , McKenzie 1970). With the advance of computer technology, an ever increasing number of publications appeared, in which perturbations of shear flows with a smooth velocity profile were calculated (Michalke 1964 , Moskvin and Frank-Kamenetsky 1967). Among them, publications (Blumen 1970 , Blumen et al. 1975 , Drazin and Davey 1977) deserve mention, which address the shear flow instability of an unbounded fluid flow. A significant number of publications are devoted to the study of plasma shear flows in the presence of an external magnetic field. Such work was stimulated mostly by problems related to solar wind flow around the Earth's magnetosphere (McKenzie 1970 , Miura and Pritchett 1982 , Mishin and Morozov 1983), and to cometary plasma tails (Ray and Ershkovich 1983), as well as to the velocity difference between flows inside the solar wind itself (Parker 1964).

In each of the cited references, boundary conditions far from the shear layer in analytical problems were specified to ease the analysis of the resulting solutions; as regards numerical problems, this was dictated either by the convenience of the numerical calculation itself or by a possibility of comparing their results with findings of previous analytical work. Furthermore, boundary conditions can be specified in different ways. The authors of (Blumen et al. 1975 , Drazin and Davey 1977) considered the shear flow of an unbounded fluid. The absence of oscillations arriving at the shear layer is a natural boundary condition for unstable modes in such a problem. In other words, far from the shear layer there are only waves escaping from it.

The authors of (Miura and Pritchett 1982 , Miura 1992), for a certain identical distance on each side of the shear layer, required that the normal oscillation velocity component is vanishing. In other words, the presence of solid walls was assumed. There are some works in which the problem of shear layer instability was considered at the presence of the wall on one side of the shear layer (Kolykhalov 1984). The set of parameters of the medium and of the oscillation characteristics considered in different problems is usually different. Besides, results obtained in them are presented differently. It is therefore not possible to identify the influence of boundary conditions on shear flow stability by comparing results from different work.

In this paper we investigate by using a unified approach the influence of the aforementioned boundary conditions on shear flow stability of a conducting fluid (or plasma) in the presence of a magnetic field. Let the coordinate axis y be directed along the velocity direction of the flow stratified in coordinate x; next, we introduce the axis z to complement the coordinate system to a right-handed one (see Fig.1). We shall consider the perturbed quantities in the form $\sim f(x)\exp\{i(k_y y + k_z z - \omega t)\}$, where ω is a wave frequency of single Fourier-harmonic. We define the tangential wave vector as $\mathbf{k}_t = (k_y, k_z)$. Let us consider two types of oscillations. The equation of oscillations of the first type, the tangential wave vector of which is normal to the magnetic field direction, is similar to the equation describing shear flow oscillations of an inviscid compressible fluid (Blumen et al. 1975 , Drazin and Davey 1977). For the oscillations of the second type, the angle between the directions of the tangential wave vector and the magnetic field is different from $\pi/2$. In this case the presence of a magnetic field plays its special role and this role increases with the decreasing angle. In this paper we consider the oscillations, the tangential wave vector of which is parallel to the magnetic field. In this manner we investigate two limiting cases between which all possible situations are realized.

Calculations that are performed in this paper, mostly refer to shear flow with a velocity profile of the form of hyperbolic tangent function. To gain a better understanding of the resulting numerical solutions, for each type of boundary conditions we solved the flow stability problem with a vortex sheet in the velocity profile which allows us to advance sufficiently far analytically.

2. Model of the medium and basic equations

Let us consider the model of the medium presented in Fig.1. We choose the velocity profile of shear flow along the axis x such that $v_0(x) \to \pm\bar{v}_0$ when $x \to \pm\infty$. The other parameters of the medium: density ρ_0, pressure P_0, and magnetic field strength $\mathbf{B}_0 = (0, B_y, B_z)$ will be considered homogeneous. The system of equations of ideal magnetohydrodynamics, linearized for small perturbations, has the form

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v_0}\nabla\mathbf{v} + \mathbf{v}\nabla\mathbf{v_0}\right) = -\nabla\mathbf{P} + \frac{\mathbf{1}}{\mathbf{4}\pi}[\mathrm{curl}\mathbf{B}\times\mathbf{B_0}],$$

$$\frac{\partial \mathbf{B}}{\partial \mathrm{t}} = \mathrm{curl}([\mathbf{v_0} \times \mathbf{B}]+[\mathbf{v} \times \mathbf{B_0}]), \tag{2.1}$$
$$\frac{\partial P}{\partial t} + \mathbf{v_0}\nabla\mathbf{P} + \gamma\mathbf{P_0}\mathrm{div}\mathbf{v} = \mathbf{0},$$

where the quantities $\mathbf{v}, \mathbf{B}$, and P refer to the perturbation field, and γ is the adiabatic index. The system of equations (2.1) can be reduced to a single equation for the normal (along the axis x) component of the displacement of oscillating fluid which we denote as ζ: $v_x = \mathrm{d}\zeta/\mathrm{dt} = \partial\zeta/\partial\mathrm{t} + (\mathbf{v_0}\nabla)\zeta$. On expanding the total oscillation field in terms of Fourier-harmonics of the form $\exp[i(k_y y + k_z z - \omega t)]$ for the displacement velocity along the axis x, we have $v_x = -i\bar{\omega}\zeta$, where $\bar{\omega} = \omega - (\mathbf{k}_t\mathbf{v}_0)$ is the oscillation frequency modified by Doppler effect, and $\mathbf{k}_t = (k_y, k_z)$ is a tangential wave vector. An equation for ζ has the form (Duhau and Gratton 1975):

$$\frac{\partial}{\partial x}\frac{\Omega^2}{K^2}\frac{\partial\zeta}{\partial x} - \Omega^2\zeta = 0, \tag{2.2}$$

where

$$\Omega^2 = \bar{\omega}^2 - (\mathbf{k_t A})^{\mathbf{2}},$$

$$K^2 = k_t^2 - \frac{\bar{\omega}^4}{\bar{\omega}^2(s^2+A^2) - s^2(\mathbf{k_t A})^{\mathbf{2}}}. \tag{2.3}$$

Here $s^2 = \gamma P_0/\rho_0$ is the velocity of sound squared, and $\mathbf{A} = \mathbf{B_0}/\sqrt{4\pi\rho_0}$ is the Alfven velocity. The components of a perturbed magnetic field are expressed in terms of ζ by the relations

$$B_x = i(\mathbf{k_t B_0})\zeta,$$

$$B_y = \frac{(k_t^2 - K^2)B_{0y} - k_y(\mathbf{k_t B_0})}{K^2}\frac{\partial\zeta}{\partial x}, \tag{2.4}$$

$$B_z = \frac{(k_t^2 - K^2)B_{0z} - k_z(\mathbf{k_t B_0})}{K^2}\frac{\partial\zeta}{\partial x},$$

and perturbed pressure

$$P = \gamma P_0 \frac{\bar{\omega}^2[\bar{\omega}^2 - (\mathbf{k_t A})^{\mathbf{2}}]}{K^2[\bar{\omega}^2(A^2+s^2) - (\mathbf{k_t A})^{\mathbf{2}}\mathbf{s^2}]}\frac{\partial\zeta}{\partial x}. \tag{2.5}$$

Let the variables be represented as dimensionless; to do this, we follow the nomenclature used in (Blumen 1970 , Blumen et al. 1975). Nomenclature: $\alpha = k_t a$ is the dimensionless tangential wave vector, $c = \omega/(k_t\bar{v}_0\cos\varphi)$ is the oscillation phase velocity that is dimensionless for the flow velocity $\mathbf{v}$ projected onto the direction $\mathbf{k}_t$, and φ is the angle between the positive direction of the axis y and the vector $\mathbf{k}_t$. We introduce the Mach number $M = \bar{v}_0\cos\varphi/s$ determined from the projection of $\mathbf{v}$ onto the direction $\mathbf{k}_t$ (see Fig.1), and a modified Mach number $\bar{M} = M\sqrt{\beta/(1+\beta)} = \bar{v}_0\cos\varphi/\sqrt{s^2+A^2}$ inferred from the velocity of magnetosound. The parameter $\beta = s^2/A^2$ is proportional to the ratio of thermal pressure P_0 to magnetic field pressure $B_0^2/8\pi$. Let us consider two limiting cases with a different mutual orientation of the vectors $\mathbf{k}_t$ and $\mathbf{B_0}$. When $\mathbf{k}_t \perp \mathbf{B_0}$ ($\psi - \varphi = \pi/2$, see Fig.1), equation (2.2) becomes

$$\left(\frac{(c-u)^2}{\bar{M}^2(c-u)^2-1}\zeta'\right)' + \alpha^2(c-u)^2\zeta = 0, \tag{2.6}$$

which corresponds to the equation used in (Blumen et al. 1975 , Drazin and Davey 1977), developed for the displacement ζ. In (2.6) the prime denotes the derivative with respect to a dimensionless coordinate $\xi = x/a$, where a is the characteristic scale of variation of the sheared flow profile (see Fig.1) which is described by the function $u(\xi) = v_0(\xi)/\bar{v}_0$, and $\bar{v}_0$ is the half-difference of the velocity profile. Letting the magnetic field strength tend to zero, we obtain $\bar{M} = M$, and letting it tend to infinity, we have $\bar{M} \to 0$, which corresponds to an infinite velocity of magnetosound. In the other limiting case $\mathbf{k}_t \parallel \mathbf{B_0}$ ($\psi = \varphi$), equation (2.2) reduces to

$$\left[\left(1+\beta+\frac{\beta}{M^2(c-u)^2-1}\right)\zeta'\right]' + \alpha^2\left[M^2\beta(c-u)^2-1\right]\zeta = 0. \tag{2.7}$$

In subsequent calculations we have to use the expression for total perturbed pressure which when $\mathbf{k}_t \| \mathbf{B_0}$ is related to the displacement ζ by the relation

$$\tilde{P} \equiv P + \frac{\mathbf{B_0 B}}{4\pi} = -\rho_0 A^2\left(\frac{M^2\beta(c-u)^2}{M^2(c-u)^2-1}+1\right)\zeta'. \tag{2.8}$$

When $\beta \to \infty$, equations (2.6) and (2.7) coincide, describing a usual hydrodynamic flow.

3. Boundary conditions

For formulating the problem of generation of unstable oscillations described by equations (2.6),(2.7), these must be supplemented by corresponding boundary conditions. In the presence of a solid wall at a certain distance Δ from the point $x = 0$, a natural requirement on it is the condition of impermeability. This is equivalent to the vanishing of the v_x–component of the oscillation velocity on the wall (or $\zeta(\Delta) = 0$).

We next consider the case of an infinite medium. Away from the shear layer on the asymptotics $|x| \gg a$ the medium is homogeneous. The solution of equation (2.2), which can be sought here in the form

$$\zeta = \bar{\zeta}\exp(ik_x x), \tag{3.1}$$

gives a dispersion equation that defines the function $\omega(\mathbf{k})$, where $\mathbf{k} = (k_x, k_y, k_z)$ is a total wave vector of oscillations. Two variants of boundary conditions are possible here. 1) If for the oscillations under consideration the medium at the asymptotics is opaque (for the neutral mode $c_i \equiv \mathrm{Im}(c) = 0$ this is equivalent to the condition $k_x^2 = -K^2 < 0$), the solution should be chosen, the amplitude of which decreases exponentially with the distance from the shear layer. 2) If the medium is transparent (for the neutral mode $k_x^2 = -K^2 > 0$), then the radiation condition should be imposed when $|x| \to \infty$. The radiation condition is defined by the sign of the x-component of the group velocity of oscillations $v_{gx} = \partial\omega/\partial k_x$.

For unstable oscillations ($c_i > 0$), both k_x and v_{gx} become complex. In this case, formally for any weakly unstable oscillations, we may introduce the notion of the waves escaping from the shear layer, in which $\mathrm{Re}(v_{gx}) > 0$ when $x > 0$, and

$\mathrm{Re}(v_{gx}) < 0$ when $x < 0$. In accordance with the equation for energy flux

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial x}(v_{gx} W) = 0,$$

which holds far from the shear layer $|x| \gg a$, for monochromatic unstable ($\partial W/\partial t > 0$) oscillations the relation $k^+_{xi} \equiv \mathrm{Im} k_x > 0$ holds for $x > 0$, and $k^-_{xi} < 0$ for $x < 0$. This warrants an exponential decrease of the amplitude of unstable oscillations running away from the shear layer. Here W is quadratic (in amplitude) density of wave energy. The oscillations considered here represent magnetosonic waves. The expression for energy density of these waves for $|x| \gg a$ has the form (Anderson 1963)

$$W = |\zeta|^2 \rho_0 \frac{\omega k^2}{k_x^2 \bar{\omega}^3}(2\bar{\omega}^2 - k^2(A^2 + s^2))(\bar{\omega}^2 - (\mathbf{k_t} A)^2).$$

Thus the radiation condition for unstable oscillations is equivalent to choosing the solution that decreases exponentially in amplitude with the distance from the shear layer. We must remark, however, that such a correspondence holds only for a well- defined group velocity. The notions of the well- and ill- defined group velocity are introduced in the Appendix in which we consider an example where the use of an ill-defined group velocity in the boundary condition leads to an erroneous result.

Next we present the asymptotic expressions for the group velocity which we used in a numerical calculation in order to check the boundary conditions used for correctness. In the case of $\mathbf{k}_t \perp \mathbf{B_0}$, for the group velocity we have

$$v_{gx} = s\frac{1+\beta}{\beta}\frac{k_x}{k_t}\frac{M^2}{(c-u)}, \tag{3.2}$$

where $k_x = \pm k_t M(c-u)\sqrt{\beta/(1+\beta)}$, and in the case of $\mathbf{k}_t \| \mathbf{B_0}$

$$v_{gx} = \frac{s}{\beta}\frac{k_x}{k_t}\frac{\left[M^2(c-u)^2(1+\beta) - 1\right]^2}{(c-u)^3\left[M^2(c-u)^2(1+\beta) - 2\right]}, \tag{3.3}$$

where

$$k_x = \pm k_t \sqrt{\frac{(M^2(c-u)^2 - 1)(M^2\beta(c-u)^2 - 1)}{M^2(c-u)^2(1+\beta) - 1}}, \tag{3.4}$$

the $\pm$ signs are chosen according to boundary conditions (so that $\mathrm{Re}(v_{gx}) > 0$ when $x > 0$, and $\mathrm{Re}(v_{gx}) < 0$ when $x < 0$).

As has been pointed out in the Introduction, three kinds of boundary conditions are possible, which are found in some or other variation in almost all shear flow stability problems. If the medium is infinite, then from the radiation condition for unstable oscillations we have the following boundary conditions when $x \to \pm\infty$:

$$\frac{\partial \zeta}{\partial x} = i k_x \zeta, \tag{3.5}$$

and the sign of k_x is chosen such that $\mathrm{Re}(v_{gx}) > 0$ when$x \to \infty$, and $\mathrm{Re}(v_{gx}) < 0$ when $x \to -\infty$. If a fixed wall is on one side of the shear layer (at the point $x = -\Delta$ for example), and on the other side the medium is infinite, then the boundary conditions have the form

$$\begin{cases} \partial\zeta/\partial x = i k_x \zeta \ , & x \to \infty, \\ \zeta = 0, & x = -\Delta. \end{cases} \tag{3.6}$$

The sign of k_x is chosen here such that $\mathrm{Re}(v_{gx}) > 0$. If the fixed walls are on either side of the shear layer (at the points $x = \pm\Delta$), then the boundary conditions have the form

$$\zeta(x = \pm\Delta) = 0. \tag{3.7}$$

The boundary conditions for the components of a perturbed magnetic field and perturbed pressure are expressed in terms of ζ in accordance with the relations (2.4),(2.5).

4. Shear flow instability in an infinite medium

Let us consider the instability of shear layer oscillations in the absence of bounding walls. It should be noted that the presence of a magnetic field manifests itself differently in the two limiting cases under consideration. In the case of $\mathbf{k}_t \perp \mathbf{B_0}$ the magnetic field can be taken into account simply by overdetermining the Mach number $\bar{M}$. In this case equation (2.6) exactly corresponds in its form to the equation describing a usual compressible hydrodynamic flow. On the contrary, in the case of $\mathbf{k}_t \parallel \mathbf{B_0}$ the role of the magnetic field does not imply a simple overdetermination of the individual characteristics of the flow but adds qualitatively new effects.

4.1. The case of $\mathbf{k}_t \parallel \mathbf{B_0}$

Oscillations of this kind are less well understood in general compared to perturbations with $\mathbf{k}_t \perp \mathbf{B_0}$. Such references as (Syrovatsky 1957 , Chandrasekhar 1962) deserve mention, where the problems of shear flow stability in an infinite medium are considered. To gain a more complete understanding of the results from a subsequent numerical investigation of flows with the velocity profile $\sim \tanh(\xi)$ we now solve the problem of flow stability with a velocity profile in the form of a tangential discontinuity $v_0(x) = \bar{v}_0\mathrm{sign(x)}$, which in equations (2.6), (2.7), (3.2) and (3.3) corresponds to

$$u(x) = \begin{cases} 1, & x > 0, \\ -1, & x < 0. \end{cases} \tag{4.1}$$

We seek the solution of (2.7) in the form (3.1) in each of the regions. As the matching conditions, we adopt the requirement of continuity of the displacement ζ and of total perturbed pressure $\tilde{P}$ (2.8) on the interface plane $x = 0$. As a result, we obtain the dispersion equation

$$\sqrt{\frac{(M^2(c-1)^2(1+\beta)-1)(M^2\beta(c-1)^2-1)}{M^2(c-1)^2-1}} - \sqrt{\frac{(M^2(c+1)^2(1+\beta)-1)(M^2\beta(c+1)^2-1)}{M^2(c+1)^2-1}} = 0. \tag{4.2}$$

On transferring one of the terms to the left-hand side of (4.2) and squaring the two sides, after some manipulation, we obtain

$$c^4M^4 - 2c^2M^2(1+M^2) + \left(M^4 - 2M^2 + \frac{2}{1+\beta}\right) = 0. \tag{4.3}$$

The solution of this equation has the form

$$c^2 = \frac{1 + M^2 \pm \sqrt{4M^2 - (1-\beta)/(1+\beta)}}{M^2}. \tag{4.4}$$

When $\beta > 1$ the expression under the radical in (4.4) is larger than zero. The minus sign of the radical in (19) corresponds to unstable oscillations ($c_i > 0$), provided that the condition (see Parker 1964 , Sen 1964 , McKenzie 1970)

$$1 - \sqrt{\frac{(\beta-1)}{(\beta+1)}} < M^2 < 1 + \sqrt{\frac{(\beta-1)}{(\beta+1)}}. \tag{4.5}$$

is satisfied. When $\beta \to \infty$ the inequality on the right-hand side of (4.5) gives a well-known instability criterion of hydrodynamic shear flows (Landau 1944): $M^2 < 2$. In proceeding to the limit of an incompressible medium ($M \to 0$), leaving the quantity $M_A^2 = M^2\beta$ finite, from the inequality on the left-hand side of (4.5) we obtain another well-known instability criterion of incompressible conducting fluid in a magnetic field (Syrovatsky 1957 , Chandrasekhar 1962): $M_A > 1$.

When $\beta < 1$ the expression under the radical in (4.4) is less than zero for $M^2 < M_0^2 = (1-\beta)/4(1+\beta)$. Furthermore, it appears at first glance that one of the roots must represent an unstable mode of oscillations. Conceivably it is this root that was detected in (Sen 1964), where the solution was found for unstable oscillations for $\beta \to 0$. However, a detailed analysis of calculations that yield the dispersion equation (4.4) (see the Appendix) shows that the formal solution, describing the instability when $\beta < 1$, is fictitious, since it does not correspond to the necessary boundary conditions for $|x| \to \infty$. Thus, if on both asymptotics the solution has the form of a wave escaping from the shear layer (which is determined by the choice of the sign of $\mathrm{Re}\,(v_{gx})$), then when $\beta < 1$, on one of the asymptotics, it is exponentially decreasing, and on the other, exponentially increasing. This is a consequence of the fact that the group velocity of the oscillations is ill-defined, and in choosing the boundary conditions, the requirement of the decreasing amplitude of the solution on both asymptotics is better to use. This requirement is not satisfied by the solutions of (4.2) for $\beta < 1$. This means that in an infinite medium in which $\beta < 1$ there are no unstable oscillations of the shear flow under consideration.

Let us now consider the solution of equation (2.7) for a flow with the velocity profile

$$u(\xi) = v_0(x)/\bar{v}_0 = \tanh(x/a) \equiv \tanh(\xi) \tag{4.6}$$

and with the boundary conditions of (3.5). We solve it numerically. First we choose the integration contour parallel to the real axis in the lower half-plane of the complex variable ξ such that it passes below all singularities of equation (2.7) which corresponds to the resonance interaction of different modes of MHD oscillations (Southwood 1974 , Leonovich 2001). Such an integration contour was chosen in (Blumen et al. 1975) for solving a similar problem for hydrodynamic flow.

Since for unstable oscillations the singularities of equation (2.7) lie in the upper half-plane of a complex ξ, it would appear that it would suffice to perform integration along the real axis. However, such an approach is applicable only for oscillations with a sufficiently large value of the growth rate. For such oscillations, the singularities of equation (2.7) are far from the real axis. In the case of weakly unstable oscillations the singularities approach the real axis. Therefore, integrating along the real axis gives an error that can turn out to be larger than the desired value of the growth rate c_i itself. The boundary conditions of (3.5) are formulated for $\xi \to \pm\infty$. In a numerical integration, they should be placed at a finite distance from the shear layer where the flow becomes virtually homogeneous, and the solution of (2.7) has the form (3.1). In our calculations we imposed the boundary conditions for $\xi = \pm 5$. A further moving-off of the initial and final points of the integration interval does virtually not influence the ultimate result.

The resulting equation is illustrated by Fig.2, showing the distribution of contours of the growth rate c_i in the plane (α, M). Fig.2a corresponds to the value of $\beta = 10$, and Fig.2b corresponds to $\beta = 1.1$. Note that the existence domain of unstable oscillations with $\mathbf{k}_t \parallel \mathbf{B_0}$ is bounded on the side of small values of M. The existence interval of unstable long-wavelength perturbations ($\alpha \to 0$) is determined by the condition (4.5) obtained within the tangential discontinuity approximation.

Stabilization of shear flow oscillations with parameters outside of this interval is due to the influence of compressibility of the medium and of magnetic field force line tension (Maxwellian tensions). Difference of the influence of a magnetic field on subsonic ($M < 1$) and supersonic ($M > 1$) flows it is possible to explain as the follows. Compressibility of medium does not influence practically on the subsonic surface oscillations, and magnetic field influence only through Maxwellian tensions. Influence of compressibility of medium is great in domain of supersonic radiating oscillations. The effect of amplification of grows rate due to decrease of compressibility is more strong than its weakening due to Maxwellian tensions.

4.2. The case $\mathbf{k}_t \perp \mathbf{B_0}$

This problem, as pointed out above, is analogous to the stability problem of a usual hydrodynamic flow. Its solution for a flow with a velocity profile in the form of a tangential discontinuity is similar to the solution of the problem considered in the preceding section for the case $\mathbf{k}_t \parallel \mathbf{B_0}$. The solution for unstable radiating modes of oscillations and the domain of their existence are defined in this case by the expressions (4.4) and (4.5), in which we should put $\beta = \infty$.

To seek the solution in the problem with the velocity profile of (4.6) we integrate numerically equation (2.6) with the boundary conditions of (3.5). Fig.3 presents the distribution of contours of the growth rate c_i in the plane $(\alpha, \bar{M})$. It corresponds to the distribution obtained in (Blumen et al. 1975). The region $\bar{M} < 1$ is determined by the unstable surface mode of oscillations, the neutral solution for which was obtained in (Blumen 1970). The region $\bar{M} > 1$ is determined by the unstable radiating mode of oscillations (Landau 1944). As is evident from the figure, within the tangential discontinuity approximation ($\alpha \to 0$) the existence domain of the radiating mode-associated unstable oscillations is bounded by the limiting value of $\bar{M} < \bar{M}_c = \sqrt{2}$ which gives the expression (4.5) in the limit $\beta \to \infty$. For hydrodynamic flows this condition for radiating modes of oscillations was obtained in (Landau 1944). Stabilization of the oscillations when a critical value of $\bar{M}_c$ is exceeded, is caused by the compressibility of the medium determined, among other things, by magnetic field pressure.

5. Instability of shear flow bounded by a single fixed wall

From general considerations it is obvious that the presence of a solid wall must lead to a change of the mode of unstable oscillations. These changes would affect to a greater extent the radiating modes of oscillations which are reflected from such a wall. Furthermore, the location of the wall becomes an independent factor that determines the distribution and value of the oscillation growth rate. Let us consider, as done in the problem with infinite flow, two limiting cases.

5.1. The case $\mathbf{k}_t \parallel \mathbf{B_0}$

First we consider a shear flow in the form of a tangential discontinuity (4.1), at a distance $x = -\Delta$ from which the solid wall is located. This problem is described by equation (2.7) with the boundary conditions of (3.6). In the semispace $x > 0$ we seek the solution in the form of an escaping wave $\zeta = \bar{\zeta}\exp(ik_x x)$, and on the side of the wall - in the form of the sum of the wave escaping from the layer and the wave reflected from the wall:

$$\zeta = \zeta_1 \exp(ik_x x) + \zeta_2 \exp(-ik_x x). \tag{5.1}$$

From the condition of continuity of the displacement ζ and total perturbed pressure $\tilde{P}$ (2.8) in the plane $x = 0$ we obtain the dispersion equation

$$\frac{M^2(c+1)^2(1+\beta)-1}{M^2(c-1)^2(1+\beta)-1}\;\frac{M^2\beta(c+1)^2-1}{M^2\beta(c-1)^2-1}\;\frac{M^2(c-1)^2-1}{M^2(c+1)^2-1} = -\tan^2(k_x^-\Delta),$$

where k_x^- is determined by (3.4) when $u = -1$. The numerical solution of this equation is presented in Fig.4.

Fig.4a plots the dependence $c_i(M)$ for $\beta = 1$ and for five different values of the parameter $\kappa = k_t\Delta = 0.1,\ 0.5,\ 1,\ 5$, and 10. All of the curves have a limiting value of M_c, below which the oscillations are stable. The presence of the lower critical value of M_c, as in the case of an infinite medium, is due to the stabilizing influence of Maxwellian tensions. Small-scale oscillations of $c_i(M)$ occur for such values of M where the argument of the tangent on the right-hand side of the dispersion equation becomes large (i.e. $|k_x^-\Delta| \gg 1$).

By varying the magnetic field strength (the parameter β), one can follow the way in which the critical value of M_c varies. Fig.4b plots the curves $c_i(M)$ for $\kappa = 1$ and for five different values of $\beta = \infty,\ 10,\ 1,\ 0.5$, and 0.2. When $\beta = \infty$ (the regime of hydrodynamic flow) there is no critical value of M_c. When $\beta \neq \infty$ there appears a critical M_c that is shifted with a decrease of β to $M = 1$. When $\beta < 1$ the existence domain of unstable oscillations breaks down into two. One of them, corresponding to the surface mode, is enclosed between two critical points $M_c < 1$. The other region corresponds to the radiating mode and is bounded at the left by the point $M_c > 1$. It should be noted that, unlike the infinite flow (without walls), no complete stabilization of oscillations sets in for any, arbitrary small, values of β.

The numerical solution of equation (2.6) for a flow with the velocity profile (4.6) and the boundary conditions (3.6) (when $\Delta/a = -20$) is presented in Fig.5. The cases **a** and **b** correspond to different values of the parameter $\beta = 10$ and $\beta = 1$. When $\alpha \to 0$ the existence domains of unstable oscillations correspond to the ones which were obtained in the problem of tangential discontinuity instability. The presence of a fixed wall results in two considerable differences from the case of unbounded shear layer. Firstly, unlike the infinite flow, the existence domain of unstable oscillations of a shear flow, in the presence of a single wall, is not bounded in M above. Secondly, as is seen from this figure, an additional broad region of unstable oscillations with a supersmall growth rate $c_i < 0.01$ was formed. The formation of this region is associated with oscillations reflected from the wall and transmitted through the shear layer. Such oscillations were absent in the infinite medium. By analogy with the surface and radiating modes of oscillations, this unstable mode can be referred to as the reflective mode.

This can be given the following interpretation. It is known that the tangential discontinuity generates, in addition to unstable oscillation modes, the radiating neutral mode (Miles 1957 , Ribner 1957). In particular, it results to the phenomenon of superreflection of oscillations incident on the shear layer. In other words, the waves that are reflected from the shear layer and transmitted through it have an amplitude larger than the incident wave amplitude. This is taking place just in the range of values of $M > M_c$, for which in the infinite case (without wall) the tangential discontinuity is stable. The presence of the wall leads to the fact that there appears a wave reflected from it, which is incident on the shear layer and is reflected from it with a large amplitude. With a multiple recurrence of this process, there arises an effective amplification of the oscillations.

An increase in magnetic induction causes stabilization of the surface modes to a much greater extent compared to the that for radiating and reflective modes. A definite periodicity in the distribution of reflective mode growth rate is associated with the formation of standing waves, partially captured between the wall and shear layer. The further the wall is located from a shear layer, the larger is the number of unstable standing waves that can be excited by the shear layer.

5.2. The case $\mathbf{k}_t \perp \mathbf{B_0}$

As it has been told, this oscillation regime is similar to oscillations in a usual hydrodynamic flow. The only distinction implies that a modified Mach number $\bar{M}$ is used, which takes into account the presence of magnetic pressure. The solution for a flow with a velocity profile in the form of a tangential discontinuity is represented by curve 1 in Fig.4b, corresponding to $\beta = \infty$. For the flow with the velocity profile (4.6) we solve numerically equation (2.6) with the boundary conditions (3.6). Fig.6 presents the distribution of the growth rate c_i in the presence of a solid wall at a distance $\xi = -15$. The major difference from the case of an infinite medium, both for flows with a tangential velocity discontinuity and for flows with a velocity profile of the form (4.6), implies the disappearance of the critical value of $\bar{M} = M_c$. The infinite flow (without wall) would go stable when this value is exceeded. The domain of existence of the reflective mode of oscillations in this case increases considerably due to the appearance of unstable oscillations with a supersmall growth rate.

As has already been pointed out, this is likely to be accounted for by an effective instability of the radiating mode of oscillations reflected from the wall and transmitted through the shear layer. Unstable oscillations in this region are not stabilized with increasing magnetic field strength. This means that stabilization of such oscillations can only occur under the action of magnetic field force line tension. It is interesting to note that this region disappears as one passes to a limiting case of tangential discontinuity ($\alpha \to 0$), in both the presence, and absence of a magnetic field.

6. Shear flow instability between two bounding walls

The dependence $c_i(\alpha)$ for separate values of the Mach number M and the parameter β was investigated in (Miura and Pritchett 1982 , Miura 1992) in shear flows with a velocity profile of the form (4.6). Boundary conditions in the cited references were chosen in the form of two solid walls on both sides of the shear layer, and integration was performed along the real axis ξ. With such an approach, as pointed out above, regions with a small value of the growth rate are described with a large error. Using the same approach as in the two previous sections, we now consider two limiting cases.

6.1. *The case* $\mathbf{k}_t \parallel \mathbf{B_0}$

Let us consider a shear flow in the form of a tangential discontinuity between two solid walls ($x = \pm\Delta$). In this case the solutions of (2.6) that satisfy the boundary conditions of (3.7), are sought on both sides of the shear layer in the form (5.1). Matching the displacement ζ and total perturbed pressure $\tilde{P}$ (2.8) for $x = 0$ gives a dispersion equation

$$\frac{M^2(c+1)^2(1+\beta)-1}{M^2(c-1)^2(1+\beta)-1}\;\frac{M^2\beta(c+1)^2-1}{M^2\beta(c-1)^2-1}\;\frac{M^2(c-1)^2-1}{M^2(c+1)^2-1}=\frac{\tan^2(k_x^-\Delta)}{\tan^2(k_x^+\Delta)},$$

where $k_x^\pm$ are determined by (3.4) for $u = \pm 1$, respectively. The numerical solution of this equation is presented in Fig.7. Fig.7a plots $c_i(M)$ for $\beta = 1$ for three different values of $\kappa = k_t\Delta = 0.1,\ 1,$ and 5. The existence domains of unstable oscillations are bounded on both sides. With a sufficient distance of the walls from the shear layer, there is a clear-defined periodicity in the distribution of the oscillation growth rate associated with the formation of standing waves. As in the case with a single wall, there are small-scale oscillations of the growth rate associated with an increase of the arguments of the tangents to values of $|k_x^\pm\Delta| \gg 1$.

The periodicity in the distribution of the oscillation growth rate can be given the following interpretation. Between the walls there arise standing waves with a well-defined eigenfrequency dependent on the tangential wave vector of oscillations and on the distance between the walls. With a change of the flow parameters (Mach number M), the various harmonics of standing waves are at resonance with the neutral mode of oscillations. Furthermore, there is a maximum in the distribution of the oscillation growth rate. If the walls are sufficiently close to each other, then even the frequency of the fundamental eigen- harmonic of standing waves becomes higher than the frequency of the neutral mode emitted by the shear layer. In this case, there are no unstable radiating modes, and the growth rate is determined only by the unstable surface mode.

Let us consider the growth rate behavior for different values of the parameter β in order to be able to follow the way in which the magnetic field strength influences the value of the critical Mach number M_c. Fig.7b plots the dependencies of the growth rate $c_i(M)$ at $\kappa = 1$ for different values of $\beta = \infty,\ 10,\ 1,$ and 0.1. When $\beta = \infty$, corresponding to the hydrodynamic flow case, the lower critical Mach number is absent. With a decrease of β, it is shifted toward $M = 1$, and the existence domain of the surface mode of unstable oscillations decreases. The walls in this case are sufficiently close to each other, which manifests itself in complete stabilization of the radiating mode of oscillations. However, at no, arbitrary small, values of β does the flow become entirely stable.

The numerical solution of equation (2.6) for a flow with a velocity profile of the form (4.6) between two fixed walls ($\Delta/a = \pm 10$) is presented in Fig.8. The contour distribution of the growth rate c_i in Fig.8**a,b** corresponds to the values of $\beta = 10$ and $\beta = 1$. With a decrease of β, the existence domain of the unstable surface mode becomes narrow, and the absolute value of the growth rate decreases. The existence domains of the radiating and reflective modes make up separate bounded islands. When the beta decreases the size of these islands decreases also. The domain of existence of the reflective mode of oscillations decreases much more than that of the radiating mode. However, the shear flow does not become fully stable for any values of β. As in the case with a single wall, the unstable reflective mode disappears in the limit of tangential discontinuity ($\alpha \to 0$). However, as opposed to the flow with one bounding wall, in the case of a flow bounded by two walls as one passes to the tangential discontinuity approximation the radiating mode disappears as well.

6.2. The case $\mathbf{k}_t \perp \mathbf{B_0}$

The solution of equation (2.6) for a flow with a velocity profile in the form of a tangential discontinuity is represented by curve 1 in Fig.7b, corresponding to $\beta = \infty$. It describes the unstable surface mode of oscillations bounded at the right by a critical value of the Mach number $M_c = 1$. Let us take a look at what would result if the walls are placed on both sides of the shear flow with a smooth velocity profile.

We solve numerically equation (2.6) for a flow with a velocity profile of the form (4.6) and the boundary conditions (3.7). Accurate to overdetermination of the Mach number $\bar{M}$, it describes hydrodynamic flow. The contour distribution of the oscillation growth rate for distance of the walls from the shear layer $\Delta/a = \pm\ 20$ is presented in Fig.9. In the region where in the infinite flow there are unstable oscillations of the radiating mode ($\bar{M} > 1$), the growth rate distribution has a quasi-periodic "island" structure. This applies to a greater degree for the reflective mode of oscillations. This is explained by the formation of standing waves between the walls. The growth rate of the radiating and reflective modes has maxima for harmonics of standing waves whose frequencies are close to the frequency of neutral mode of oscillations radiated by the shear layer.

As the walls approach the shear layer, the islands of unstable radiation oscillations disappear first, followed by the disappearance of the region of unstable oscillations of the surface mode. When the walls approach closer than by $\xi \approx \pm 1.2$, the shear flow becomes fully stable even in the incompressible limit ($\bar{M} \to 0$).

Thus we can speak about the stabilizing influence of the walls, between which the shear flow lies. The fact that the radiating mode of oscillations becomes stable when the walls come closer together is due to a change in harmonic eigenfrequencies of the standing (between the walls) waves. The stabilization of the surface mode of oscillations can be explained as follows. The characteristic scale of unstable oscillations of the surface mode in the direction across the shear layer, in the presence of the walls, is determined by the distance between them. If the walls are sufficiently far from the shear layer, then both the characteristic scale and the region of unstable oscillations of the surface mode remain virtually the same as in the infinite flow. As the walls come closer together, this scale decreases. Furthermore, the range of values of the transverse wave number α and of the Mach number $\bar{M}$, at which the surface mode oscillations become unstable, decreases also. When the walls approach as much as a scale on the order of the characteristic scale of the shear flow profile, the flow becomes fully stable.

7. Conclusion

We now summarize the main results of this study.

1. A comparative analysis of the influence of different boundary conditions on shear flow stability of compressible fluid in a magnetic field has been carried out. Two limiting cases of the longitudinal and transverse mutual orientation of the vectors $\mathbf{k_t}$ and $\mathbf{B_0}$ were considered. In the case of $\mathbf{k_t} \perp \mathbf{B_0}$, equation (2.6), describing the oscillations, coincides in form with the equation describing the oscillations of a usual hydrodynamic shear flow of inviscid compressible fluid. In the case of $\mathbf{k_t} \parallel \mathbf{B_0}$, the magnetic field has an additional stabilizing influence at the expense of Maxwellian tensions.

2. In the case of $\mathbf{k_t} \perp \mathbf{B_0}$, in an infinite (unbounded) medium the distribution of the oscillation growth rate in the plane $(\alpha, \bar{M})$ is similar to the distribution of the oscillation growth rate of shear flow of inviscid compressible fluid in the plane (α, M), where α is a dimensionless tangential wave vector, and M and $\bar{M}$ are the usual and modified Mach numbers. In the limit of tangential discontinuity ($\alpha \rightarrow 0$) the oscillations are stabilized when a critical value of $\bar{M} > \sqrt{2}$ is exceeded.

In the case of $\mathbf{k_t} \parallel \mathbf{B_0}$, an additional influence of Maxwellian tensions manifests itself in a stabilization of the oscillations when the shear flow velocity difference is below some minimum critical value. The value of this difference depends on magnetic field strength. The flow is completely stabilized when the magnetic field reaches such a value at which $\beta = 1$ (β is the ratio of the velocity of sound squared to the Alfven velocity squared, proportional to the thermal to magnetic pressure ratio).

3. Shear flow in the presence of a single fixed wall is less stable than in an infinite (unbounded) medium. For it there disappears the upper critical value of the velocity difference, and when this value is exceeded, the flow in an infinite medium becomes stable. Moreover, a new mode of unstable oscillations associated with the wave reflected from a wall and transmitted through the shear layer arises. Generation of this unstable mode is explained by a resonance of wave reflected from a wall and the neutral mode emitted by the shear layer.

The case $\mathbf{k_t} \parallel \mathbf{B_0}$ differs from the case $\mathbf{k_t} \perp \mathbf{B_0}$ in that the oscillations, as in the case of an infinite medium, become stabilized when the shear flow velocity difference is below some critical value. Therein lies an additional stabilizing factor that is associated with magnetic field force line tension. However, unlike the infinite flow, no complete stabilization of the oscillations occurs, whatever the finite values of the magnetic field strength.

4. The shear flow is the stablest between two fixed walls. The domain of existence of unstable oscillations in such a shear flow is divided into three bounded regions, associated with the surface mode, the radiating and reflective modes. The last two regions have a peculiar island structure when the magnetic field strength is small enough. Such a structure of the growth rate is associated with the formation of standing waves between fixed walls. As the walls come closer together, this region of unstable oscillations disappears. If the walls are closer than $\Delta = \pm 1.2a$, then the flow with $\mathbf{k_t} \perp \mathbf{B_0}$ becomes fully stable.

In the case of $\mathbf{k_t} \parallel \mathbf{B_0}$, the action of force line tension leads to a stabilization of the oscillations when the shear flow velocity difference is below a certain critical value. However, if the walls are at a sufficient distance from the shear layer, then, as in the case of flow with a single wall, a complete stabilization of the oscillations does not occur, whatever the finite values of the magnetic field strength.

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Appendix: Analyzing the solution for unstable oscillations when $\beta < 1$ in an unbounded medium

If we reverse the sign before the second radical in (4.2), then the form of the solution (4.4) remains unchanged. Consequently, the solution (4.4) contains roots corresponding to two different signs before the radicals in (4.2). To determine to which sign the chosen unstable (when $\beta < 1$) solution (4.4) corresponds, we consider it near the neutral point $M = M_0$, where $c_i = 0$. In this case $c^2 = 1 + M_0^{-2}$. On substituting this expression into (3.4), we find that when $M_0 < M_{01} = \sqrt{\sqrt{5} - 2}/2$ the square of the x–component of the wave vector $k_x^2 > 0$, and when $M_0 > M_{01}$ we have $k_x^2 < 0$. In the former case the medium is transparent to the waves under consideration, and this medium is supported by a well-defined notion of the group velocity, with which wave energy is transferred. By receding slightly along M inside the region where the imaginary component $c_i > 0$ ($M = M_0(1-\varepsilon)$, where $0 < \varepsilon \ll 1$) appears, we obtain a complex expression for k_x. The sign before the radical in (3.4) should be chosen such that $k_{xi} = \mathrm{Im}(k_x) > 0$ when $x > 0$ and $k_{xi} < 0$ when $x < 0$, which corresponds to the exponentially decreasing (in amplitude) solution at a larger distance from the shear layer. In this case the signs of the group velocity correspond to the oscillations which carry the energy away from the shear layer. However, such a choice of the signs of k_x leads to the dispersion equation with like signs before the radicals, which does not correspond to (4.2).

In the latter case ($M_0 > M_{01}$) the medium is opaque for the oscillations under consideration, and the choice of the signs of k_x is determined at the neutral point $M = M_0$ by the requirement of an exponential decrease of their amplitude with a distance from the shear layer. The group velocity for such oscillations is not determined. If we recede inside the region of $M < M_0$ there appear a positive imaginary component $c_i > 0$ and the associated small component of the group velocity $\mathrm{Re}(v_{gx})$. It is easy to verify that, as in the preceding case, the signs of the group velocity correspond to the oscillations which carry energy away from the shear layer; however, the signs before the radicals in the dispersion equation do not correspond to (4.2). These signs can be reconciled only by choosing the increasing solution on one side of the shear layer, which is in conflict with the physical statement of the problem.

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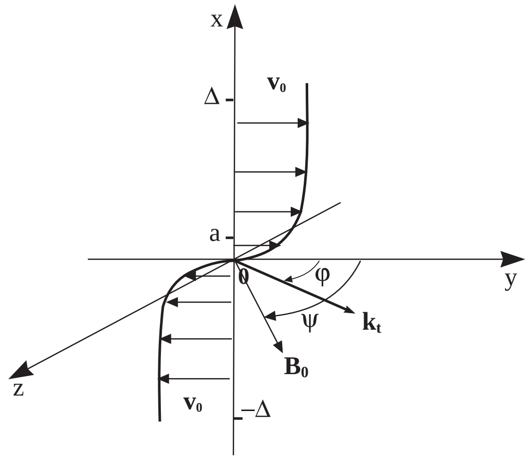

Figure .1. Model of the medium, and coordinate system. Designations: a – characteristic scale of the shear layer, $\pm\Delta$ – location of the boundaries in the form of walls, $\mathbf{v_0}$ and $\mathbf{B_0}$ – vectors of the unperturbed velocity and magnetic field of the medium, $\mathbf{k_t}$ – tangential wave vector of oscillations.

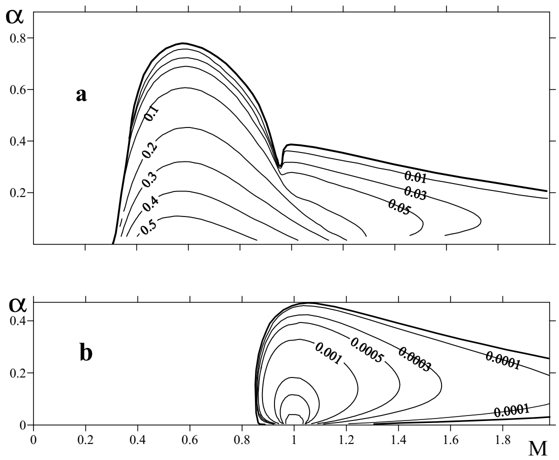

Figure .2. Contour distribution for the growth rate ($c_i \equiv \mathrm{Im}c$) of the oscillations with $\mathbf{k_t} \parallel \mathbf{B_0}$ of a sheared MHD flow in an unbounded medium for two values of the parameter β: a - $\beta = 10$, b - $\beta = 1.1$. In the domain $M < 1$ unstable oscillations are associated with surface mode, and in the domain $M > 1$ they are associated with radiating mode.

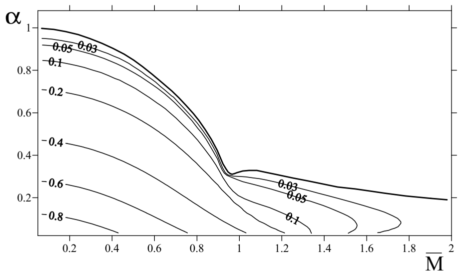

Figure .3. Contour distribution for the growth rate ($c_i \equiv \mathrm{Im}c$) of the oscillations with $\mathbf{k_t} \perp \mathbf{B_0}$ in an unbounded medium. In the domain $M < 1$ unstable oscillations are associated with surface mode, and in the domain $M > 1$ they are associated with radiating mode.

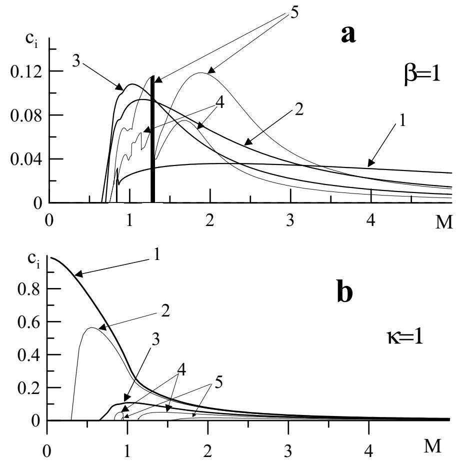

Figure .4. Dependence of the growth rate $c_i(M)$ of the oscillations with $\mathbf{k_t} \parallel \mathbf{B_0}$ of a sheared MHD flow with the velocity profile as tangential discontinuity bounded by a fixed wall on one side of the shear layer. Plots 1-5 corresponds to different values of the parameters β and κ: a - $\beta = 1$, $\kappa = 0.1, 0.5, 1, 5, 10$, b - $\kappa = 1$, $\beta = \infty, 10, 1, 0.5, 0.2$.

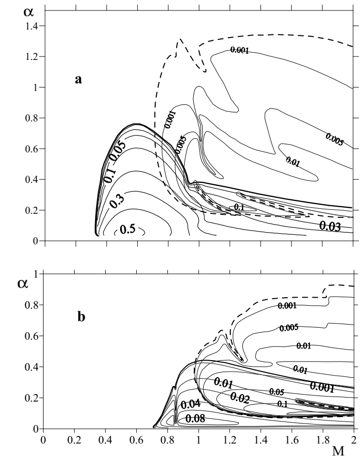

Figure .5. Contour distribution for the growth rate c_i of the oscillations with $\mathbf{k_t} \parallel \mathbf{B_0}$ of a sheared MHD flow bounded by a fixed wall on one side of the shear layer ($\xi = 20$) for two different values of the parameter β: a - $\beta = 10$, b - $\beta = 1$. The bold isolines corresponds to the surface and radiating modes. Growth rate distribution of the reflective mode is shown by the thin isolines.

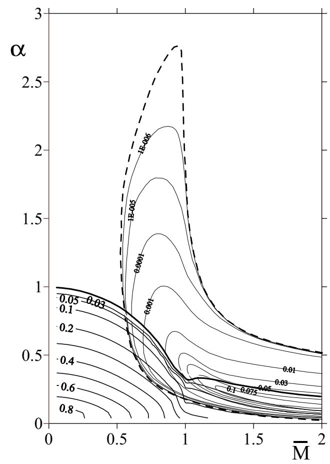

Figure .6. Contour distribution for the growth rate c_i of the oscillations with $\mathbf{k_t} \perp \mathbf{B_0}$. Presented are the unstable oscillations of a sheared MHD flow bounded by a fixed wall on one side of the shear layer $\xi = 15$. The bold isolines corresponds to the surface and radiating modes. Growth rate distribution of the reflective mode is shown by the thin isolines.

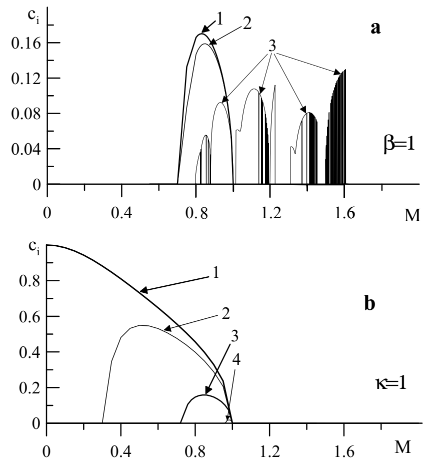

Figure .7. Dependence of the growth rate $c_i(M)$ of the oscillations with $\mathbf{k_t} \parallel \mathbf{B_0}$ of a sheared MHD flow with the velocity profile as tangential discontinuity between two fixed walls for different values of the parameters β and κ: a – $\beta = 1$, plots 1-3 correspond to the values of $\kappa = 0.1, 1, 5$, b – $\kappa = 1$, plots 1-4 correspond to the values of $\beta = \infty, 10, 1, 0.1$.

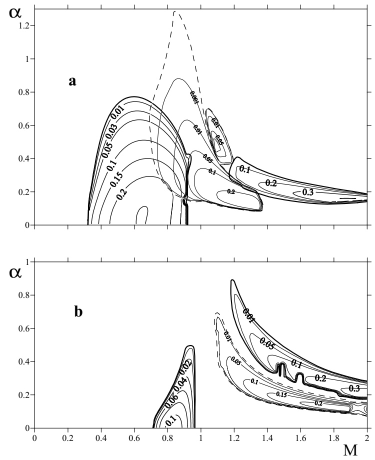

Figure .8. Contour distribution for the growth rate c_i of the oscillations with $\mathbf{k_t} \parallel \mathbf{B_0}$ of a sheared MHD flow between two fixed walls ($\xi = \pm 10$) for two different values of the parameter β: a – $\beta = 10$, b – $\beta = 1$. The bold isolines corresponds to the surface and radiating modes. Growth rate distribution of the reflective mode is shown by the thin isolines.

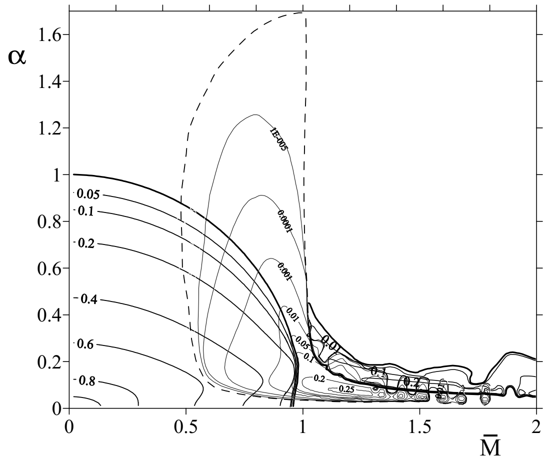

Figure .9. Contour distribution for the growth rate c_i of the oscillations with $\mathbf{k_t} \perp \mathbf{B_0}$ of a sheared MHD flow between two fixed walls $\xi = \pm 20$. The bold isolines corresponds to the surface and radiating modes. Growth rate distribution of the reflective mode is shown by the thin isolines.