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Long-term evolution of the magnetic field generated by an ensemble of Rossby vortices

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The evolution of the large-scale component of the magnetic field generated by an ensemble of Rossby vortices is numerically simulated. The distribution of the Rossby vortices excited at the beginning of each Carrington rotation is determined from the analysis of Kitt Peak synoptic maps. Our model also considers 11-year hydrodynamic and 22-year magnetic field oscillations. In the vicinity of the Rossby vortices, the toroidal magnetic field is significantly amplified and the sign of the angle between the rope of the field lines and the equator is in accordance with observations for "normal" sunspots. We also suggest the possibility of the interpretation by our model of "abnormal" sunspot phenomena. We find that an inverse cascade, namely, the merging of Rossby vortices, gives rise to the formation of large-scale hydrodynamic structures with a life-time on the order of a solar cycle period. We conclude from this that the formation of such structures can thus explain the appearance of long-lived, large-scale component in the distribution of the magnetic field.

Key words: sun spots – magnetic fields – large-scale component – Rossby vortices

1. Introduction

Results obtained from recent helioseismology research show the existence of variations in the hydrodynamic flow near the base of the convection zone with a characteristic time of 1 year (Howe 2000). In the frame of the context we have recently developed (Tikhomolov 1995; Tikhomolov and Mordvinov 1996) such oscillations can be explained by the excitation of the large-scale Rossby vortices at the interface between the convection zone and the radiative interior with the same characteristic life-time. Another test for this theory is to compare the results of numerically simulating long-term evolution of the magnetic field generated by Rossby vortices with data obtained from synoptic maps.

Classic-style dynamo models consider long-lived large-scale solar magnetic field structures as a manifestation of the excitation of non-axisymmetric components by the dynamo mechanism (Stix 1971). In these models oscillations of the magnetic field appear in the background of steady (non-oscillating on the time-scale of solar cycle) flows. Rather interesting results were obtained in the framework of the semi-empirical Leighton model by specifying bipolar magnetic groups as the sources of magnetic fields (Wang, Nash and Sheeley 1989). However, none of these models are able to explain a whole range of observed phenomena in the evolution of large-scale solar magnetic fields.

In our hydrodynamic approach the sources of such long-lived large-scale structures are Rossby vortices. We suggest that they can be excited due to shear instability of the zonal flow near the base of the solar convection zone. One well-known example of the excitation of such kinds of vortices is the Great Red Spot of Jupiter (Nezlin 1994). Using this analogy with Jupiter we can expect that, in the sun, large-scale cyclones can appear because solar differential rotation has cyclonic shear in latitude. Anticyclones rapidly decay in such conditions and can thus appear only as outstanding events that manifest themselves in some powerful phenomena (Tikhomolov 1998).

The possibility of shear instability in the sun has been widely discussed (Dziembowski and Kosovichev 1987; Charbonneau, Dikpati and Gilman 1999) but currently there is no consensus on this question. We think that the element playing the essential role in the development of shear instability is the non-stationarity of zonal flow at the base of the solar convection zone. The conditions for shear instability in low- and mid-latitudes appear only at specific time intervals. The well-known manifestations of such non-stationarity of the shear are torsional waves that drift from high to low latitudes (Howard and LaBonte 1980). The hydrodynamic model for the formation of torsional waves was described in our recent paper (Tikhomolov 2001). In the present paper we explore the idea of the excitation of Rossby vortices due to shear instability in torsional waves as well as study the formation of the resulting large-scale magnetic structures at the time scale of the solar cycle.

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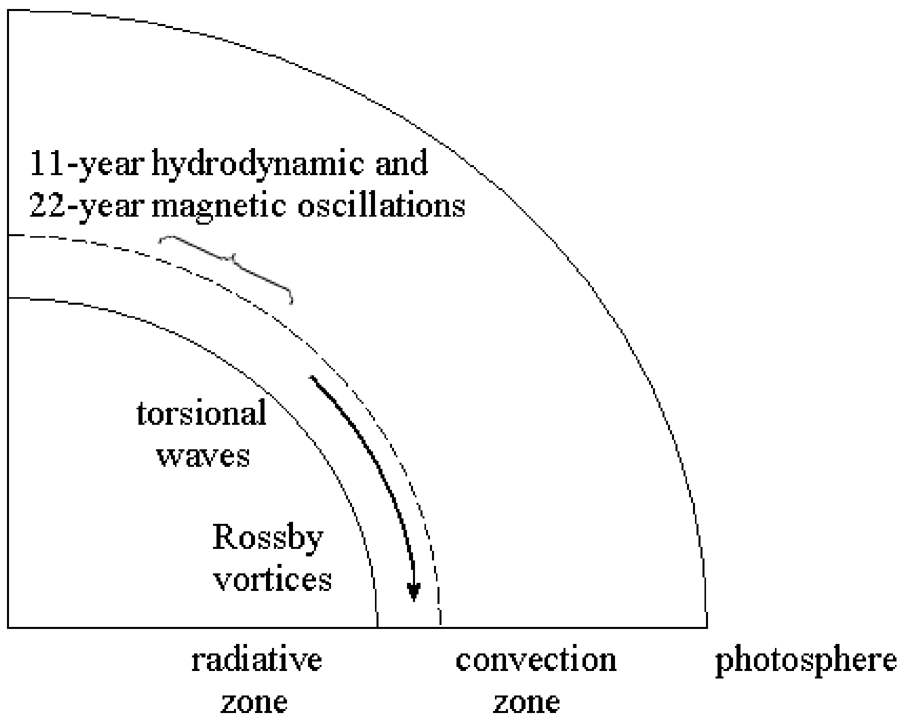

Fig. 1: The location of the layers of the excitation of 11-year and 22-year variations, torsional waves and Rossby vortices.

In considering the evolution of the non-axisymmetric magnetic field component on the time scale of the solar cycle, it is also necessary to take into account the 11-year variability of the axisymmetric part of both the hydrodynamic flow and the magnetic field. Thus we present a complex model which includes both short-term (on the order of several Carrington rotations) and long-term (on the order of a solar cycle) components.

2. Model

Fig. 1 shows the basic processes operating near the base of the convection zone. 11-year hydrodynamic oscillations and 22-year magnetic oscillations are excited due to deformational long-wave instability in the interfacial layer. The formation of torsional waves travelling from high to low latitudes in this layer can lead to the appearance of conditions for shear instability and the excitation of large-scale Rossby vortices. Rossby vortices act as modulators for the magnetic fields flowing to the surface. In addition, they can also induce transfer of the magnetic field up to the solar surface and thus initiate the formation of solar complexes of activity (Tikhomolov 1998).

For large-scale flows near the bottom of the convection zone, even for a rather slowly rotating sun, it is possible to use the quasigeostrophic approximation (which considerably simplifies the mathematical formulation of the problem). The upper and lower boundaries of the layer are considered to be stress-free, and at the boundaries corresponding to the poles the velocity is specified to be zero. The equations are written in Cartesian coordinate system which is rotating with the velocity of plasma at latitude 30°. Established results of helioseismology show that the upper part of the radiative zone rotates with this velocity at all latitudes.

For all magnetic field components on the lower boundary of the layer, the condition of perfect conductivity is specified. On the side boundaries (corresponding to polar regions) conditions are specified in such a way that the magnetic field is parallel to the polar axis. On the upper surface of the layer, for the horizontal components of the magnetic field, the condition of perfect conductivity is assumed, and for the vertical component an "open" boundary condition is used (Yoshimura, 1981) i.e. it is supposed that this component freely emerges through the upper surface of the layer.

There are both calculated and free parameters in the model. The values for the free parameters are adjusted to produce results that are in the best agreement with experimental data. For coefficients of effective turbulent kinematic viscosity and coefficient of magnetic diffusion near the bottom of the convection zone we take the values of $2 \cdot 10^{11}$ cm^2 s^{-1}. The thickness of the layer near the interface between the convective and radiative zones is taken to be $\mathrm{H} = 10^9$ cm.

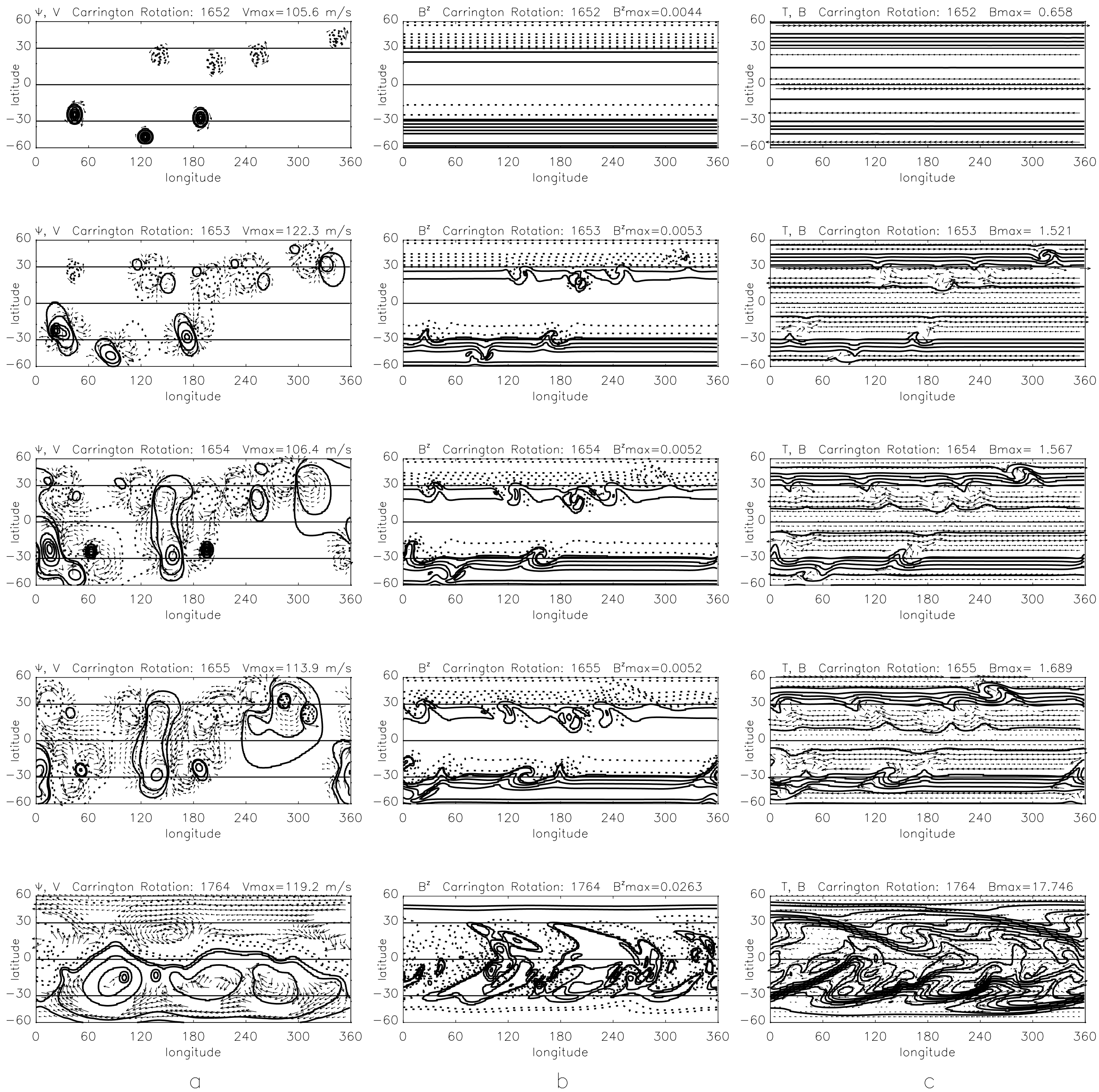

Fig. 2: Numerical simulations of the generation of the magnetic field by Rossby cyclones near the base of the convection zone. Solid and dashed lines show, respectively, positive and negative values of the quantities. All values for each figure are normalized by their maximum value. Carrington rotation number and maximum values of velocity (Vmax), vertical (B^zmax) and toroidal magnetic field (Bmax) components are indicated at the top of the figures. (a) The evolution of the Rossby vortices, specified at the beginning of each Carrington rotation. Shear flow is subtracted. Shown are the contours of stream function ψ: ± 0.05, ± 0.1, ± 0.3, ± 0.5, ± 0.7, and ± 0.9. Arrows show distribution of the velocity V. (b) The evolution of the vertical component B^z for flows depicted in (a). (c) The evolution of the toroidal component for flows depicted in (a). Shown are the contours of stream function T: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. Arrows show distribution of the toroidal field B.

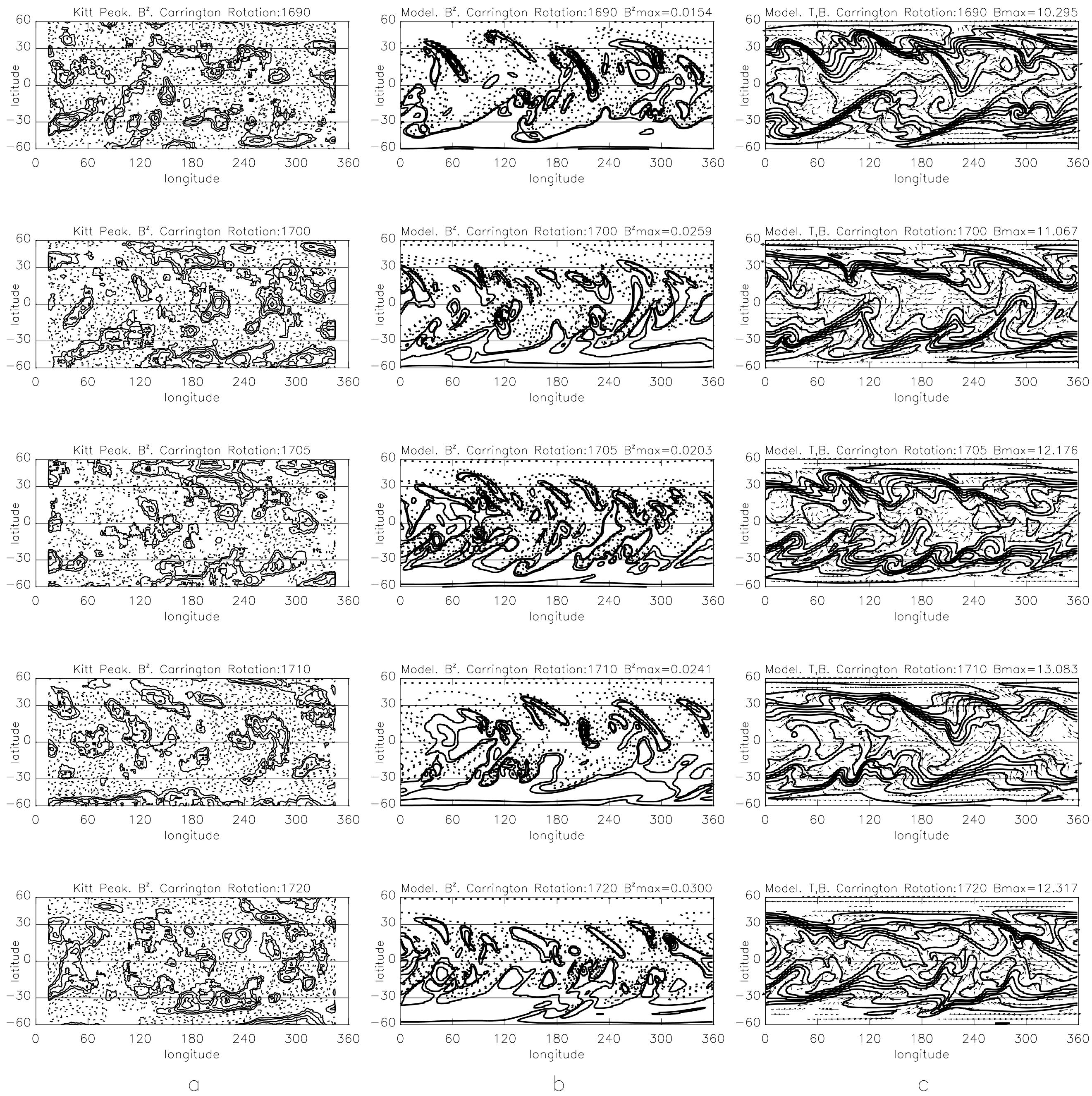

Fig. 3: The observed and simulated distributions of the solar magnetic field. Solid and dashed lines show, respectively, positive and negative values of the quantities. All values are normalized by their maximum value. Carrington rotation number and maximum values of vertical (B^zmax) and toroidal magnetic field (Bmax) components are indicated at the top of the figures. (a) Smoothed distributions of the magnetic field obtained from Kitt Peak synoptic maps. Shown are the contours: ±0.15, ±0.3, ±0.5, ±0.7, ±0.9. (b) The evolution of the simulated vertical component B^z near the base of the convection zone. Shown are the contours: ±0.05, ±0.1, ±0.3, ±0.5, ±0.7, and ±0.9. (c) The evolution of the simulated toroidal component near the base of the convection zone. Shown are the contours of stream function T: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. Arrows show distribution of the toroidal field B.

The present numerical model is, in fact, a combination of two models described in previous papers; namely, the two-dimensional Rossby Vortex Model (RVM) (Tikhomolov 1995; Tikhomolov and Mordvinov 1996) and the one-dimensional Hydrodynamical Oscillations Model (HOM) (Tikhomolov 2001). For purpose of simplicity the excitation of Rossby vortices and generation of the long-term 11-year solar oscillations are considered as separate processes. At each time step the axisymmetric components in RVM are replaced by values calculated in HOM. Such a method permits us to study the processes by which the Rossby vortices produce the large-scale non-axisymmetric magnetic field component on the time-scale of a solar cycle.

A priori we don't have clear features which can give us precise co-ordinates for the Rossby vortices and instances of the time of their excitation due to shear instability. Thus their co-ordinates are evaluated by visual analysis of Kitt Peak's synoptic maps, and correspond to areas of large-scale active regions which were observed during several Carrington rotations. The co-ordinates of Rossby vortices are specified in the numerical model at the beginning of each Carrington rotation.

3. Results

Our calculations cover the period from Carrington rotation 1652 to 1764. Fig. 2 shows the evolution of stream functions and the magnetic field during the first four Carrington rotations, as well as the final distributions near the base of the convection zone. At the initial instant of time seven Rossby cyclones of Gaussian forms were specified in the northern and southern hemispheres. Then, at the beginning of each Carrington rotation, new vortices of the same shape and amplitude are added. The size of initial vortices is chosen to be ten helio-degrees and their maximum velocity is $1.05 \cdot 10^4$ cm s^{-1}. All distributions are presented in the Carrington frame of reference and the differential rotation is subtracted from the distribution of stream function. The axis of ordinate shows the sinus of latitude (but corresponding notations are in degrees on this axis).

One of the main processes occurring is the merging of vortices. Thus, after several Carrington rotations, large-scale disturbances of differential rotation are formed. One can clearly see in the first column for the Carrington rotation 1764 three well-formed structures in the southern hemisphere and two structures in the northern hemisphere. The maximum value for the disturbance of the velocity has approximately the same order of magnitude for the whole period of calculations, i.e. 10^4 cm s^{-1}.

After initialization, Rossby vortices twist magnetic field lines and under some conditions this leads to the formation of closed configurations that are obvious in the distribution of the toroidal magnetic field component in the last column for Carrington rotation 1764. The initial non-disturbed toroidal magnetic field (Carrington rotation 1652) has a west to east direction in the low latitudes of the northern hemisphere (this corresponds to the direction from the right side of the figures to the left) . In the low latitudes of the southern hemisphere it has the opposite direction. The first four Carrington rotations demonstrate the process for the formation of closed configurations. The maximum value of the toroidal magnetic field strength significantly increases in the areas of actions of Rossby vortices.

The vertical magnetic field component (middle column) changes sign in high latitudes during Carrington rotations 1652 to 1764. One of the interesting features to note is the concurrent stretching of the magnetic field lines in a direction which has some angle to the equator. This effect is caused by the simultaneous action of Rossby vortices and differential rotation.

In Fig. 3 we compare the results of our numerical simulations with data from Kitt Peak magnetic synoptic maps. The synoptic maps (first column) at each latitude are smoothed on the spatial scale of ten helio-degrees. We present distributions with a time interval of ten Carrington rotations. The middle column shows the distribution of the calculated vertical magnetic field component near the base of the convection zone. The most noticeable feature of both the experimental and model results is the banana-like distribution of the large-scale structures.

The figures for the toroidal component (last column) also show the arc-like (or banana-like) shape. As we mentioned above, in the areas of actions of Rossby vortices toroidal magnetic field becomes rather intense. This can lead to the initialization of transfer to the surface of the magnetic field due to magnetic buoyancy. The intense toroidal magnetic fields in most of these areas have the "right" direction: the sign of the angle between the rope of field lines and the equator is in accordance with observations of the "normal" sunspots. However it is important to note that in some areas the strong magnetic fields can also have an "anomalous" sign, as one can see on each of the figures in the last column of the Fig. 3.

Thus, the formation at the solar surface of large-scale Ω-loops with "normal" and "abnormal" angles of their plane to the equator(with the dominance of "normal" angle) seems rather natural in our model. Note that the effect of stretching is a result of the action of the ensemble of Rossby vortices during several Carrington rotations and is much more visible if several vortices are excited consequently near the same place. The relation of the maximum of the toroidal component to the maximum of the vertical component in our calculation is on the order of ≈ 500. But, it is necessary to note that this is the relationship at the base of the convection zone; during the transfer of the magnetic field to the surface it can change.

A more detailed comparison shows some discrepancies, however. One can notice that in our simulations, magnetic structures are located in a more narrow equatorial band than they are in experimental data. This is likely caused by neglecting the meridional circulation that leads to the transfer of the magnetic fields to high latitudes. One also should keep in mind that during the transfer of the magnetic field through the convection zone, the magnetic structures deform, and become wider and less regular. In addition, we specify initial vortices of a uniform shape and size; this is a rather crude approximation. In spite of these limitations at some Carrington rotations, say, 1710, one can see rather good correlation between the observed and computed magnetic fields.

4. Discussion

For a long time investigations into the evolution of large-scale solar magnetic fields attracted much attention from the "observational" point of view. Many interesting phenomenological laws governing the behaviour of large-scale solar formations over a time interval of several tens of Carrington rotations were developed (Ambroz 1987, Bumba 1987,1996, Mordvinov and Tikhomolov 1992). These laws demonstrate themselves most clearly at the largest spatial scales. Several different approaches in interpreting this "large-scale" effect currently exist (Stix 1971). Our point of view is that this fact is evidence of the existence of the large-scale sources for the large-scale magnetic structures, namely, Rossby vortices.

In our previous papers we showed that a whole range of solar large-scale phenomena can be explained by the excitation of Rossby vortices near the base of the solar convection zone (Tikhomolov 1995, 1998; Tikhomolov and Mordvinov 1996, 1997). In present paper we tried to reproduce the behaviour of the large-scale magnetic field on the time-scale of a solar cycle by specifying the distribution of the Rossby vortices at the beginning of each Carrington rotation. The co-ordinates for the Rossby vortices were determined by the analysis of synoptic magnetic maps. It would be better if we could use the distribution of the velocity field at the base of the convection zone and correct the evolution of the velocity field at each Carrington rotation. Such a method can provide a prediction of the long-term evolution of the large-scale magnetic structures in more detail. We expect that in the near future helioseismology will be able to provide us such information. Nevertheless, even in this situation we have some physical limitations on the predictability of the evolution of the large-scale magnetic fields.

Shear instability produces vortices in a first approximation in a random manner. However large-scale modulation of the velocity field distribution can, obviously, influence the appearance of vortices. In fact, we have a complex system with a feedback, because vortices produce large-scale velocity component by merging. Thus the next step in the development of our model is the inclusion of the consideration of the influence of the large-scale modulation of the velocity field on the distribution of vortices. We can not entirely exclude, of course, the probability nature of Rossby vortex excitation, but a model with such a property will be able to predict at much higher level of the large-scale magnetic field behaviour.

The next problem is: How does the excitation of initial vortex influence the appearance of the other vortices in the neighbourhood? We know from observations that a powerful complex of activity doesn't appear as a single formation, but as several consequent structures. This fact provides reason enough to suggest that the excitation of the first initial Rossby vortex leads to an increase in the likelihood of the excitation of other vortices near it. To model this effect it is necessary to take into account the details of velocity distribution and changes in the local shear instability conditions after the excitation of vortices. Because of difficulties in the analysis of such situations, the most promising way to solve this problem continues to be numerical simulations.

The results we obtained in this and previous papers show that our "hydrodynamic" model is able to reproduce the basic observable large-scale solar phenomena. We can expect that inclusion the effects mentioned above will lead to a more elaborate, self-consistent model that will have a forecasting ability comparable to the modern terrestrial atmosphere global models.

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