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GEOMAGNETIC PULSATIONS ASSOCIATED WITH MHD-WAVEGUIDE IN MAGNETOSPHERIC DUCTS

A. V. BUZEVICH, A. S. LEONOVICH and V. A. PARKHOMOV
Siberian Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation (SibIZMIR), Siberian Branch of the U.S.S.R. Academy of Sciences, Irkutsk 33, P.O. Box 4, U.S.S.R.

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Abstract—A theory of waveguide propagation of MHD waves in magnetospheric ducts (detached plasma regions) is presented. It is shown that Alfvénic waveguide modes can be excited due to the resonance interaction with the waveguide FMS modes. The Alfvénic modes penetrate to the ground and lead to electromagnetic field variations, registered as local geomagnetic Pc1–2 pulsations. The problem of the Alfvénic waveguide modes propagation from the magnetosphere to the Earth's surface is considered and a comparison is made of theoretical results with experimental data.

1. INTRODUCTION

During local daytime, regular geomagnetic Pc1–2 pulsations with periods ranging from 2–15 s (Buzevich and Potapov, 1979; Inhester *et al.*, 1984) to 25 s (Parkhomov *et al.*, 1983) have been observed on satellites in regions of enhanced density of the background plasma (Kaye and Kivelson, 1979). These pulsations are usually registered in places where the related regions are projected on to the Earth's surface. The pulsations can be excited in the magnetosphere due to an ion-cyclotron instability of the ring current protons or due to oxygen ions (Inhester *et al.*, 1984; Kalisher *et al.*, 1982).

A characteristic feature of these pulsations is their high spatial localization. This is due to their propagation in magnetospheric ducts, i.e. in dense-plasma regions which detach themselves from the plasmasphere and stretch along geomagnetic field lines (Kaye and Kivelson, 1979; Inhester *et al.*, 1984).

Leonovich (1984) suggested a model of localization of Pc1–2 pulsations in magnetospheric ducts by a plasma waveguide associated with cross dispersion of Alfvén waves due to the inertia of plasma ions. The model is valid, however, only within a narrow, near-equatorial region, rather than throughout the entire length of a plasma filament. Gul'elmi and Polyakov (1983) and Leonovich *et al.* (1985) described another type of waveguide associated with the MHD-wave dispersion due to the finiteness of the Larmor radius of the ions. Limitations on the existence of such waveguides are imposed by the requirement $\beta > m_e/m_i$, where β is the ratio of kinetic to magnetic plasma pressure. It is easy to show that in magnetospheric ducts this condition can be satisfied from the equator to the ionosphere itself, which enables the effects associated directly with this waveguide to be observed on the ground.

Distortions which the waveguide modes receive when they pass through the ionosphere and atmosphere must be taken into account for a correct comparison of geomagnetic pulsation structure in the magnetosphere and on the ground.

In the present paper a theoretical analysis of the waveguide propagation of MHD-waves in magnetospheric ducts is carried out and the penetration of the waves from the magnetosphere to the Earth's surface is treated in the comparison of theoretical results with experimental data.

The resonance interaction of waveguide Alfvénic and FMS modes is treated in Section 2. The structure of the waveguide Alfvénic modes in ducts near the ionosphere is determined. The problem of penetration of the modes from the magnetosphere to the ground is investigated in Section 3. In the Discussion a comparison of the theoretical results with experimental data is made.

2. THE WAVEGUIDE PROPAGATION OF MHD-MODES IN MAGNETOSPHERIC DUCTS

The plasma model which was used in Gul'elmi and Polyakov (1983) and Leonovich *et al.* (1985) is inappropriate for modelling of ducts. For that purpose we used a plasma cylinder in a straightforward magnetic field (the curvature of field lines was neglected) and when $\beta \ll 1$ the field is considered uniform (Fig. 1a). The plasma density is maximum at the center of the duct ($r = 0$) and decreases with distance from the center. Let us now investigate the structure of axially symmetric waveguide modes of the form

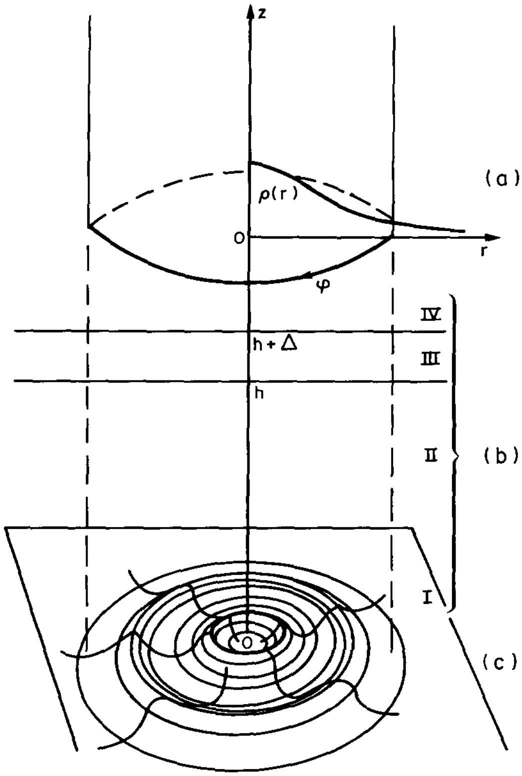

FIG. 1. THE MODEL WAVEGUIDE MHD-MODES PENETRATING FROM THE MAGNETOSPHERE TO THE GROUND.
(a) The magnetospheric-duct model; (b) the model of the medium in the vicinity the Earth (I—ideally conducting ground, II—neutral atmosphere, III—ionosphere, IV—uniform magnetosphere); (c) amplitude profile of waveguide geomagnetic pulsations on the ground.

$\exp(\mathrm{i}k_z z - \mathrm{i}\omega t)$ in magnetospheric ducts, for which the structure can be obtained analytically.

Let us suppose that the waveguide modes are localized near the axis of the plasma cylinder. Then, the plasma density may be represented by the expansion

$$\rho(r) = \rho(0)(1 - r^2/a^2),$$

which is valid when $r \ll a$. In a cylindrical coordinate system, we obtain a system of MHD-equations:

$$\frac{\partial}{\partial \xi}\frac{1}{\xi}\frac{\partial \xi B_r}{\partial \xi} - (\xi^2 - \lambda)B_r = -\mathrm{i}\varepsilon B_\varphi, \tag{1}$$

$$\kappa^4\frac{\partial}{\partial \xi}\frac{1}{\xi}\frac{\partial \xi B_\varphi}{\partial \xi} - (\xi^2 - \lambda)B_\varphi = \mathrm{i}\varepsilon B_r, \tag{2}$$

where $\mathbf{B}$ is the wave's magnetic field, and $\xi = r\sqrt{k_z/a}$. The parameters $\kappa^4 = u^2\beta$, $\varepsilon = uk_z a$, and $u = \omega/\omega_i$ (ω_i is ion frequency) near the ionosphere are small: $\kappa, \varepsilon, u \ll 1$. The parameter λ is introduced by the relation $\omega = k_z A(0)(1 + \lambda/k_z a)$, where $A = B_0/\sqrt{4\pi\rho}$ is the Alfvén velocity. The system of equations (1) and (2) can be solved by the method of Green's functions, as was done by Leonovich et al. (1985). The Green's function of equation (1) is

$$G(\xi, \xi') = \sum_{n=0}^{\infty} \frac{y_n(\xi)\, y_n(\xi')}{\lambda - \lambda_n},$$

where $y_n(x) = a_n x \exp(-x^2/2) L_n^{(1)}(x^2)$ is the orthonormal eigenfunction of the operator on the left-hand side of (1), a_n is the normalization factor, $L_n^{(1)}$ are the generalized Laguerre polynomials, and $\lambda_n = 4(n+1)$ are the corresponding eigenvalues of the parameter λ. We then obtain a solution of (1)

$$B_r(\xi) = -\mathrm{i}\varepsilon \sum_{n=0}^{\infty} \frac{C_n y_n(\xi)}{\lambda - \lambda_n}, \tag{3}$$

where

$$C_n = \int_0^\infty B_\varphi(\xi)\, y_n(\xi)\, \mathrm{d}\xi. \tag{4}$$

Similarly, for (2) we obtain

$$B_\varphi(\xi) = \mathrm{i}\frac{\varepsilon}{\kappa} \sum_{m=0}^{\infty} d_m \frac{y_m(\xi/\kappa)}{\lambda - \kappa^2 \lambda_m}, \tag{5}$$

where

$$d_m = -\mathrm{i}\varepsilon \sum_{n=0}^{\infty} C_n \frac{\alpha_{mn}}{\lambda - \lambda_n}, \quad \alpha_{mn} = \int_0^\infty y_m\left(\frac{\xi}{\kappa}\right) y_n(\xi)\, \mathrm{d}\xi. \tag{6}$$

The waveguide modes of our interest are $|\lambda - \kappa^2\lambda_m| \ll \kappa^2$ (in particular, this includes the main waveguide modes). If simultaneously the inequality $|\lambda - \lambda_n| \ll 1$, i.e. $|\lambda_n - \kappa^2\lambda_m| \ll \kappa^2$ is satisfied, then we can leave only one main term in (3)–(6). Substituting (5) into (4), we obtain the quadratic equation for eigenvalues of $\lambda^{(n)}$ of the system (1) and (2) that gives:

$$\lambda_{1,2}^{(n)} = \frac{\lambda_n + \kappa^2\lambda_m}{2} \pm \sqrt{\frac{(\lambda_n - \kappa^2\lambda_m)^2}{4} + \frac{\varepsilon^2}{\kappa}\alpha_{mn}^2}. \tag{7}$$

It is easy to show that in the aforementioned case $\alpha_{mn} = t_m \tau_n \kappa^2$, where $t_m, \tau_n \sim 1$. If $|\lambda_n - \kappa^2\lambda_m| \gg \varepsilon\kappa^{3/2}$, then the relations between amplitudes B_φ and B_r corresponding to two values of $\lambda^{(n)}$ from (7) are different:

$$\frac{|B_\varphi|_{(1)}}{|B_r|} \sim \frac{\varepsilon\kappa^{1/2}}{\lambda_n - \kappa^2\lambda_m} \ll 1, \quad \frac{|B_\varphi|_{(2)}}{|B_r|} \sim \frac{\lambda_n - \kappa^2\lambda_m}{\varepsilon^2} \gg 1.$$

The first solution corresponds to the FMS waveguide mode, and the second one corresponds to the Alfvén mode. Whereas if $|\lambda_n - \kappa^2\lambda_m| \ll \varepsilon\kappa^{3/2}$, the relation between the B_φ and B_r components in both cases is identical

$$\frac{|B_\varphi|_{(1,2)}}{|B_r|} \sim \frac{1}{\kappa^{1/2}} \gg 1$$

and corresponds to the resonance of Alfvén and FMS waveguide modes. The structure of the FMS mode indicates that the waveguide FMS mode, when passing through the resonance, is restructuring spatially and transforms into an Alfvén mode and vice versa. This mechanism can effectively excite Alfvén waves in the magnetosphere with a typical size across the geomagnetic field of the order of tens of kilometers. The main waveguide modes and waves, as reported by Dubinin *et al.* (1985), are among them.

The eigenvalues of the parameter λ for the main waveguide modes ($m \sim 1$) are close to $\kappa^2\lambda_m$:

$$\lambda^{(m)} \approx \kappa^2\lambda_m - \varepsilon^2\kappa^3\tau_m^2 \sum_{n=0}^{\infty} \frac{\tau_n^2}{\lambda_n}.$$

The corresponding solution of (1)–(2) is

$$B_r^{(m)}(\xi) = -C_m\varepsilon\kappa^2 t_m \sum_{n=0}^{\infty} \frac{\tau_n y_n(\xi)}{\lambda_n}, \tag{8}$$

$$B_\varphi^{(m)}(\xi) = \mathrm{i}C_m y_m\left(\frac{\xi}{\kappa}\right), \tag{9}$$

where C_m is the amplitude of waveguide mode with index m. As $|B_\varphi/B_r| \gg 1$, the main waveguide modes are of the Alfvén type. Assume that geomagnetic pulsations consist, in principle, of the main modes. The characteristic features of the magnetospheric waveguide modes (8) and (9) penetrating to the ground are treated in Section 3.

3. THE PENETRATION OF WAVEGUIDE MHD-MODES FROM THE MAGNETOSPHERE TO THE EARTH'S SURFACE

In order to obtain the electromagnetic field variations on the ground produced by waveguide modes of the form (8) and (9), we consider the problem of their passage through the ionosphere and atmosphere. For this purpose, we appeal to the following model of a medium that is generally accepted for problems of this kind (Fig. 1b) (Belyaev and Polyakov, 1980; Lyatsky and Maltsev, 1983). The magnetic field is normal to the surface of a perfectly conducting Earth (I) and is directed along the Z-axis. In between the Earth and the ionosphere (III), there is a layer of nonconducting atmosphere (II) of thickness h. We assume the ionosphere is represented by a non-uniform Hall's layer of thickness Δ. When $Z > h+\Delta$, the magnetosphere (IV) is uniform. This last supposition is not quite consistent with the aforegoing. However, it seems to be justified because near the ionosphere, the transverse inhomogeneity of the magnetospheric duct is largely smoothed out so that the main plasma inhomogeneity is associated with the ionosphere and is directed along the Z-axis.

In order to investigate the distribution of axially symmetric waveguide modes it is convenient to apply the Fourier–Bessel transformation (Titchmarsh, 1960)

$$\mathbf{B}(r) = \int_0^\infty \mathbf{B}(k)\,J_1(kr)k\,\mathrm{d}k, \quad \mathbf{B}(k) = \int_0^\infty \mathbf{B}(r)\,J_1(kr)r\,\mathrm{d}r, \tag{10}$$

where $J_1(z)$ is the first-kind Bessel's function. Hence, from MHD-equations for tangential components of the electric field we obtain

$$\frac{1}{k_0^2}\frac{\partial^2 E_r}{\partial z^2} + \varepsilon_P(z)E_r = -\varepsilon_H(z)E_\varphi, \tag{11}$$

$$\frac{1}{k_0^2}\frac{\partial^2 E_\varphi}{\partial z^2} + \left(\varepsilon_P(z) - \frac{k^2}{k_0^2}\right)E_\varphi = \varepsilon_H(z)E_\varphi, \tag{12}$$

where $k_0 = \omega/c$, and ε_P and ε_H are the components of the dielectric constant tensor associated with Pedersen and Hall ionospheric conductivities. In the homogeneous magnetosphere and atmosphere the relationship between the electric and magnetic components of field has the form

$$E_r = (k_0/k_z)B_\varphi, \quad E_\varphi = -(k_0/k_z)B_r. \tag{13}$$

In the magnetosphere, the waves $(0, E_\varphi, 0)$ and $(E_r, 0, 0)$ are linearly independent, the former corresponding to fast magnetosound oscillations (FMS) and the latter to Alfvénic oscillations. From equations (11) and (12) it is evident that as the Alfvén waves pass through the ionosphere, they receive a certain small addition of the E_φ-component of field, while the FMS oscillations receive that of the E_r-component. In this case, whereas the electromagnetic field variations produced on the ground by FMS-waves are virtually independent of this addition, it is of key importance for Alfvén waves.

In solving equations (11) and (12) we assume that the condition $c/\sqrt{4\pi\omega\sigma_P} \gg \Delta$, where σ_P is the Pedersen ionospheric conductivity, is satisfied. This implies that for the pulsations under consideration, the ionosphere is optically thin and we may neglect the terms of the equations proportional to ε_P. Hence, to an accuracy of the correction introduced above, we have $\mathbf{E}(h) \approx \mathbf{E}(h+\Delta)$. In the atmosphere, the solutions to equations (11) and (12) that satisfy uniform boundary conditions on the ground have the form

$$E_{r,\varphi} = G_{r,\varphi} \sinh(\sqrt{k^2-k_0^2}z). \tag{14}$$

In order to derive the field of FMS-waves from the waveguide mode on the ground, the solution for the magnetosphere (8) can be combined with that of (14). On specifying for the magnetosphere

$$E_\varphi = \tilde{E}_\varphi e^{ik_z z} + D e^{-ik_z z}$$

where $\tilde{E}_\varphi$ is the amplitude of an incident FMS-wave and D is that of a reflected FMS-wave, and combining the tangential electric and magnetic wave field components for $z = h$, we obtain

$$G_\varphi^{(F)} = \frac{2\tilde{E}_\varphi e^{ik_z h}}{\sinh(\sqrt{k^2-k_0^2}h) + i(\sqrt{k^2-k_0^2}/k_z)\cosh(\sqrt{k^2-k_0^2})}$$

The index F represents a variation associated with FMS-waves.

The spatial structure of the magnetic field variations on the ground is determined through inverse transformation (10) to be

$$B_r(r) = \frac{i}{k_0}\int_0^\infty G_\varphi(k)\sqrt{k^2-k_0^2}\,J_1(kr)k\,dk. \tag{15}$$

Before proceeding with the calculations, we adopt the following characteristic parameters of the problem: $a \sim 10^3$ km, $k_z = \omega/A \sim 10^{-4}$ km^{-1}, $k_0 = \omega/c \sim 10^{-6}$ km^{-1}, and $h \sim 10^2$km. Hence, there results the inequality

$$k_0 \ll \bar{k} \ll h^{-1} \tag{16}$$

where $\bar{k}$ is the characteristic scale of localization of the function $\tilde{E}_\varphi(k)$ which, as is apparent from (8), is $\bar{k} = \sqrt{k_z/a}$. Using (16), the integral of (15) is quite easily evaluated with the result that

$$B_r^{(F)}(r) \approx -2B_r^{(n)}(r) \tag{17}$$

where $B_r^{(n)}(r)$ follows from (8). A twofold increase in amplitude is due to the combined contribution from the incident and reflected wave fields.

By determining in a similar way the field of the Alfvén portion of the variations at ionospheric heights, we find

$$E_r(h) = \frac{2\tilde{E}_r(h)}{1 + i(k_0^2/k_z\sqrt{k^2-k_0^2})\coth(\sqrt{k^2-k_0^2}h)}, \tag{18}$$

where $\tilde{E}_r$ is the amplitude of the incident Alfvén wave given by (9) with (13) taken into account. In order to determine the E_φ-component of the Alfvén portion of waveguide modes, we solve equation (12), with (18) appearing on the right-hand side. Using Green's function we seek the solution which has the form

$$E_\varphi^{(A)} = Ce^{kz} + Be^{-kz} + E_r(h)\frac{k_0^2}{2k}\int_{-\infty}^{\infty} e^{-k|z-z'|}\varepsilon_H(z')\,dz'.$$

In the magnetosphere, a non-propagating FMS-variation of the form $\exp(-kz)$ corresponds to this solution. Upon combining with the solution of magnetosphere and atmosphere, we obtain

$$G_\varphi^{(A)} = \frac{k_0^2\tilde{E}_r(h)}{k\sinh(\sqrt{k^2-k_0^2}h) + \sqrt{k^2-k_0^2}\cosh(\sqrt{k^2-k_0^2}h)} \times \int_h^{h+\Delta} e^{-(z-h)}\varepsilon_H(z)\,dz. \tag{19}$$

It is apparent from (4) that for $\tilde{E}_r$, the characteristic scale of localization in k-space is $\bar{k} = \sqrt{k_z/a}/\kappa$, where $\kappa = u^{1/2}\beta^{1/4} \sim 10^{-2}$, i.e. inequality (16) changes to $k_0 \ll h^{-1} \ll \bar{k}$. Then, on performing inverse transformation (10), we have

$$B_r^{(A)}(r) \approx (-1)^n \frac{C_n a_n}{k_z h}\left(\frac{a\kappa^2 k_0^2}{k_z}\right)^{3/2} E_H \frac{(r/h)}{[1+(r/h)^2]^{3/2}} \tag{20}$$

where

$$E_H = \frac{1}{h}\int_h^{h+\Delta} \varepsilon_H(z)\,dz.$$

Hence, it can be seen that the localization of the Alfvén part of a waveguide mode during its passage towards the Earth varies from $\bar{r} = \sqrt{a/k_z}/\kappa$ to $\bar{r} \sim h$. Also, the spatial structure of the waveguide modes is smoothed out and is independent of number m. So, the amplitude profile of a geomagnetic pulsation penetrating to the Earth from the magnetospheric duct is independent of its composition and is universal.

The general variation of the magnetic field on the ground produced by the waveguide modes (8) and (9) incident on the ionosphere, is determined by a superposition of the contributions from the Alfvénic (20) and FMS (17) parts of these modes. A characteristic form of the spatial distribution of the amplitude is given in Fig. 1c showing two coaxial "cirques", the outer of which has a crest when $r \sim \sqrt{a/k_z}$ and the inner having a crest when $r \sim h$. The polarization of such pulsations is nearly linear and is directed toward the center of the spot produced by the duct cross-section projected on to the Earth.

4. DISCUSSION

It should be noted that the resonace interaction of waveguide Alfvén and FMS modes studied in this paper can be one of the most important sources of

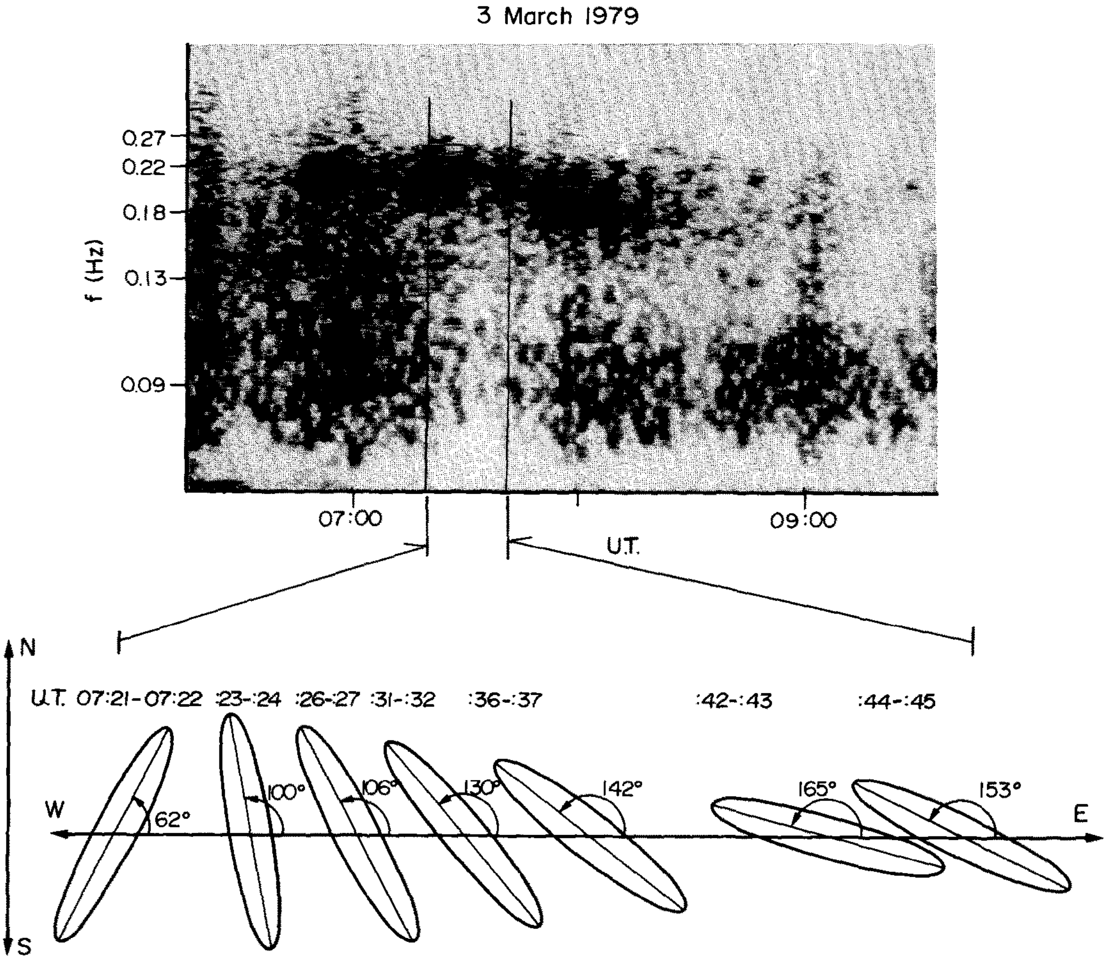

FIG. 2. THE CHANGE OF POLARIZATION ELLIPSES OF LOCAL Pc1–2 PULSATIONS AS OBSERVED AT THE STERLEGOVO STATION ($\phi = 69°$N, 3 MARCH 1979, 07:22–08:09 U.T.).

small-scale Alfvén waves in the magnetosphere. The mechanism of ion-cyclotron instability (the most important source of Pc1 pulsations) is ineffective for small-scale Alfvén waves because the maximum of its increment is in the range $k_\perp < \sqrt{uk_\parallel} \ll k_\parallel$, and the opposite inequality $k_\perp \gg k_\parallel$ is typical of the waves under study. These waves do indeed exist in the magnetosphere and are associated with regions of enhanced plasma density which stretch along the geomagnetic field (Dubinin *et al.*, 1985). We do not yet know of any other sources for these waves (nor for the waveguide modes under study).

The common feature of the waveguide geomagnetic pulsations is the essential stretching of their polarization ellipse towards the center of the "illumination spot". We must note that this result depends strongly on the magnetospheric-duct model. If the duct is not an ideal cylinder, then the ratio of major to minor axes of the polarization ellipse will be different. Nevertheless, it might be anticipated that there will remain the general tendency for the major axis to be directed across the duct's projection on to the ground.

At this point we should mention a paper of Hayashi *et al.* (1981), who investigated experimentally the amplitude and polarization characteristics for local Pc1 pulsations from the Canadian network of stations. In particular, the conclusion drawn in that paper is that the major axis of the polarization ellipses is directed across the isolines of the amplitude profile of the pulsations. A common feature of these pulsations is that the hodograph's rotation changes from left to right with increasing distance from the center of the source.

This phenomenon in the paper just cited is interpreted within the framework of theoretical models (Altman and Fijalkov, 1968; Greifinger, 1968). The geomagnetic pulsations incident on the ionosphere have left-hand polarization and excite oscillations with a right-hand polarization at the location of incidence. The left-hand polarization predominates in the neighbouring zone. Right-hand polarizations run away through the MHD-waveguide in the ionospheric $F2$-region and predominate in a distant zone.

Within the context of the waveguide propagation theory of geomagnetic pulsations under consideration this phenomenon can be explained from a different point of view. Waveguide modes contain both Alfvén waves with left-hand polarization and FMS-modes with right-hand polarization. The former penetrating to the Earth's surface give the inner crest of the amplitude profile and the latter give the outer crest.

It may be expected that due to the distortion of the duct's cross-section from an ideal circle, the polarization ellipse is not so stretched as predicted by our model, and the direction of the hodograph's rotation corresponds to the related Alfvén and FMS modes. Thus, the left-hand polarization can be observed on the inner crest of the profile, while the right-hand polarization can be registered on the outer crest, in agreement with the observations.

The scale of the polarization transition region is in agreement with observations as well. The radius of the inner crest is 100 km from the center and that of the outer crest is of the order of a few hundred kilometers. But data reported by Hayashi *et al.* (1981) do not indicate the presence of crests in the amplitude profile, which is natural because the station network in their study is not sufficient to resolve the fine profile structure. Another interesting feature of the pulsations under study is associated with the motion of the duct's end along the Earth's surface due to the Earth's rotation. In this case, rotation of the polarization ellipse that indicates the motion of the "illumination spot" on the ground from East to West can be registered.

To verify this result experimentally, 68 time intervals of Pc1–2 pulsations have been selected from observations obtained at a meridional chain of stations during the winter months of 1976, 1979 and 1982 (Eruschchenkov *et al.*, 1977). The average statistical ellipses of Pc1–2 polarization, as inferred at 1 min intervals using observations from the Sterlegovo station ($\phi = 69°$N, 3 March 1979, 07:22–08:09 U.T.) are presented in Fig. 2. The study shows that the polarization of the pulsation is an elliptic one and the rotation of the polarization ellipses corresponds to the motion of the source from East to West. Similar regularities are observed in other cases as well.

The same typical features of the Pc1–2 polarization were obtained at a dense network of stations by Inhester *et al.* (1984). According to that paper, a number of instantaneous meridional sections of the pulsation amplitude profile may be constructed. A typical feature of such sections is the presence of one maximum or more. A distribution with one maximum is commonly observed at the onset of pulsations and the observation begins, first, at the station near the center of the source, spreading then to the periphery.

This may be explained in terms of our theory. As is shown in Fig. 1, when the "illumination spot" approaches the meridional chain of stations, it must be registered at first by stations nearest to the center of the spot and later by stations in the periphery. Thus, one-maximum distributions can be observed just at the beginning of the observation run and, then, the number of maxima may increase to four. The observations mentioned above all provide indirect support for our theory.

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