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ALFVÉN WAVES: TO THE 80th ANNIVERSARY OF DISCOVERY

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Abstract. The article is dedicated to the anniversary of the discovery of Alfvén waves. The concept of Alfvén waves has played an outstanding role in the formation and development of cosmic electrodynamics. A distinctive feature of Alfvén waves is that at each point in space the group velocity vector and the external magnetic field vector are collinear to each other. As a result, Alfvén waves can carry momentum, energy, and information over long distances. We briefly describe two Alfvén resonators, one of which is formed in the ionosphere, and the second presumably exists in Earth's radiation belt. The existence of the ionospheric resonator is justified theoretically and confirmed by numerous observations. The second resonator is located between reflection points located high above Earth symmetrically with respect to the plane of the geomagnetic equator.

Keywords: Alfvén velocity, dispersion law, group velocity, geometrical optics, heavy ions.

Waves with the dispersion law $\omega = c_A k_\parallel$ are known as Alfvén waves in honor of Hannes Alfvén, who discovered them 80 years ago [Alfvén, 1942]. Here, ω is the wave frequency; $k_\parallel = \mathbf{kB}/|\mathbf{B}|$; $\mathbf{k}$ is the wave vector; $\mathbf{B}$ is the external magnetic field; $c_A = B/\sqrt{4\pi\rho}$ is the Alfvén velocity; ρ is the plasma density. The concept of Alfvén waves has played a significant role in the formation and development of cosmic electrodynamics [Alfvén, 1952]. It is impossible to cite here even selectively the prodigious amount of literature on Alfvén waves. In honor of the outstanding discovery, we only briefly describe two Alfvén resonators, one of which is formed in the ionosphere, and the second presumably exists in Earth's radiation belt.

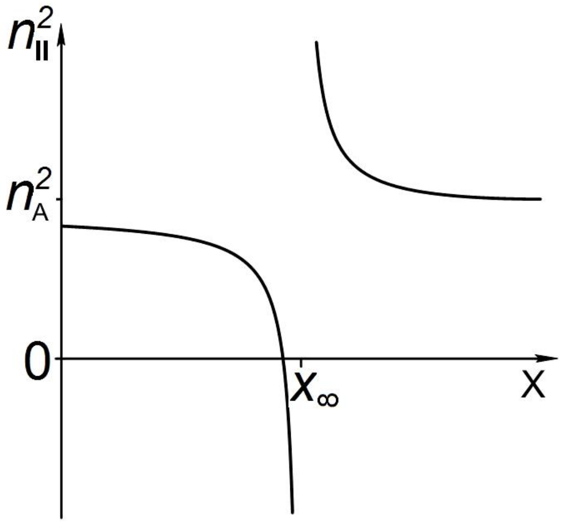
A distinctive feature of Alfvén waves is that the vector $\mathbf{B}$ and the group velocity vector $\mathbf{v} = d\omega/d\mathbf{k}$ are collinear: $\mathbf{v} = \pm\mathbf{B}/\sqrt{4\pi\rho}$. The signs + and − correspond to the inequalities $\mathbf{kB}>0$ and $\mathbf{kB}<0$. Surprising consequences follow from the collinearity. Alfvén waves can carry momentum, energy, and information over long distances. Recent observations suggest that quasi-monochromatic oscillations with a carrier frequency of 3.3 MHz are transferred by Alfvén waves over 150 million km, from the solar surface to the Earth surface [Guglielmi et al., 2015; Guglielmi, Potapov, 2021].

Another consequence of the collinearity is directly related to Alfvén resonators. In the geometrical optics approximation, an Alfvén wave in an inhomogeneous medium experiences refraction, but does not lose its directivity properties. In other words, if, for example, $\mathbf{kB}>0$, this inequality can change to the opposite only if conditions for the applicability of the geometrical optics approximation are violated. Suppose that in the Northern Hemisphere in the F2 layer the geometrical optics works and $\mathbf{kB}>0$. The wave propagates downward and reaches a layered structure consisting of the lower ionospheric layers, the atmosphere, and the upper lithosphere, whose thickness is of the order of skin depth. Obviously, in these layers there is a sharp violation of the conditions for the applicability of the geometrical optics. An upward reflected Alfvén wave ($\mathbf{kB}<0$) appears. It crosses the F2 layer and penetrates into the exosphere. Here, the Alfvén velocity $c_A = B/\sqrt{4\pi\rho}$ rapidly increases due to a rapid decrease in the plasma density ρ with height. Conditions of the geometrical optics approximation break down again, and a downward wave appears. It can readily be understood that the standing Alfvén waves which have a harmonic spectrum structure and are trapped in the ionospheric resonator are formed in this way. The physico-mathematical calculation and observation of geoelectromagnetic ULF oscillations in the Pc1 range (0.2–5 Hz) clearly demonstrate the existence of the ionospheric Alfvén resonator [Belyaev, Polyakov, 1980; Potapov et al., 2014]. It is interesting that the resonator is characterized by the distribution of spectral lines over odd harmonics [Potapov et al., 2022].

To imagine the structure of the second resonator located high above Earth in the near-equatorial region of the radiation belt, we will make a brief excursion into crystal optics [Landau, Lifshits, 2003]. Consider a strongly anisotropic uniaxial crystal and bring the permittivity tensor to the diagonal form. It follows from the Fresnel equation that the dispersion equation for extraordinary wave is similar to the dispersion equation for Alfvén wave [Guglielmi, 1979]. This analogy suggests the following question: how will the dispersion equation change if we take into account the non-diagonal terms in the permittivity tensor, but leaving the tensor to be Hermitian? In other words, we want to impart gyrotropic properties to the medium, and we immediately indicate how this can be done in hydrogen magnetoactive plasma. It turns out to be enough to imagine that hydrogen plasma contains a small admixture of heavy ions, for example, oxygen ions O$^+$ [Gintsburg, 1963].

We use this general information to analyze Alfvén waves in the outer radiation belt in a small neighborhood of the geomagnetic equator. Figure shows the dispersion curve in a hydrogen plasma containing a very small admixture of oxygen ions. The wave frequency is assumed to be close to the gyrofrequency of O$^+$ ions in this region. Along the horizontal axis is the distance along the geomagnetic field line. The origin of the coordinates coincides with the point of intersection of the field line with the equator. Along the X-axis, the magnetic field strength and hence the ion gyrofrequency change. Along the vertical axis is the squared longitudinal refractive index $n_\parallel = ck_\parallel / \omega$. It almost coincides with the squared refractive index of Alfvén waves $n_\mathrm{A} = c / c_\mathrm{A}$ everywhere except for the narrow band in the vicinity of the resonance in which the local gyrofrequency of O$^+$ ions matches the wave frequency. Notice that a little to the left of the resonance is a turning point $\left(n_\parallel = 0\right)$ in the vicinity of which approximation of the geometrical optics is invalid. So, the wave running from left to right is reflected from the turning point. The reflected wave propagates from right to left, crosses the equator, and is reflected from a completely similar turning point located in the opposite hemisphere of the magnetosphere. The process repeats and the resonator is formed [Guglielmi et al., 2000] (see also [Guglielmi, Potapov, 2012, 2021; Mikhailova, 2017]). Thus, the theory predicts that between the reflection points located high above Earth symmetrically with respect to the plane of the geomagnetic equator there is a subequatorial Alfvén resonator.

In conclusion, we should say that the Alfvén theory has radically enriched not only linear, but also nonlinear space physics. In particular, satellite observations provide convincing evidence that Alfvén waves have a significant effect on plasma in Earth's magnetosphere due to ponderomotive forces [Lundin, Guglielmi, 2006].



Dispersion curve in the vicinity of the equator of the geomagnetic field line (see the text)

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