

## A method for restoration of radio channel transfer function by chirp sounding of the ionosphere

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[1] Analysis of both in-phase and quadrature components of received frequency-modulated continuous wave (FMCW) signal at the output of intermediate frequency (IF) band-pass filter of the chirp ionosonde demonstrates a new possibility of recovering the transfer function of the HF radio channel within the band of operating frequencies and the interval of time delays that are defined by the IF filter characteristics. The transfer function is determined by the parameters of all parts comprising the radio channel: transmitter, receiving-transmitting antenna-feeder devices, Earth surface, ionosphere, as well as the receiver circuitry participating in the signal processing. However, characteristics of all parts except the ionosphere can be considered stationary and well-known or controlled. It is the ionosphere that displays significant temporal variability. A new technique is proposed for recovering the transfer function of the ionospheric radio channel, together with its implementation as a signal preprocessing circuit containing a correcting digital filter that adapts to the current amplitude frequency and phase frequency characteristics of the channel. *INDEX TERMS*: 2441 Ionosphere: Ionospheric storms; 2407 Ionosphere: Auroral ionosphere; 2447 Ionosphere: Modeling and forecasting; *KEYWORDS*: FMCW sounder, chirp sounding technique, ionospheric radio channel, transfer function, digital signal processing.

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### 1. Introduction

[2] The ionosphere plays an important role in the physical processes of the solar-terrestrial environment. The ionospheric characteristics are primarily controlled by the solar radiation that is characterized by various scales of time variability. Generally, during the periods of high solar activity the ionization level is increased and visa versa. At dawn and dusk time in the ionosphere strong gradients of the electron concentration are developed. The solar bursts generate ionosphere perturbations on a variety of spatial-temporal scales.

[3] Temporal variability of the ionosphere can be caused by other physical processes in the solar-terrestrial environment. For example, magnetic storms can trigger the auroral and midlatitude ionospheric and magnetospheric substorms with consequent auroral effects in the high-latitude ionosphere. The variability of the ionospheric conditions can

be considered as a viable indicator of the active processes operating in the solar-terrestrial environment.

[4] The frequency-modulated continuous wave (FMCW) chirp sounder, originally introduced by *Barry and Fenwick* [1965], still remains among modern techniques and instrumentation for ionospheric research. It is characterized by a broad bandwidth of the sounding signal, low power, and high-resolution and noise immunity. Over the past two decades the chirp sounding technique undergoes an actively modernization with the help of new digital signal processing concepts for waveform generation, digitization, and processing of the chirp signal [*Ivanov et al.*, 2003a; *Salous*, 1992, 1997; *Salous and Shearman*, 1986]. The modern FMCW sounder has become a digital radiophysical observatory for sophisticated diagnostics of the HF radio channels and ionospheric monitoring. *Ivanov et al.* [2003b] have shown that digital methods applied to the chirp signal processing considerably expand its ability to compensate signal distortion in the dispersive ionospheric channels and to correct the phase frequency characteristic (PFC) of the channel, thus improving the group delay resolution of the FMCW sounder. This paper presents a technique that analysis the chirp signal at the output of the IF filter to reconstruct the channel transfer function of the ionospheric path.

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## 2. Window Processing of the FMCW Signal

[5] The basic concept of the FMCW sounder operations is known for decades. To date, several approximate theoretical methods have been developed to model the FMCW sounder operations [Gurgel *et al.*, 2006; Lundborg and Lundgren, 1992; Philipp *et al.*, 1991; Salous, 1997]. One of the approaches, as discussed by Ilyin *et al.* [1996] and further developed by Davydenko *et al.* [2002], bases its methodology in systematic formal examination of the impact produced by the ionospheric radio channel and the sounder receiver circuitry on the sounding FMCW signal. We briefly describe the major points of the method below.

[6] The transmitted FMCW signal with constant unit amplitude can be represented as:

$$e_{\text{tr}}(t) = \begin{cases} \sin \chi t^2 & t > 0 \\ 0 & t < 0 \end{cases} \quad (1)$$

where  $\chi$  is the chirp sweep rate. Assuming that the characteristics of the ionospheric radio channel are linear, signal at the input of the receiver may be written as a convolution  $e_{\text{tr}}(t)$  with the impulse response function of the radio channel  $h(\tau)$ :

$$e_{\text{rec}}(t) = \int_0^{\infty} h(\tau) e_{\text{tr}}(t - \tau) d\tau \quad (2)$$

In the chirp receiver, signal  $e_{\text{rec}}(t)$  is mixed with the replica of transmitted signal  $e_{\text{tr}}(t)$ , and passed through a low-pass filter. We assume in further discussion that the filter has ideal characteristics. The low-frequency signal is sampled within a particular time window  $w(t)$  placed at times  $t_k$ . As the window slides with time, the samples are fed to the spectrum analyzer to obtain set of output spectra  $S_k^+(F)$ , corresponding to the positive values of frequency  $F$ . The set of spectra  $S_k^+(F)$  is the result of one FMCW chirp sounder measurement session:

$$S_k^+(F) = \frac{1}{4} e^{iFt_k} \int_0^{\infty} h(\tau) W(F - 2\chi\tau) e^{i(\chi\tau^2 - 2\chi t_k\tau)} d\tau \quad (3)$$

Here  $W$  is the spectrum of the window  $w(t)$ . Ilyin *et al.* [1996] showed that processing of the individual time domain samples of the received chirp signal is equivalent, from the mathematical point of view, to analysis of the complex narrow-band pulse signals propagating the radio channel at the group delays determined by the peak positions in the observed spectrum  $S_k^+(F)$ . On the basis of this representation of the chirp ionosonde signal, Mikhaylov [2001] proposed a methodology for reconstructing the transfer function of the ionospheric radio channel in the  $E$  layer's region of semi-transparency from the near-vertical FMCW sounding spectra. Davydenko *et al.* [2002] obtained an expression for the dispersive distortion for individual observed spectra depending on dispersion of the group time delay, time window

length, and FMCW sweep rate. Calculated widths of the spectral lines due to dispersion are consistent with the experimental data obtained at the FMCW chirp sounder [Brynko *et al.*, 1988] of Institute of Solar-Terrestrial Physics, Siberian Branch, Russian Academy of Sciences (ISTP SB RAS).

## 3. Modeling of the Digitized FMCW Signal at Intermediate Frequency

[7] The proposed technique for reconstructing transfer function of the ionospheric radio channel is based on a possibility of digitizing FMCW signal at the intermediate frequency (IF). Let us analyze the IF signal shape assuming known band-pass filter properties.

[8] Received chirp signal is mixed with the signal of the reference oscillator  $e_0(t)$ :

$$e_0(t) = \sin(\omega_i t + \chi t^2) \quad (4)$$

where  $\omega_i$  is the intermediate frequency. The beat tones are transferred to the IF amplifier that amplifies the low-frequency part and cuts off the high-frequency part by a band-pass filter with the impulse response function  $f(t)$ . Without limiting generality of our reasoning, we will assume that the IF amplifier's gain is 1. Then signal at the band filter output  $e_i(t)$  has the following form:

$$e_i(t) = \frac{1}{2} \int_0^{\infty} f(\xi) \int_{-\infty}^{\infty} h(\tau) \cos[\omega_i(t - \xi) + 2\chi\tau(t - \xi) - \chi\tau^2] d\xi d\tau \quad (5)$$

The signal  $e_i(t)$  can be split into its in-phase and quadrature components  $e_{ai}(t)$  and  $e_{bi}(t)$  by multiplying it by  $\cos(\omega_i t)$  and  $\sin(\omega_i t)$  in two different channels while simultaneously suppressing the high-frequency components:

$$e_{ai}(t) = \frac{1}{4} \int_0^{\infty} f(\xi) \int_{-\infty}^{\infty} h(\tau) \cos[\omega_i \xi - 2\chi\tau(t - \xi) + \chi\tau^2] d\xi d\tau \quad (6)$$

$$e_{bi}(t) = \frac{1}{4} \int_0^{\infty} f(\xi) \int_{-\infty}^{\infty} h(\tau) \sin[\omega_i \xi - 2\chi\tau(t - \xi) + \chi\tau^2] d\xi d\tau$$

By the causality principle,  $f(t) = 0$  when  $t < 0$  and therefore the lower limit of integrals can be extended down to  $-\infty$ .

[9] In the conventional approach to the chirp waveform processing the signal is demodulated to the low-frequency band, resulting in a single output signal that is a linear combination of both quadrature components. However, as it will become clear from the following discussion, the proposed method for reconstructing the channel transfer function requires both quadrature components to be acquired

individually. For convenience of further analysis, we express the signal in the complex form:

$$\tilde{e}_i(t) = e_{ai}(t) - ie_{bi}(t) \quad (7)$$

Expressions (6) and (7) describe signal at the intermediate frequency. Substituting (6) into (7) and interchange of integrations yields:

$$\tilde{e}_i(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} h(\tau) F(-\omega_i - 2\chi\tau) e^{-i(\chi\tau^2 - 2\chi\tau t)} d\tau \quad (8)$$

Here  $F(\omega)$  is the transfer function of band-pass filter at the intermediate frequency.

[10] By changing the integration variable in (8) and turning to  $\omega = 2\chi\tau$ , we can conclude that (8) is structurally analogous to a mathematical description of signal with spectrum  $h(\omega/2\chi)$  passing through a band-pass filter with transfer function

$$\tilde{F}(\omega) = F(-\omega_i - \omega) \exp(-i\omega^2/4\chi)$$

Or, in other words, signal (8) is the response of the band-pass filter with transfer function  $\tilde{F}(\omega)$  to a signal with spectrum  $h(\omega/2\chi)$ . Dispersive properties of this filter considerably differ from the properties of the typical band-pass filter installed in a chirp sounder.

#### 4. Method of Reconstructing Channel Transfer Function Using FMCW Ionospheric Sounding

[11] Modern digital signals processing techniques can be used to build a corrective filter that can reverse the dispersive signal distortion in the radio propagation channel by modifying the transfer function  $\tilde{F}(\omega)$  [e.g., *Ivanov et al.*, 2003a, 2003b]. To that end, the circuitry for preprocessing of the received signal is proposed to include an additional corrective filter with a transfer function  $F_{\text{cor}}(\omega)$  that is related to the transfer function  $F(\omega)$  of the band-pass filter, within its band  $[\omega_i, \omega_i + \Omega]$ , by the following expression:

$$F(\omega)F_{\text{cor}}(\omega) = \begin{cases} e^{i\frac{(\omega_i + \omega)^2}{4\chi}} & \omega \in [\omega_i, \omega_i + \Omega] \\ 0 & \omega \notin [\omega_i, \omega_i + \Omega] \end{cases} \quad (9)$$

Correcting filter with such characteristics can be implemented in the digital form. Then the signal passed sequentially through the band-pass and correcting filters is equivalent to the signal passed through an "effective" filter whose transfer function is the product of two transfer functions in (9). The resulting output signal can then be written as:

$$\tilde{e}_{\text{cor}}(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} h(\tau) F(-\omega_i - 2\chi\tau) \times$$

$$F_{\text{cor}}(-\omega_i - 2\chi\tau) e^{-i(\chi\tau^2 - 2\chi\tau t)} d\tau \quad (10)$$

Representing the impulse response function of the ionospheric channel as a sum  $h = h_1(\tau) + h_2(\tau)$ , where  $h_1(\tau)$  is not zero for  $\tau = [0, \tau_1 = \Omega/2\chi]$  and zero for  $\tau > \tau_1$ , and  $h_2(\tau)$  is not zero only for  $\tau > \tau_1$ , we obtain

$$\tilde{e}_{\text{cor}}(t) = \frac{\pi}{2} \int_{-\infty}^{\infty} h_1(\tau) e^{i2\chi t\tau} d\tau \quad (11)$$

Expression (11) is a Fourier transform of the impulse function of the ionospheric channel  $h_1(\tau)$ , which means that its transfer function is equal to

$$\tilde{e}_{\text{cor}}(t) = \pi^2 H(2\chi t) \quad (12)$$

The expression obtained above signifies the following. Adding the corrective filter with characteristics described by equation (9) to preprocessing circuitry of the chirp ionosonde at its intermediate frequency produces signal  $\tilde{e}_{\text{cor}}(t)$  that is uniquely and directly determined by the part of the ionospheric channel's transfer function responsible for the group time delays from 0 to  $\tau_1$ . To determine the transfer function for the ionosphere's response corresponding to delays larger than  $\tau_1$ , the start time of reference oscillator must be shifted up, similar to what is done in order to observe, for example, round-the-globe signals.

[12] The restored transfer function reflects the current ionosphere state. Calculation of the ionospheric characteristics (the electron concentration profile  $N_e(h)$ , the peak density  $N_{em}(h)$ ) by means of the transfer function is the inverse problem that is not considered here. The results of continuous chirp sounding séances allow us to investigate variability of the ionosphere and its relation with the physical processes operating in the solar-terrestrial environment.

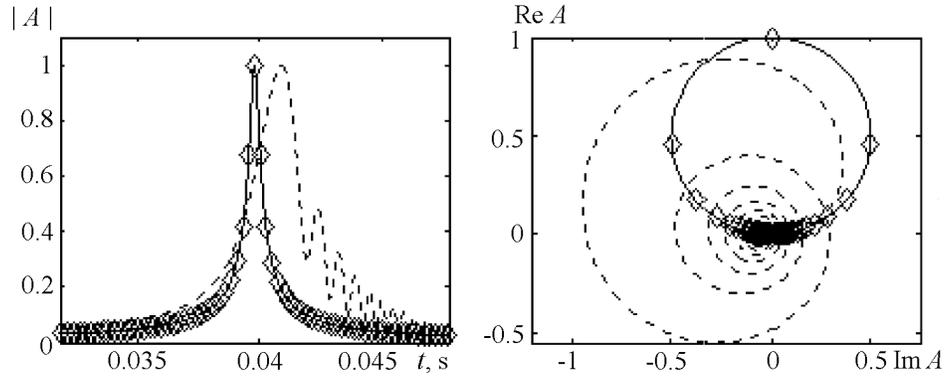
[13] An addition of the correcting filter does not limit the usual capabilities of the chirp ionosonde in its operating modes. The reconstruction of the ionospheric channel transfer function can be done prior to the typical chirp waveform processing techniques, in particular those requiring the windowing operation applied to the received signal. The chirp sounder measurement interval of group time delays in propagation modes  $[0, \tau_1]$  remains unchanged by corrective filtering (as determined by the pass filter bandwidth at the intermediate frequency and the chirp sweep rate).

#### 5. Method Validation

[14] The proposed method for reconstructing the transfer function of the ionospheric radio channel using digitized FMCW signal at the intermediate frequency has been validated in two ways.

[15] First, we modeled the FMCW chirp ionosonde operations in its ionospheric sounding mode assuming the perfect mirror reflection:

$$h(\tau) = \delta(\tau - \tau_m) \quad H(\omega) = \frac{1}{2\pi} e^{i\omega\tau_m} \quad (13)$$



**Figure 1.** Signals' envelopes (left) and phase diagrams (right) after the band (dashed lines) and correcting (solid lines) filters. Diamonds mark the values of the given transfer function characteristics (16).

where  $\delta$  is a delta function. Then the received FMCW signal is

$$e_{\text{rec}}(t) = e_{\text{tr}}(t - \tau_m) \quad (14)$$

and after completing all processing stages in the receiver we obtain:

$$\tilde{e}_{\text{cor}}(t) = \frac{\pi}{2} e^{i2\chi t \tau_m} \quad (15)$$

Substituting the value of transfer function (13) into (12) provides the expression identical to (15).

[16] Second, we conducted a theoretical numerical simulation of the transfer function reconstruction in a resonant circuit with known characteristics. An RC resonant circuit was arbitrary chosen with the transfer function in the following form:

$$H(\omega) = ((1 - \omega^2 LC) + i\omega RC)^{-1} \quad (16)$$

Amplitude frequency characteristics of the band-pass and correcting filters were chosen to be rectangular. Figure 1 shows the envelopes  $|A|$ , where  $|A| = u(t)/|u_{\text{max}}(t)|$  and the signal phase diagrams at the output of the band-pass filter (dashed lines) and the correcting filter (solid lines). The phase diagrams are plotted as  $\text{Re}(A)$  versus  $\text{Im}(A)$ . The exact transfer function for the selected resonant circuit (16) is labeled with the diamond symbols. Good agreement is observed between the original transfer function and the output of the correcting filter.

## 6. Conclusion

[17] Analysis of both in-phase and quadrature components of the chirp signal at the intermediate frequency of the FMCW ionosonde receiver shows possibility to reconstruct the part of the ionospheric radio channel transfer function that determines the group time delays of interest. To obtain the transfer function, a digital correcting filter can be implemented as an additional preprocessing circuit. The described

method is applicable in any of the FMCW chirp sounder operating modes such as the vertical, oblique and backscatter sounding. At the same time, the method does not require other changes of the receiver, and conventional windowing operations can still be applied to the reconstructed transfer function to produce standard ionograms. The expressions obtained for the received chirp signal can be used to consider alternative methods to analyze “frequency-time” variables in order to evaluate the dependence of group time delays on operating frequency, such as, for example, the wavelet analysis. These methods shall determine the delay and frequency resolution.

[18] Practical implementation of the proposed techniques in the chirp sounding requires analysis of the noise characteristics that will affect the outcome of preprocessing in the correcting filter. We will target this topic in our future research.

[19] One FMCW ionosonde measurement under conventional signal processing, produces the frequency dependence of the group delay, that is the ionogram. The practical usage of proposed technique allows us to reconstruct the part of the ionospheric radio channel transfer function that directly characterizes the state of ionosphere. Further investigations can be expected in this region, e.g., the solution of the inverse problem for the restoration of the ionospheric characteristic parameters from the derived transfer function, or the exploration of the hidden associations between the temporal variability of the ionosphere and the physical processes in the solar-terrestrial environment.

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