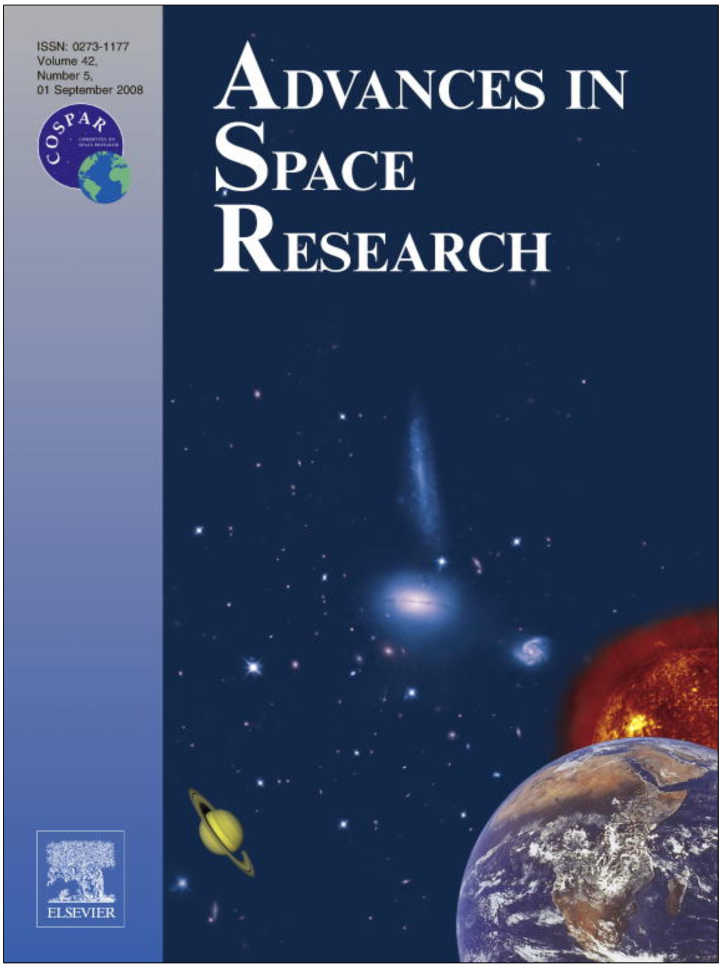
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Standing Alfvén waves with $m \gg 1$ in a dipole magnetosphere with moving plasma and aurorae

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Abstract

The structure of standing Alfvén waves with large azimuthal wave numbers ($m \gg 1$) is studied in a dipole model of the magnetosphere with rotating plasma. In the direction across magnetic shells the structure of such waves is determined by their dispersion associated with curvature of geomagnetic field lines and corresponds to the travelling wave localized between toroidal and poloidal resonant surfaces. In projection into the ionosphere (along geomagnetic field lines) this structure is similar to the structure of a discrete auroral arc. The azimuthal structure of an auroral arc is similar to azimuthal structure of Alfvén waves with $m \sim 100$. Possible interaction mechanisms between the Alfvén waves and energetic electron fluxes forming auroral arcs are discussed.

Keywords: Magnetosphere; Alfvén waves; Aurora

1. Introduction

The best known of magnetospheric MHD-oscillations are the resonant Alfvén waves excited by the mechanism of field line resonance (Tamao, 1965; Southwood, 1974; Chen and Hasegawa, 1974). The monochromatic fast magnetosonic wave penetrating inside the magnetosphere from outside excites the Alfvén oscillations on the resonant magnetic shell where their local frequency coincides with the frequency of the magnetosonic wave. Such mechanism can be effective enough only for the large-scale (in the azimuthal direction) oscillations. Arbitrary oscillation in an axisymmetrical magnetosphere, for example in the model with dipole magnetic field, may be represented as a sum of azimuthal harmonics of the form $\exp(\mathrm{i}m\phi)$, where m is the azimuthal wave number, ϕ is the azimuthal angle. Oscillations excited in field line resonance have $m \sim 1$. The magnetosonic oscillations with $m \sim 1$ are able to generate the resonant Alfvén waves because it can penetrate inside the magnetosphere with rather large amplitude.

Azimuthally small-scale ($m \gg 1$) fast magnetosonic oscillations practically are unable to penetrate inside the magnetosphere and could not be source of the Alfvén waves in the magnetosphere. Alfvén oscillations with $m \gg 1$ can be excited inside the magnetosphere by the external currents located on the same resonant shells (Leonovich and Mazur, 1993). For example, it can be external currents in the conductive layer of the ionosphere. Alfvén oscillations in a magnetosphere are known to enjoy two types of polarization (Dungey, 1954; Radoski, 1967; Radoski and Carovillano, 1969). Oscillations with $m = 0$ are called toroidally polarized, while those with $m \to \infty$ are termed poloidally polarized oscillations. In toroidal Alfvén oscillations, the disturbed magnetic field and plasma oscillate azimuthally, while the electric field oscillates radially – across magnetic shells. In poloidal oscillations, on the

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contrary, the magnetic field and plasma oscillate radially, while the electric field oscillates azimuthally.

Each geomagnetic field line crosses the Earth's ionosphere in both northern and southern hemispheres. Alfvén oscillations, propagating chiefly along these field lines, form standing waves between the magnetoconjugated ionospheres (Radoski, 1967; Radoski and Carovillano, 1969). The polarization of standing Alfvén waves with $m \sim 1$ generated by the field line resonance mechanism is almost toroidal. The structure of standing Alfvén waves with $m \gg 1$ is more complicated. They are excited by a monochromatic source as nearly poloidal oscillations on the magnetic shell where the source frequency coincides with a poloidal eigenfrequency of the Alfvén oscillations. Later on, they slowly (much slower than the Alfvén velocity) move across magnetic shells to the shell where the toroidal eigenfrequency of Alfvén oscillations equals the source frequency. Alfvén oscillations are entirely absorbed in the neighbourhood of this shell because of their dissipation in the ionosphere. In the process of this displacement, standing Alfvén waves change their polarization from nearly poloidal to nearly toroidal (Leonovich and Mazur, 1993).

In all earlier papers on the subject, the structure and spectrum of standing Alfvén waves were determined by using magnetosphere models in which the plasma was assumed to be immobile. However, the actual magnetosphere is a dynamically equilibrium plasma configuration. In other words, motion is an inherent property of the magnetospheric plasma and undoubtedly plays an important role in the formation of the structure and spectrum of Alfvén waves in the magnetosphere. In the present paper, we make attempt to investigate how the motion of the magnetospheric plasma influences the spectrum and structure of standing Alfvén waves with $m \gg 1$ and its signature in discrete auroral arcs.

On the Earth Alfvén oscillations may be shown up as geomagnetic pulsations. The oscillations with $m \sim 1$ penetrate on the Earth's surface well enough and their structure can be studied on a ground net of magnetometers. Oscillations with $m \gg 1$ practically does not penetrate to the ground and their structure is rather difficult for study. However, from our point of view, there is at least one phenomenon where the structure of small-scale Alfvén waves can be visualized. It is a question relevant to the discrete auroral arcs. The comparison of the transverse structure and dynamics of azimuthally small-scale Alfvén waves with the structure and dynamics of discrete auroral arcs, as we will see, gives the basis to suggest that such connection probably exists.

The paper is structured as follows. In Section 2, we briefly describe the magnetosphere model and derive basic equations for the spatial structure and spectrum of standing Alfvén waves with $m \gg 1$ in the vicinities of the resonant surfaces. In Section 3, the equations for standing Alfvén waves near the toroidal and poloidal resonant surfaces are solved. Section 4 produces a solution for the model equation describing the structure of monochromatic standing Alfvén waves across magnetic shells in the entire region of existence. Section 5 discusses features of standing Alfvén wave structure in the magnetosphere with moving plasma and compares it to the structure and characteristic dynamics of discrete auroral arcs. Section 6 lists the principal results of this research.

2. Model medium and major equations

We employ the model of a magnetosphere with dipole-like magnetic field (Leonovich et al., 2004) and introduce the curvilinear coordinate system associated with magnetic field lines. The x^1 coordinate is directed across the magnetic shells, x^3 coordinate – along a field lines, and x^2 coordinate – in the azimuthal direction (see Fig. 1). Both plasma and magnetic field are assumed to be homogeneous along the azimuthal coordinate x^2. The motion of plasma in this model is simulated by its azimuthal rotation, v_ϕ is the velocity of azimuthal motion of the plasma.

To examine Alfvén oscillations in this model, we will use the system of ideal MHD equations:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{1}{4\pi}[\mathrm{curl}\,\mathbf{B} \times \mathbf{B}], \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}\,[\mathbf{v} \times \mathbf{B}], \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0, \tag{3}$$

$$\frac{d}{dt}\frac{P}{\rho^\gamma} = 0, \tag{4}$$

where $\mathbf{B}$ and $\mathbf{v}$ are magnetic field vector and plasma motion velocity, P and ρ are the plasma pressure and density, and γ is the adiabatic index. In (1) and (4), $d/dt = \partial/\partial t + (\mathbf{v}\nabla)$ represents the Lagrangian derivative in a moving plasma. In steady state ($\partial/\partial t = 0$), the set of Eqs. (1)–(4) describes the distribution of the equilibrium plasma parameters $\mathbf{B}_0$, $\mathbf{v}_0$, P_0 and ρ_0.

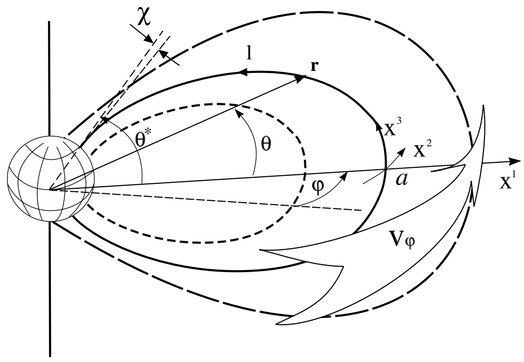

Fig. 1. A model of magnetosphere with a dipole magnetic field and azimuthally rotating plasma ($\mathbf{v} = (0, v_\phi, 0)$). Coordinate systems tied to magnetic field lines are shown: a curvilinear orthogonal system of coordinates (x^1, x^2, x^3) and a non-orthogonal system of coordinates (a, ϕ, θ) used in numerical calculations.

It is convenient to investigate Alfvén waves by expressing the components of their electromagnetic and velocity fields in terms of potentials. According to the Helmholtz decomposition theorem, any differentiable vector field can be represented as the sum of irrotational and solenoidal fields. We represent the perturbed electric field as

$$\mathbf{E} = -\nabla\varphi + \mathrm{curl}\,\boldsymbol{\Psi},$$

where φ is the scalar potential and $\boldsymbol{\Psi} = (\psi_1, \psi_2, \psi_3)$ is the vector potential. Obviously, the field is invariant with respect to adding an arbitrary constant to φ, so, without loss of generality, the constant can be set equal to zero. The field is also invariant with respect to adding the gradient of an arbitrary function to the vector potential, $\boldsymbol{\Psi} \to \boldsymbol{\Psi} + \nabla\chi$. Choosing the function $\nabla\chi$ to satisfy the equation $\psi_1 + \nabla_1\chi = 0$, we can write $\boldsymbol{\Psi} = (0,\xi,\psi)$, where $\xi = \psi_2 + \nabla_2\chi$ and $\psi = \psi_3 + \nabla_3\chi$. In the linear approximation, the components of the electric field are related to the components of the magnetic field and velocity field by the equation

$$\mathbf{E} = -\frac{1}{c}(\mathbf{v} \times \mathbf{B}_0 + \mathbf{v}_0 \times \mathbf{B}). \tag{5}$$

This equation, as well as Eqs. (1) and (2) allow us to express electromagnetic field and oscillation velocity components through potentials φ, ξ, ψ. Based on (3) and (4), one can also express the disturbed density $\tilde{\rho}$ and the pressure $\tilde{P}$ in terms of the potentials.

We will focus our interest on oscillations with $m \gg 1$ in the vicinity of resonant surfaces. We will search for the solution of the system Eq. (1)–(4) in the form of decomposition into harmonics of the form $\exp(-\mathrm{i}\omega t + \mathrm{i}k_2x^2)$, where ω is the source-induced oscillation frequency, k_2 is azimuthal wave number (if $x^2 = \phi$ is azimuthal angle, $k_2 \equiv m = 0, 1, 2, \ldots$). As further calculations will show, the potential φ has a singularity on these surfaces. The field of resonant Alfvén oscillations can be expressed through it:

$$\begin{aligned}
&E_1 = -\nabla_1\varphi, \quad E_2 = -\mathrm{Im}\varphi, \quad E_3 = \frac{m\Omega}{\bar{\omega}}\nabla_3\varphi,\\
&B_1 = \frac{mc}{\bar{\omega}}\frac{g_1}{\sqrt{g}}\nabla_3\varphi,\\
&B_2 = \mathrm{i}\frac{c}{\bar{\omega}}\frac{g_2}{\sqrt{g}}\nabla_3\left(\nabla_1 + m\frac{\Omega'}{\bar{\omega}}\right)\varphi, \quad B_3 = 0,\\
&v_1 = -\mathrm{i}\frac{m}{p}\frac{c}{B_0}\varphi, \quad v_2 = \frac{cp}{B_0}\nabla_1\varphi,\\
&v_3 \approx \mathrm{i}\frac{cp}{B_0}\frac{\Omega}{\bar{\omega}}(\nabla_3 \ln g_2)\nabla_1\varphi,
\end{aligned} \tag{6}$$

where $\bar{\omega} = \omega - m\Omega$, g_i $(i = 1, 2, 3)$ is metric tensor components, $p = \sqrt{g_2/g_1}$, $g = g_1g_2g_3$. Using the azimuthal angle ϕ as the azimuthal coordinate yields $v_{02} \equiv v_\phi = \sqrt{g_2}\Omega$, where Ω is the angular speed of the rotating plasma, $\Omega' \equiv \nabla_1\Omega$.

The Eq. (6) were derived using the equation

$$\frac{g_3}{\sqrt{g}}\nabla_1\xi = \nabla_3\left(\frac{\omega}{\bar{\omega}}\varphi - \mathrm{i}\frac{\Omega}{\bar{\omega}}\frac{g_2}{\sqrt{g}}\nabla_1\Psi\right),$$

obtaining from the third Eq. (5). For specific numerical calculations we used model of the magnetosphere with dipole magnetic field. Thus the system of coordinates (a, ϕ, θ) has been chosen, where the equatorial radius of a field line a is unambiguously associated with transverse coordinate x^1, the azimuthal angle ϕ – with coordinate x^2, and the role of longitudinal coordinate x^3 is played by the latitude θ (Fig. 1). Note that the coordinate system (a, ϕ, θ) differs from (x^1, x^2, x^3) in being non-orthogonal. However, its more convenient to employ for numerical calculations.

In this system of coordinates the intensity of a dipole magnetic field is

$$B_0(a,\theta) = \bar{B}\left(\frac{\bar{a}}{a}\right)^3\frac{\sqrt{1+3\sin^2\theta}}{\cos^6\theta},$$

where $\bar{a}$ – equatorial radius of a magnetic shell on which equatorial intensity of a magnetic field is $\bar{B}$, and the element of length along a field line is $\mathrm{d}l = a\cos\theta\sqrt{1+3\sin^2\theta}$. Components of metric tensor are

$$g_1 = \frac{\cos^6\theta}{1+3\sin^2\theta}, \qquad g_2 = a^2\cos^6\theta.$$

There is no similar simple expression for component g_3, however it can be determinate through the relation of the segments which are cutting off by two close located coordinate surfaces $x^3 = \text{const.}$ at the latitudes θ and θ_0 on two field lines with equatorial radiuses a and a_0 accordingly (Leonovich et al., 2004):

$$\frac{g_3(a,\theta)}{g_3(a_0,\theta_0)} = \left(\frac{a}{a_0}\right)^6\left(\frac{\cos\theta}{\cos\theta_0}\right)^{12}\frac{1+3\sin^2\theta_0}{1+3\sin^2\theta}.$$

The method of construction of an orthogonal coordinate system associated with curvilinear field lines of shear-free magnetic field is presented in Klimushkin et al. (1995). Proehl et al. (2002) propose a method of constructing a non-orthogonal coordinate system associated with curvilinear magnetic field lines. The work demonstrated this method as applied for investigating low-frequency resonant Alfvén oscillations in a model magnetosphere with dipole magnetic field. A similar investigation of the field line resonance mechanism in a dipole model magnetosphere was carried out in Leonovich and Mazur (1989). The same method was used in Lysak (2004) for studying higher-frequency Alfvén oscillations in the Pc1-geomagnetic pulsation frequency range. For these oscillations, details of both plasma and magnetic field distribution near the ionosphere are even more important than for low-frequency oscillations, so that application of the non-orthogonal coordinate system becomes especially convenient.

Note that a stationary system of coordinates associated with Earth witnesses the emergence of longitudinal (relative to background magnetic field) components of the electric field of oscillations $E_3 \neq 0$ and of the velocity field $v_3 \neq 0$. It is worth noting that this parallel electric field cannot accelerate charged particles of the magnetospheric plasma because it appear only in the coordinates system moving with background medium. But this effect can prove of importance in dynamics of the magnetospheric plasma ions and electrons and in their precipitation into the ionosphere.

The equation for monochromatic Alfvén waves may be formulated for a scalar potential φ. If an interaction with fast magnetosonic waves is negligibly small the resulting equation for transverse small-scale ($m \gg 1$) Alfvén waves has the form:

$$\nabla_1 \widehat{L}_T \nabla_1 \varphi - k_2^2 \widehat{L}_P \varphi + k_2 \frac{\nabla_1 \Omega}{\bar{\omega}} \nabla_1 \widehat{L}_{T0} \varphi = 0, \tag{7}$$

where

$$\widehat{L}_T = \widehat{L}_{T0} - \beta_T,$$
$$\widehat{L}_P = \widehat{L}_{P0} - \beta_P$$

are the toroidal and poloidal longitudinal operators,

$$\widehat{L}_{T_0} = \frac{1}{\sqrt{g_3}} \nabla_3 \frac{p}{\sqrt{g_3}} \nabla_3 + p \frac{\bar{\omega}^2}{A^2},$$
$$\widehat{L}_{P_0} = \frac{1}{\sqrt{g_3}} \nabla_3 \frac{p^{-1}}{\sqrt{g_3}} \nabla_3 + p^{-1} \frac{\bar{\omega}^2}{A^2}$$

are the zero approximation operators in a cold ($P_0 = 0$) stationary ($\Omega = 0$) plasma,

$$\beta_T = p \frac{\Omega^2}{A^2} \left(\frac{\nabla_3 g_2}{\sqrt{g_2 g_3}} \right)^2,$$
$$\beta_P = p \frac{\Omega^2}{2A^2} (\nabla_1 \ln g_2) \left(\nabla_1 \ln \frac{\rho_0 \sqrt{g_3}}{B_0} \right) + \frac{S^2}{A^2} \frac{\nabla_1 \ln \bar{\omega}/B_0}{\sqrt{g_1 g_2}} \left(\nabla_1 \ln \frac{P_0^{1/\gamma} \sqrt{g_3}}{B_0} \right)$$

are the corrections due to the plasma motion and finite pressure. Here $A = B_0/\sqrt{4\pi\rho_0}$ is Alfvén speed, and $S = \sqrt{\gamma P_0/\rho_0}$ is sound speed.

3. The parallel structure of Alfvén oscillations near the poloidal and toroidal resonant surfaces

As is shown in (Leonovich and Mazur, 1993), transverse small-scale Alfvén waves can be excited in the neighbourhood of a poloidal resonant surface only. The boundary condition for the potential φ has the form (Leonovich and Mazur, 1996):

$$\varphi|_{x_\pm^3} = \mp \frac{v_\pm}{\bar{\omega}} \frac{1}{\sqrt{g_3}} \frac{\partial \varphi}{\partial x^3}\bigg|_{x_\pm^3} - \frac{J_\parallel^\pm}{V_\pm} \tag{8}$$

on the ionosphere, where the notations are:

$$v_\pm = \frac{c^2 \cos \chi_\pm}{4\pi \Sigma_p^\pm}, \quad V_\pm = \frac{\Sigma_p^\pm}{\cos \chi_\pm}.$$

The signs $\pm$ refer to the northern and southern hemispheres, χ is the angle between the vertical and the field line at its intersection point with the ionosphere, Σ_p is the integrated Pedersen conductivity in the ionosphere, while the function $J_\parallel$ is related to the density $j_\parallel$ of external field-aligned currents in the topside ionosphere by

$$\Delta_\perp J_\parallel = j_\parallel,$$

where $\Delta_\perp = \nabla_1^2/g_1 - k_2^2/g_2$ is the transverse Laplacian. The right-hand terms in (8) will be assumed to be small, implying the values of the parameters $v_\pm$ and $1/V_\pm$ to be small too.

Since we assume Alfvén oscillations across magnetic shells to be small-scale, the characteristic wavelength on x^1 coordinate is much smaller than the longitudinal wavelength on x^3 coordinate. In this context, the solution to Eq. (7) can be sought in the form:

$$\varphi = U(x^1) H(x^1, x^3) \exp(\mathrm{i} k_2 x^2 - \mathrm{i}\omega t), \tag{9}$$

where the function $U(x^1)$ describes the small-scale structure of oscillations across magnetic shells, while $H(x^1,x^3)$ describes their structure along geomagnetic field lines. Justification of the desired solution in the form (9) is given in more detail in Leonovich and Mazur (1993).

In the neighbourhood of a poloidal resonant surface, the azimuthal wavelength is much smaller than the radial: $k_2^2 \gg |\nabla_1 U/U|^2$. The second term in (7), therefore, begins to greatly exceed the two others. Represent the function $H(x^1, x^3)$ in the neighbourhood of a poloidal resonant surface as

$$H(x^1, x^3) = P(x^1, x^3) + h(x^1, x^3),$$

where $P(x^1, x^3)$ is a function describing the longitudinal structure of standing Alfvén waves and satisfying, in the main order of perturbation theory, the equation:

$$\widehat{L}_P P = 0. \tag{10}$$

Boundary conditions on the ionosphere for function $P(x^1, x^3)$, in the same approximation, have the form

$$P(x^1, x_\pm^3) = 0. \tag{11}$$

The solution of the problem (10) and (11) are the poloidal eigenfunctions $P_N(x^1, x^3)$ and the corresponding eigenvalues $\bar{\omega} \equiv \Omega_{PN}(x^1)$, where $N = 1, 2, \ldots$ is the longitudinal wave number.

Near toroidal resonant surfaces, the characteristic wavelength (on x^1 coordinate) of the Alfvén oscillations across magnetic shells under study is much smaller than that on the azimuthal x^2 coordinate. The generation of Alfvén oscillations near the toroidal surface is much less effective than near the poloidal (Leonovich and Mazur, 1993). Therefore, in the neighbourhood of the toroidal surface in the boundary condition (8), the last term, related to external currents in the ionosphere, can be ignored. Represent function $H(x^1, x^3)$, describing the structure of oscillations along a field line near the toroidal surface, as:

$$H(x^1, x^3) = T(x^1, x^3) + h(x^1, x^3).$$

Here, the function $T(x^1, x^3)$ satisfies the zero-order equation

$$\widehat{L}_T T = 0, \tag{12}$$

with homogeneous boundary conditions on the ionosphere

$$T(x^1, x^3_{\pm}) = 0. \tag{13}$$

The solution of the problem (12) and (13) is represented by toroidal eigenfunctions $T_N(x^1, x^3)$ ($N = 1, 2, \ldots$ being the longitudinal wave number) with corresponding eigenvalues $\bar{\omega} = \Omega_{TN}(x^1)$.

The numerical solution of the problem Eqs. (10) and (11) is shown in Figs. 2 and 3. Fig. 2 shows the longitudinal (along a magnetic field line) structure of the first three modes of the poloidal and toroidal standing Alfvén waves at the magnetic shell $L = 6$ (here, $L = a/R_E$, with R_E being the Earth's radius, is the dimensionless parameter labelling the magnetic shells). The oscillation amplitudes are normalized to unity. It should be noted that identical modes (i.e., those with the same number N) of the poloidal and toroidal standing Alfvén waves have very similar structures.

Fig. 3 shows the eigenfrequencies of the fundamental mode of the poloidal and toroidal standing Alfvén waves. Curve 1 corresponds to toroidal waves with the azimuthal mode numbers $m = \pm 20$, ± 50 and ± 100, while curves 2–9 correspond to poloidal waves with the same azimuthal mode numbers. The upper curve in Fig. 3 shows the polarization splitting of the spectrum, $\Delta\Omega_1 = \Omega_{T1}(x^1) - \Omega_{P1}(x^1)$, for the fundamental mode of standing Alfvén waves with $m = -50$. The function $\Delta\Omega_1$ changes its sign at the point of intersection of the functions $\Omega_{T1}(x^1)$ and $\Omega_{P1}(x^1)$. It should also be noted that the absolute value of $\Delta\Omega_1$ is one or two orders of magnitude larger than that for other modes with $N = 2, 3, \ldots$ The mode $N = 1$ is the only one which can be visible in a discrete auroral arcs.

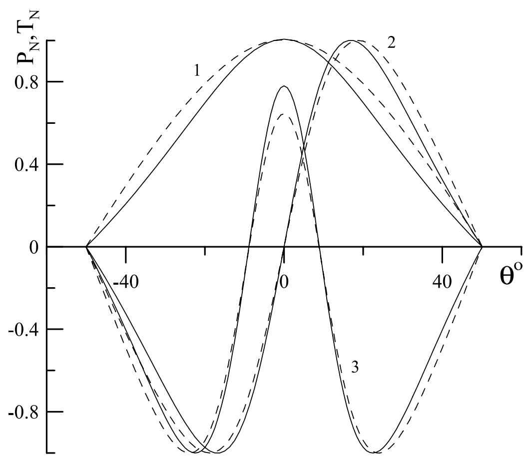

Fig. 2. The structure of standing Alfvén waves with toroidal (solid lines) and poloidal polarization (dashed lines). The curves are of the poloidal, P_N, and toroidal, T_N, eigenfunctions with unity amplitude for the first three parallel harmonics ($N = 1, 2, 3$).

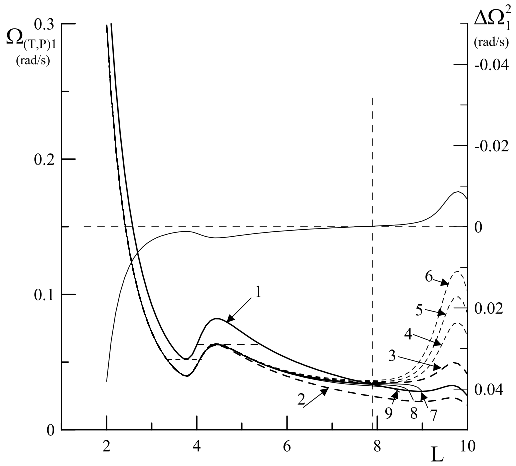

Fig. 3. Eigenfrequencies $\Omega_{(T,P)1}$ of the fundamental longitudinal modes of toroidal (curve 1) and poloidal (curves 2–9) Alfvén waves vs. magnetic shell parameter L (the family of the lower curves, referring to the left ordinate). The numerals 2–9 correspond to the poloidal eigenfrequencies (2) in a cold plasma at rest ($P_0 = 0, \Omega = 0$), (3) in a plasma with a finite gas-kinetic pressure ($P \neq 0$) and with $\Omega = 0$, and in a finite-pressure rotating plasma ($P \neq 0$, $\Omega \neq 0$) for different azimuthal mode numbers $m =$ (4) -20, (5) -50, (6) -100, (7) 20, (8) 50, and (9) 100. The upper curve, referring to the right ordinate, shows the spectral splitting of the eigenfrequencies, $\Delta\Omega_1^2$, vs. magnetic shell parameter L for $m = -50$.

4. The structure of standing Alfvén with $m \gg 1$ waves across magnetic shells

In the first order of perturbation theory near poloidal resonance surface (7) serves to obtain the equation

$$\nabla_1 \widehat{L}_T(\Omega_{PN}) P_N \nabla_1 U_N - k_2^2 U_N \widehat{L}_P(\Omega_{PN}) h_N - k_2^2 U_N \frac{p^{-1}}{A^2} \times (\bar{\omega}^2 - \Omega_{PN}^2) P_N + \frac{k_2 \Omega'}{\Omega_{PN}} \widehat{L}_T(\Omega_{PN}) P_N \nabla_1 U_N = 0.$$

Function $h(x^1, x^3)$ corrects $P(x^1, x^3)$ in the higher orders of perturbation theory. Boundary conditions for it have the form

$$h_N(x^1, \ell_\pm) = \mp \frac{v_\pm}{\Omega_{PN}} \frac{\partial P_N}{\partial \ell}\bigg|_{\ell_\pm} - \frac{J_\parallel^\pm}{U_N(x^1) V_\pm}. \tag{14}$$

Multiplying this equation by P_N and integrating it along a field line between the magnetoconjugated ionospheres yields:

$$\alpha_{PN} \nabla_1^2 U_N - k_2^2[(\bar{\omega} + \mathrm{i}\gamma_{PN})^2 - \Omega_{PN}^2] U_N + \alpha_{PN} \frac{k_2 \Omega'}{\Omega_{PN}} \nabla_1 U_N = I_N, \tag{15}$$

where the notations are:

$$\alpha_{PN} = \int_{\ell_-}^{\ell_+} P_N^2\left(\frac{\partial^2 p}{\partial\ell^2} + \beta_P\right)\mathrm{d}\ell,$$

$$\gamma_{PN} = \frac{1}{2\Omega_{PN}^2}\left[\frac{v_+}{p_+}\left(\frac{\partial P_N}{\partial\ell}\right)_{\ell_+} + \frac{v_-}{p_-}\left(\frac{\partial P_N}{\partial\ell}\right)_{\ell_-}\right],$$

$$I_N = \left[\frac{j_\parallel^{(+)}}{V_+}\sqrt{g_1g_2}\left(\frac{\partial P_N}{\partial\ell}\right)_{\ell_+} - \frac{j_\parallel^{(-)}}{V_-}\sqrt{g_1g_2}\left(\frac{\partial P_N}{\partial\ell}\right)_{\ell_-}\right].$$

One can see from (15) that γ_{PN} acts as a decrement of poloidal Alfvén waves, determined by their Joule dissipation in the ionosphere. Function I_N represents a source of oscillations associated with an external currents in the ionosphere. Eq. (15) describes the structure of standing Alfvén waves across magnetic shells near a poloidal resonant surface.

In the same way we can derive the equation near toroidal resonance surface:

$$\nabla_1[(\bar\omega + i\gamma_{TN})^2 - \Omega_{TN}^2]\nabla_1 U_N - k_2^2\alpha_{TN}U_N = 0, \tag{16}$$

where

$$\alpha_{TN} = -\int_{\ell_-}^{\ell_+} T_N^2\frac{\partial^2 p^{-1}}{\partial\ell^2}\mathrm{d}\ell,$$

$$\gamma_{TN} = \frac{1}{2\Omega_{TN}^2}\left[v_+p_+\left(\frac{\partial T_N}{\partial\ell}\right)_{\ell_+} + v_-p_-\left(\frac{\partial T_N}{\partial\ell}\right)_{\ell_-}\right].$$

The function γ_{TN} constitutes a decrement for toroidal Alfvén waves due to their dissipation in the ionosphere.

A solutions to (15) and (16) may be found by linearizing the coefficients in the neighbourhood of the poloidal $x^1 = x^1_{PN}$ and toroidal $x^1 = x^1_{TN}$ resonant surfaces and these solutions later matched together across the gap using the Wentzel–Kramers–Brillouin method on x^1 coordinate. This rather cumbersome technique was implemented in (Leonovich and Mazur, 1993). This paper applies a different approach.

A technique was developed in (Leonovich and Mazur, 1997) for describing the structure of standing Alfvén waves across magnetic shells by means of a model equation. Combining Eqs. (15) and (16), describing the structure of oscillations in the neighbourhood of the poloidal and toroidal resonant shells, one can construct a model equation

$$\sqrt{\alpha_{PN}}\nabla_1[(\bar\omega + i\gamma_N)^2 - \Omega_{TN}^2]\nabla_1U_N -$$
$$\sqrt{\alpha_{PN}}\frac{k_2\Omega'}{\bar\omega}[(\bar\omega + i\gamma_N)^2 - \Omega_{TN}^2]\nabla_1U_N -$$
$$\sqrt{\alpha_{TN}}k_2^2[(\bar\omega + i\gamma_N)^2 - \Omega_{PN}^2]U_N = \sqrt{\alpha_{TN}}I_N, \tag{17}$$

applicable for the entire region of existence of oscillations under study.

Consider just such regions of the magnetosphere where $\Omega_{PN}(x^1) > \Omega_{TN}(x^1)$. Since the functions $\Omega_{PN}(x^1)$ and $\Omega_{TN}(x^1)$ are similar enough, the typical scale of their variation on x^1 coordinate is virtually the same. Near the poloidal resonant surface, where $\bar\omega = \omega - m\Omega = \Omega_{PN}(x^1)$ we have approximately

$$\Omega_{PN}(x^1) \approx \bar\omega\left(1 - \frac{x^1 - x^1_{PN}}{L}\right), \tag{18}$$

where L is the characteristic scale of the variation of $\Omega_{PN}(x^1)$ near $x^1 = x^1_{PN}$ and in the neighbourhood of the toroidal resonant surface (where $\bar\omega = \Omega_{TN}(x^1)$):

$$\Omega_{TN}(x^1) \approx \bar\omega\left(1 - \frac{x^1 - x^1_{TN}}{L}\right). \tag{19}$$

Denote the equatorial distance between the toroidal and poloidal resonant surfaces as $\Delta_N = x^1_{TN} - x^1_{PN}$. In Fig. 3 Δ_1 corresponds to the distance between the intersection points of the line $\bar\omega = \mathrm{const}$ with the $\Omega_{P1}(x^1)$ and $\Omega_{T1}(x^1)$ curves. Eqs. (18) and (19) imply that, in linear approximation, $\Delta_1 \approx \Delta\Omega_1 L/\bar\omega$, where $\Delta\Omega_1 = \Omega_{T1} - \Omega_{P1}$ is the polarization splitting of the spectrum.

Turning to the dimensionless transverse coordinate $\xi = (x^1 - x^1_{TN})/\Delta_N$, we obtain

$$\frac{\partial}{\partial\xi}(\xi + i\varepsilon)\frac{\partial U_N}{\partial\xi} + q(\xi + i\varepsilon)\frac{\partial U_N}{\partial\xi} - \kappa_N^2(\xi + 1 + i\varepsilon)U_N = b_N, \tag{20}$$

where the following dimensionless parameters are introduced

$$\varepsilon = 2\frac{\gamma_N}{\bar\omega}\frac{L}{\Delta_N}, \quad q = k_2\Delta_N\Omega'/\bar\omega, \quad \kappa_N^2 = \sqrt{\frac{\alpha_{TN}}{\alpha_{PN}}}k_2^2\Delta_N^2,$$
$$b_N = I_N\sqrt{\frac{\alpha_{TN}}{\alpha_{PN}}}\frac{L\Delta_N}{\bar\omega^2}.$$

A solution to (20) may be found using a Fourier transform:

$$U_N(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar U_N(k)e^{ik\xi}\mathrm{d}k. \tag{21}$$

Substituting (21) into (20) can produce a first-order equation for the Fourier harmonic $\bar U_N$, to be solved easily (Leonovich and Mazur, 1997). The expression obtained for $\bar U_N(k)$ can be substituted into (21) to yield a solution to the initial Eq. (20) in the form:

$$U_N(\xi) = i\frac{b_N}{\kappa_N}\int_0^\infty \frac{\exp[ik(\xi + i\varepsilon) + i\alpha_N\arctan\psi(k)]}{\sqrt{k^2 - ikq + \kappa_N^2}}\mathrm{d}k, \tag{22}$$

where

$$\alpha_N = \frac{\kappa_N^2 + q/2}{\sqrt{\kappa_N^2 + q^2/4}}, \qquad \psi(k) = k\frac{\sqrt{\kappa_N^2 + q^2/4}}{\kappa_N^2 - ikq/2}.$$

Let us consider the behaviour of the solution (22) near the toroidal resonant surface ($\xi \to 0$), as well as asymptotics on $|\xi| \to \infty$.

With $\xi \to 0$, the bulk of the integral in (22) accumulates due to high values of $k \gg \kappa_N$. Letting $k \to \infty$ in the denominator (22) and in $\psi(k)$ produces an expression for both the derivative

$$\frac{\partial U_N}{\partial\xi} \underset{\xi\to 0}{\approx} \frac{b_N}{\kappa_N(\xi + i\varepsilon)}e^{i\alpha_N\arctan\psi(\infty)},$$

and the function itself

$$U_N \underset{\xi\to 0}{\approx} \frac{b_N}{\kappa_N} e^{\mathrm{i}\alpha_N \arctan\psi(\infty)} \ln(\xi + \mathrm{i}\varepsilon).$$

One can see that, for $\varepsilon = 0$, $U_N(\xi)$ on the toroidal resonant surface ($\xi \to 0$) has a logarithmic singularity known for resonant Alfvén waves.

Conversely, with $|\xi| \to \infty$, it is the values of $k \ll \kappa_N$ that make the greatest contribution into the integral (22). Therefore, we could set $k = 0$ in the denominator of the integrand in (22) and in $\psi(k)$, resulting in

$$U_N(\xi) \underset{|\xi|\to\infty}{\approx} -\frac{b_N}{\kappa_N^2} \frac{1}{\xi + \mathrm{i}\varepsilon}$$

the asymptotics for high values of ξ. Thus, while $\varepsilon = 0$, the function $U_N(\xi)$ has a singularity on the toroidal resonant surface, and linearly decreases in amplitude when moving away from it.

Fig. 4 shows a typical behaviour of the function $U_N(\xi)$, describing the structure of the 1-st harmonic of standing Alfvén waves across magnetic shells, for the azimuthal wave number $m = 100$. The decrement was chosen to be very small ($\varepsilon = 10^{-3}$) in order to reveal the role of the parameter κ_N, acting as the wave number on ξ coordinate. The solution is a wave travelling from the poloidal ($\xi = -1$) to the toroidal resonant surface ($\xi = 0$). This is evident from the phases $\mathrm{Re}\, U_N$ and $\mathrm{Im}\, U_N$ differing by $\sim\pi/2$ in the interval $-1 < \xi < 0$. This can easily be shown analytically by computing the integral (22) for $\kappa_n \to \infty$ using the saddle point method. With $\kappa_n \ll 1$ the oscillations' harmonic structure vanishes, and they become typical resonant oscillations analogous to those in field line resonance. In the neighbourhood of a resonance peak the polarization of oscillations is toroidal.

5. Discussion

Let us now discuss the main features of standing Alfvén waves with $m \gg 1$ examined in this paper. First of all, let us note that in the magnetosphere with moving plasma having an azimuthal component of speed it is impossible to form a monochromatic Alfvén wave with $m \neq 0$, standing in the azimuthal direction. In motionless plasma such a wave can be generated, because waves with azimuthal wave numbers $\pm m$ have identical spatial structures. It is evident from Fig. 3, the position of poloidal resonant surface in moving plasma essentially depends on azimuthal wave number and velocity of background plasma. Thus, oscillations with different signs of m have different spatial structure. Hence, oscillations with $m \gg 1$ observable in the real magnetosphere, that propagate in the convective magnetospheric plasma are necessarily waves running in the azimuthal direction.

This effect can be noticed even better if we pass from frequency $\bar{\omega}$ to the real frequency of the oscillation source. Fig. 5 presents frequencies which the source of oscillations should have in order to excitate the first five harmonicas of standing Alfvén waves with $m = \pm 50$. When passing into the outer magnetosphere, where the effects of plasma movement begin to be noticeable, the eigen-frequencies of such oscillations differ strongly. However, even if oscillations with such azimuthal wave numbers have identical amplitudes in the source spectrum, they are unable to form a standing wave in the azimuthal direction because they have different poloidal resonant surfaces.

It is visible from Fig. 3 that resonant surfaces intersect in the outer magnetosphere (at $L \approx 8$). This effect is due to increased temperature and velocity of plasma rotation. This means that the above-described Alfvén oscillation structure in the form of a wave running across the magnetic cannot occur in the areas adjoining the magnetopause. From the same figure it is evident that for oscillations having different signs of azimuthal wave numbers m and velocity Ω the poloidal resonant surface disappears entirely. This means that Eq. (10) has no solutions satisfying the

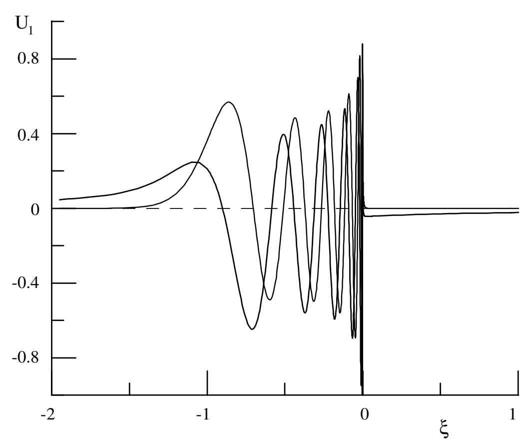

Fig. 4. The structure of standing Alfvén waves across magnetic shells – the U_N function's real (bold line) and imaginary (thin line) components describing the structure of travelling-wave-type oscillations with $\kappa_N = 20$.

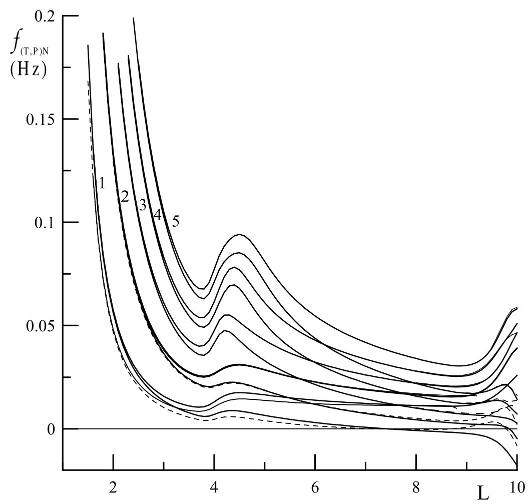

Fig. 5. Dependency of the toroidal (thick lines) and poloidal (thin lines) eigenfrequencies $\Omega_{TN}(x^1)$, $\Omega_{PN}(x^1)$ on the magnetic shell parameter L for the first five longitudinal harmonics ($N = 1, \ldots, 5$), in the source reference frame. The poloidal eigenfrequency values shown correspond to the azimuthal harmonics $m = 50$ (solid lines) and $m = -50$ (dashed lines).

boundary conditions. It is only in the inner magnetosphere ($L < 8$) that the structure in the form of a wave running across magnetic shells can be observed for the basic harmonic of standing Alfvén waves $N = 1$.

Finally, as is implied in (6), difference between Earth rotation and magnetospheric plasma (where the standing Alfvén wave field is concentrated) convection velocities, the oscillations enjoy the appearance of an electric field component directed along the background magnetic field, in the transitive layer at 200–400 km. This longitudinal electric field appears to be unable to accelerate charged particles into the ionosphere, but it can make their flow to become structured and to generate a visible structure of auroral arc. The same effect can result from the modulation of electric potential drop across of existing double layer by the field of an Alfvén wave (Lysak and Song, 2003), or owing to plasma density cavities forming in the Alfvén wave field-aligned current, which also play the role of double layers (Bespalov et al., 2006).

Let us discuss some features to be exhibited by a discrete auroral arc generated by one of the harmonics of azimuthal-small-scale Alfvén waves examined in this work. First, this structure should consist of several parallel auroral arcs whose dynamics allow us to regard them as a unit. Separate arcs inside of such a structure will reflect the structure and dynamics of a standing Alfvén wave which has the form of a "running wave" across magnetic shells. Discrete auroral arcs appear to have a similar structure (Nadubovich, 1992; Samson et al., 1996).

Second, in order the structure of "running wave" kind might be formed in the real magnetosphere, characteristic azimuthal wave numbers of Alfvén oscillations should be in a range $m > 50$. But, azimuthal wave numbers cannot be too big ($m < 200$) for clear observation of separate arcs inside of a discrete auroral arc (its does not have the diffusive form). In the specified range $50 < m < 200$, the number of a separate structural elements (separate arcs) in a discrete auroral arc can be varied (from 2 to 8). Characteristic width of such discrete auroral arc is determined by the distance between poloidal and toroidal resonant surfaces, that for the basic harmonic of standing Alfvén waves, $N = 1$, mapped into the ionosphere is ~70 km at the auroral latitudes. The width of separate arcs depends on their number (or, that the same, from azimuthal wave number m) and can vary from 5 to 20 km. Characteristic feature of such structure is decrease of the width of a separate arcs from the widest, located at one of the edges, to the narrowest, located on the other edge. It is associated with decrease of characteristic wavelength of the oscillations across magnetic shells from the poloidal resonant surface to toroidal.

Third, sometimes aurorae have a periodic structure in the azimuthal direction. The sizes of elements of the azimuthal structure are in the 5–60 km range that corresponds to the typical azimuthal wave numbers $m \sim 100$–1000. An azimuthal velocity of such structures is ~1 km/s, which approximately corresponds to the phase velocity $v_\phi = \omega\rho/m$ of a standing Alfvén wave with $m \sim 100$. As Fig. 3 implies, at latitudes corresponding to the auroral region ($\theta \sim 65°$–$70°$, $\rho = R_E\cos\theta \sim 3000$ km), the characteristic eigenfrequency of the standing Alfvén waves' basic harmonic $\omega \sim 0.05$ rad/s in our model magnetosphere with a dipole magnetic field. Since the azimuthal velocities observed in auroral propagation $v_\phi \sim 1$ km/s. Note that kinetic Alfvén waves excited by the mechanism of field line resonance have $m \sim 1$ and they cannot exhibit such azimuthal phase velocities. In addition to phase motion of individual structural elements inside of an auroral arc, discrete arcs can move bodily, owing to plasma drift (westwards before local midnight, eastwards after midnight) at approximately the same velocities ~1 km/s.

And, finally, such a discrete auroral arc should exhibit periodic alternations of bright and dark arcs that corresponds to periodic changes in the direction of parallel currents in Alfvén wave in the neighbouring halfwaves of its transverse structure. In the dynamics of a discrete auroral arc this should manifest itself as its periodic displacement in the north–south direction with Alfvén oscillation periods, which for the basic harmonics at auroral latitudes are approximately 100 s.

6. Conclusion

Listed below are the major results of this work.

1. Eq. (7) was derived describing the structure of standing Alfvén waves with high azimuthal wave numbers $m \gg 1$ in a dipole magnetosphere with rotating plasma.
2. Solutions were found to Eq. (10) and (12), defining the longitudinal (along magnetic field lines) structure and spectrum of the eigenfrequencies of poloidal and toroidal Alfvén oscillations in the magnetosphere. Eqs. (15) and (16) have been obtained, describing the transverse (across magnetic shells) structure of standing Alfvén waves in the magnetosphere with rotating plasma near the poloidal and toroidal resonant surfaces.
3. A model Eq. (17) was constructed that helps to define the transverse structure of Alfvén oscillations under study not only near resonant surfaces but in the entire region of existence. An analytical solution (22) was produced for this equation.
4. The structure of observed discrete auroral arcs was compared to the structure of standing Alfvén waves with high azimuthal wave numbers $m \sim 100$. The characteristic transverse size of the localization region of such oscillations as mapped onto the ionosphere, ~50–70 km. The characteristic azimuthal propagation velocities of aurorae tally with the azimuthal phase velocities of Alfvén waves with $m \gtrsim 100$.

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