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MHD oscillations with a discrete spectrum?**

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Abstract. A new concept of the global magnetospheric resonator is suggested for fast magnetosonic waves in which the role of the resonator is played by the near-Earth part of the plasma sheet. It is shown that the magnetosonic wave is confined in this region of the magnetosphere within its boundaries. The representative value of the resonator's eigenfrequency estimated at $f \sim 1$ mHz is in good agreement with observational data of ultra-low-frequency MHD oscillations of the magnetosphere with a discrete spectrum ($f \sim 0.8, 1.3, 1.9, 2.6 \dots$ mHz). The theory explains the ground-based localization of the oscillations observed in the midnight-morning sector of the high-latitude magnetosphere.

A reasonable amount of research has been done to date, addressing the theory of global MHD eigen-oscillations of the magnetosphere (in addition to the references cited above, see also Zhu and Kivelson, 1989; Lee and Lysak, 1991; Leonovich and Mazur, 2001). Nevertheless, the status of the theory cannot be recognized as satisfactory. The Earth's magnetosphere is not a closed cavity. The geotail extends into the solar wind to an indefinitely large distance. Therefore, magnetosonic waves are thought of as being able to freely escape along it. In such a case, however, it is beyond reason to hope for a closed resonator with a discrete spectrum of frequencies. This difficulty of the theory has not yet received any acceptable resolution.

1 Introduction

The Earth's magnetosphere represents a low-density plasma cavity within the solar wind stream separated from it by a sharp boundary, the magnetopause. It has fairly long been hypothesized that this cavity would serve as a resonator for fast magnetosonic waves [McClay, 1970; Southwood and Kivelson, 1986; Kivelson and Southwood, 1986]. The resonator's eigenmodes (cavity modes, global modes) can be driven by instabilities at the magnetopause [McKenzie, 1970] or by unsteady processes in the solar wind [Southwood and Kivelson, 1990]. Such oscillations, which are the largest-scale and lowest-frequency MHD oscillations of the magnetosphere, must have a discrete spectrum. Through the field line resonance mechanism, the magnetosonic eigenmodes can excite standing Alfvén waves on those field lines, the local Alfvén frequency of which coincides with the resonator's eigenfrequency [Southwood and Kivelson, 1986; Leonovich and Mazur, 1989]. However, such oscillation, due to its large scale, can be observed directly on the Earth's surface without the intermediate excitation of the standing Alfvén wave.

In a meanwhile, oscillations, having the basic properties of global MHD modes, were detected. They are observed both with HF radars [Ruohoniemi et al., 1993; Samson and Harrold, 1992] and at ground-based magnetometer networks [Wanliss et al., 2002]. These oscillations have clearly pronounced maxima in the oscillation spectrum at the frequencies $f \sim 1.3, 1.9, 2.6,$ and 1.4 mHz. Sometimes oscillations with a frequency 0.8 mHz [Lessard et al., 2003] are observed. The oscillations are normally recorded in the midnight-morning sector at latitudes from 60° to 80° , with a low disturbance level of the magnetosphere $K_p \sim 1 - 3$ [Harrold and Samson, 1992; Samson and Harrold, 1992]. The frequencies of such oscillations are independent of the latitude of the observation point and remain virtually unchanged both from event to event and within a single observing interval.

The experimental detection of ultra-low-frequency hydro-magnetic oscillations with a discrete spectrum made the problem of constructing an adequate theory still more urgent. Unquestionably such a theory must include the presence of the geotail. Some authors [Harrold and Samson, 1992; Samson and Harrold, 1992; Samson and Rankin, 1994] made an attempt to treat the observed oscillations as the eigenmodes not of the resonator but of the waveguide in the geotail. However, the eigenmodes of the waveguide produce no discrete spectrum, since the wavelength along the waveguide can be an arbitrary one. The authors of the cited papers recognize

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that this difficulty has no satisfactory resolution in terms of the theory they suggested [Samson and Rankin, 1994]. Furthermore, there is a number of other problems inherent in this theory. So, inside this concept it is difficult to explain such low frequencies of the oscillations, and also why they do not go away into the geotail along the waveguide. In this paper we suggest a new concept of the resonator for ultra-low-frequency magnetosonic waves in which it is possible to overcome most of the difficulties inherent in previous work.

2 The resonator in the near-Earth part of the plasma sheet

The basic idea of this study implies that the role of the resonator is played by the near-Earth part of the plasma sheet (NEPS). This important large-scale element of the magnetosphere represents a lens-shaped region (see Fig.1), filled with a sufficiently dense plasma ($n \sim 1 \text{ cm}^{-3}$). Combined with a reasonably low value of the magnetic field ($B \sim 5 - 10 \text{ nT}$), this leads to minimum (for the entire magnetosphere) representative values of the Alfvén velocity $A = B / \sqrt{4\pi m_p n} \sim 10^2 \text{ km/s}$. It is not inconceivable that the values $A \sim (30 - 50) \text{ km/s}$ exist [Borovsky et al., 1998]. The boundary of the plasma sheet presented in Fig.1 should be understood as the surface, which divide the magnetosphere cavity on the areas with the big and small values of Alfvén velocity. Therefore it does not reach the Earth, at approach to which the values of a geomagnetic field and Alfvén velocity are growing.

Velocity of fast magnetosonic waves is represented by the following expression $C_f = \sqrt{A^2 + S^2}$, where $S = \sqrt{\gamma P / \rho}$ - sound velocity in plasmas. In contrast to Alfvén velocity, the distribution of sound velocity in the magnetosphere is much less known. It is because of sound velocity is a function of temperature which distribution in the magnetosphere is determined much worse than the distribution of the magnetic field and density of plasma. So in the neutral sheet (and in the equatorial part of the NEPS), temperature of plasma is great enough for value $\beta = B^2 / 8\pi nT \approx A^2 / S^2 \gg 1$ [Borovsky et al., 1998]. It means, that gas-kinetic pressure of plasma is in here much greater than the magnetic pressure. Plasma sheet is the hottest area of the magnetosphere. Its steady state is supported by sharp gradient of magnetic pressure in the direction from neutral sheet to tail lobes.

The conditions in the NEPS is different completely. The geomagnetic field here is a dipole-like one. Dipole field is force-free, in spite of the fact that its field lines are curvilinear. Therefore it is possible to suppose, that the geomagnetic field in the NEPS area is almost force-free one. It means, that the plasma pressure does not play essential role in the forces balance providing equilibrium of the magnetic configuration. We will consider the magnetosphere in an undisturbed quiet state. It is possible to suppose, that in such steady state the volume-average of a flux tube condition $\beta \lesssim 1$ is hold in the NEPS. It means, that on the order of magnitude $C_f \approx A$. The insufficient difference of the eigenfrequencies estimation carried out below, associated with considering of a sound veloc-

ity, does not influence on the order of value in the final result. In the tail lobes, where $\beta \ll 1$, influence of sound velocity can be neglected at all.

The NEPS continually changes to a flat neutral sheet, separating the magnetotail into two lobes. These parts of the plasma sheet form a unit plasma configuration. Nevertheless, treating them separately is quite justified, both in terms of the morphology and from the character of propagation of magnetosonic waves.

The characteristic radius of the NEPS roughly coincides with the cross-section of the geotail in this part of the magnetosphere $a \sim 10^5 \text{ km}$. The inner edge of the plasma sheet lies at (5-10) R_E from the Earth, and the outer edge of the NEPS is at the distance (20-30) R_E in the antisunward direction. Thus the characteristic size of the NEPS in this direction Δ is of the same order of magnitude $\Delta \sim 10^5 \text{ km}$. The above values of the Alfvén velocity A and of the size of the region $a \sim \Delta$ lead to the characteristic value of the frequency of the possible eigenmode $f \sim A/a \sim 1 \text{ mHz}$, in full agreement with observed values.

Of course, before speaking of the eigenmode frequency, one has to make sure that the fast magnetosonic wave can be confined within the NEPS. In the radial direction, the confinement is caused by the presence of a sharp boundary, the magnetopause, at which the Alfvén velocity changes abruptly from the values (300-500) km/s in the magnetosphere to substantially lower values (50-100) km/s in the magnetosheath. On such a boundary there is taking place partial reflection of the magnetosonic wave which increases with the increasing Alfvén velocity difference. The transition coefficient κ_t ($\kappa_t = 1 - \kappa_r$, where κ_r is the reflection coefficient) is equal, on the order of magnitude, to the ratio of the Alfvén velocities on the outer and inner side of the magnetopause - for the aforementioned values $\kappa_t \sim 0.1 - 0.3$. Thus at this boundary there is taking place a sufficiently effective confinement of magnetosonic waves. Partial penetration of the wave through the magnetopause leads to the resonator's Q factor $Q \sim 3 - 10$.

Before proceeding to the discussion of the reason why the wave is reflected by the outer and inner edges of the plasma sheet, we present some simple evidence from the theory of MHD resonators. In a homogeneous cold plasma residing in a homogeneous magnetic field, the frequency of a fast magnetosonic wave is given by the relation

$$\omega^2 = (k_x^2 + k_y^2 + k_z^2)A^2, \quad (1)$$

where k_x, k_y, k_z stand for the wave vector components. If the wave is confined (say, by reflecting walls) in a square box with the size along its axes a_x, a_y, a_z , then the wave vector components are quantized

$$k_x = \frac{\pi n_x}{a_x}, \quad k_y = \frac{\pi n_y}{a_y}, \quad k_z = \frac{\pi n_z}{a_z}, \quad (2)$$

where the wave numbers n_x, n_y, n_z can take a natural series of values. If the medium within the resonator is inhomogeneous in one coordinate, say x , and if the inhomogeneity is such that the WKB approximation is applicable, then (1)

can be treated as the equation defining a quasi-classical wave vector $k_x = k_x(x, \omega)$:

$$k_x^2 = \frac{\omega^2}{A^2(x)} - k_y^2 - k_z^2, \quad (3)$$

in which case the relations (2) hold for k_y and k_z as before. The turning point (more exactly, surface) at which $k_x^2(x) = 0$ separates the opaque region, where $k_x^2(x) < 0$, from the transparent region, where $k_x^2(x) > 0$. It is easy to see that in the opaque region the Alfvén velocity has larger values compared to the transparent region. The turning surface can be treated as a reflection surface where the wave is reflected from the opaque region. The reflection coefficient κ_r increases with the increasing width of the opaque region. If the width of the opaque region is much larger than the characteristic wavelength then $\kappa_r \approx 1$. If the transparent region is surrounded by turning points (surfaces) on both sides, then it is possible for a resonator to be produced. The frequency spectrum of such a resonator within the WKB approximation is determined by the Bohr-Sommerfeld quantization condition.

In the three-dimensional inhomogeneous case, these notions all retain their meaning. The turning surface, which is not a plane in the general case, separates the transparent region, with a relatively small value of the Alfvén velocity, from the opaque region where the values of the Alfvén velocity are larger. We now return to the discussion of the NEPS. The confinement of the magnetosonic wave within this region in the earthward direction is caused by the presence of the turning surface in this direction. The existence of this surface is caused by a significant increase of the Alfvén velocity from $A \sim 10^2$ km/s in the NEPS to $A \sim (2 - 3) \cdot 10^3$ km/s in the near-Earth region. This turning surface virtually coincides with the inner edge of the plasma sheet, where an abrupt change of the Alfvén velocity occurs. The region between the inner edge of the plasma sheet and the Earth is an opaque region for the wave under consideration.

The outer edge of the NEPS is a reflection surface for the same reason. Behind it there are the tail lobes in which the Alfvén velocity reaches even larger values than in the near-Earth region $A \sim (3 - 10) \cdot 10^3$ km/s. The only question remains with regards to the cleft-shaped region of transition from the NEPS to the neutral sheet. This region may be treated as a peculiar kind of hole in the outer edge of the NEPS on which the neutral plasma sheet "abuts". It would appear that the magnetosonic wave can freely escape to it. For the oscillations under consideration, this is not the case, however.

The neutral plasma sheet may be regarded as a planar waveguide [Siskoe, 1969]. Let the axis x be directed along the waveguide axis in the antisunward direction, and axis y azimuthally, and the axis z across the neutral sheet. Formulas (2) and (3) can be applied for such a waveguide. On the order of magnitude $a_y \sim a \sim 10^5$ km, and $a_z \sim 10^4$ km is the thickness of the neutral sheet. For the fundamental mode of oscillations in the such waveguide $k_y \sim \pi/a$, $k_z \sim \pi/a_z$, i.e. $k_y^2 \ll k_z^2$. The frequency of the oscillation (eigenmode)

under consideration, as has already been pointed out above, $\omega \sim \pi A/a$, where A is the characteristic Alfvén velocity in the NEPS. The velocity of fast magnetosonic waves in the neutral sheet because of more high temperature is greater than its average value in the NEPS. Then within the neutral sheet we have

$$\frac{\omega^2}{C_f^2} \ll k_z^2,$$

and, according to (2), (3)

$$k_x^2 = -k_z^2 \sim -\frac{\pi^2}{a_z^2}.$$

The resonator's fundamental eigenmode cannot propagate in the neutral sheet but penetrates to its depth only by about its thickness a_z . In other words, the turning surface, from which full reflection of magnetosonic waves under consideration occurs, is formed in the NEPS in the antisunward direction on whole geomagnetic tail section, including the plasma sheet. Thus the fast magnetosonic wave under consideration is confined on all the boundaries surrounding the NEPS.

3 Conclusion

We now formulate the results obtained in this study and the implications that follow directly from them.

1. The near part of the plasma sheet (NEPS) can play the role of the resonator for fast magnetosonic waves. These waves are reflected both by the resonator's side boundary, which form part of the magnetopause, and by the inner and outer edges of the NEPS. Because of small thickness of the neutral sheet, modes of the resonator cannot penetrate into its depth.

2. The characteristic value of the frequency of the resonator's fundamental modes is ~ 1 mHz, which is in good agreement with observed frequencies of ultra-low-frequency oscillations with a discrete spectrum of frequencies. A complete calculation of the spectrum of the resonator's eigenoscillations requires a definite model for the NEPS, and this will be the subject of our separate study. It is clear that by fitting the model parameters, it is always possible to achieve a good reconciliation with observations of one or two eigenfrequencies. It is of interest, however, to what extent the properties of the spectrum of such a resonator can be reproduced in general.

3. The Q factor of the resonator discussed here is not very high. The loss of energy caused by the escape of the mode through the magnetopause gives the values $Q \sim 3 - 10$. If the other loss mechanisms are important, then the Q factor will be still smaller. But this value of the Q factor is large enough for the resonator driven by a sudden impulse (a shock wave from the solar wind, say) to show a train of oscillations of 3-10 periods.

4. Standing Alfvén modes rather than the cavity modes themselves (the near-Earth part of the magnetosphere is the opacity region for them) are observed on the ground. They

are driven by the field line resonance mechanism along field lines passing through the resonator's region. The fact that the resonator is represented by the NEPS explains in a natural way the ground-based localization of observed ultra-low-frequency oscillations with a discrete spectrum, both in latitude 60° - 80° and in longitude - in the midnight-morning sector. The displacement of the oscillations relative to their symmetric midnight position can be explained by the convection of magnetospheric plasma that makes the NEPS move into the morning sector [Elphic et al., 1999].

5. Unfortunately, the concept suggested in this study can not to explain the surprising stability of the frequencies observed. Incidentally, this difficulty is inherent in any version of the theory of magnetospheric resonator. The magnetosphere is not stable, and its parameters vary over a rather wide range, including the parameters of the NEPS. One conceivable explanation for this can be found through the following reasoning. The magnetospheric cavity modes are long-period oscillations ($T \sim 10^3$ s). In order for them to stand out against the dynamics of the magnetosphere itself, this must be relatively stable on time intervals on the order of 1 hour or longer. Indeed, oscillations are most frequently observed at the level of a disturbance corresponding to $K_p \sim 1 - 3$. In this case the magnetospheric parameters lie within a much narrower range compared to their extreme values. In other words, possibilities for the existence and excitation of cavity modes are only created within a relatively narrow range of possible states of the magnetosphere, including the NEPS. Besides, it can be suggested that such an integral characteristic of the resonator as its eigenfrequency (on the order of magnitude, it is $f \sim A/a$) is more stable than separately the parameters A and a characterizing it.

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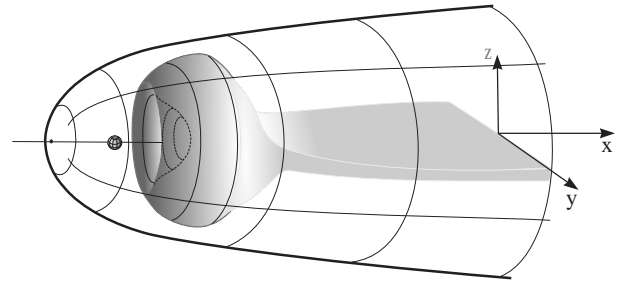


Fig. 1. Model of the magnetosphere. The plasma sheet appears gray. The figure does not show the asymmetry of the near-Earth plasma sheet about the midday-midnight meridian. The outer boundary of the figure is the magnetopause separating the magnetosphere and the magnetosheath.

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