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Magnetic-field advection in inhomogeneous turbulence

L.L. Kichatinov[1] **and G. Rüdiger**[2,★]

[1] Siberian Institute of Terrestrial Magnetism, P.O. Box 4026, Irkutsk 664033, Russia
[2] Institut für Astronomie und Astrophysik der Universität Würzburg, Am Hubland, D/W-8700 Würzburg, Germany

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Abstract. We consider the magnetic field transport by turbulence in inhomogeneous fluids taking into account the feedback of magnetic field on the motion. The magnetic influence on locally isotropic turbulence of density-stratified fluids enables the turbulence to advect the mean field. The effect is *formally* similar to magnetic buoyancy, which has also no linear counterpart. The "turbulent buoyancy", however, has no definite sign. It is "almost always" directed upwards for weak magnetic fields but acts downwards in the strong-field limit when the turbulence is close to two-dimensionality.

The non-linear regime of the already known diamagnetic transport due to non-uniform intensity in the turbulence field is also considered. The corresponding velocity is found to decrease monotonically with the field strength and becomes negligible in the strong-field case. A comparison with the so far discussed magnetic buoyancy effect shows the necessity of the treatment of the overall field-advection problem.

Key words: hydromagnetics – turbulence – Sun: magnetic fields

1. Introduction

Mean-field dynamo theory requires the knowledge of the entire alpha-tensor. While the latter's symmetric part describes the induction of an EMF along the mean magnetic field ("alpha-effect"), the antisymmetric part represents the advection of the field as a whole ("diamagnetism"). Only a few calculations of this effect are known so far, despite of its importance for the "buoyancy problem" of the dynamo theory. As is well-known, magnetic buoyancy tends to move away the magnetic field from the region of most-effective alpha-effect. The very open question here is whether a turbulence-induced compensation or even over-compensation of this uncomfortable effect exists which tends to move back the magnetic field to the bottom of the convection zone – which is supposed as the favourable site for the solar dynamo (Golub et al. 1981; Spruit & van Ballegooijen 1982).

Generally, the field advection terms, i.e. the antisymmetric part of the alpha-tensor, begin also to play a more and more important role in the non-linear dynamo theory. Numerical simulations have shown that such "buoyancy" terms can saturate the dynamo in a much more convincing way as the traditional alpha-quenching does (Moss et al. 1989; Schmitt & Schüssler 1989). The resulting magnetic fields basically possess a simpler structure compared to the other known feedback mechanisms.

The alpha-tensor exists only in inhomogeneous fluids. Its symmetric part (the "alpha-effect") forms a pseudo-tensor, so that it can only exist in rotating turbulence fields. That is not the case for the antisymmetric part of the tensor, which already in non-rotating turbulences occurs. One can generally show that it "almost always" vanishes for homogeneous fields (except higher-order correlations are involved, Krause & Rädler 1980).

If the fluid is inhomogeneous, however, the magnetic fields will be advected as a whole. A turbulent fluid, for example, with non-uniform effective diffusivity η_T behaves as diamagnetic and transports the magnetic field with the effective velocity $\boldsymbol{U}_{\rm dia} = -\nabla\eta_T/2$ (in the high-conductivity limit).

A much more important inhomogeneity capable for field-advection is density stratification. For a scalar field θ with

$$\partial\theta/\partial t + \mathrm{div}(\theta\boldsymbol{u}) = 0 \tag{1.1}$$

locally isotropic mixing of an anelastic fluid with

$$\mathrm{div}(\rho\boldsymbol{u}) = 0 \tag{1.2}$$

results in an effective transport of $\langle\theta\rangle$ along the density gradient. The second-order-correlation-approximation simply leads to the flux representation, $\mathscr{F} = \langle\boldsymbol{u}\theta\rangle$,

$$\mathscr{F} = -\int \langle \boldsymbol{u}(x,t)\,\mathrm{div}\,\boldsymbol{u}(x,t-\tau)\rangle\,\mathrm{d}\tau\,\langle\theta\rangle. \tag{1.3}$$

Then Eq. (1.2) yields

$$\partial\langle\theta\rangle/\partial t + \mathrm{div}(\boldsymbol{G}\chi_T\langle\theta\rangle) = \nabla(\chi_T\nabla\langle\theta\rangle), \tag{1.4}$$

with

$$\boldsymbol{G} = \nabla\log\rho \quad \text{and} \quad \chi_T = \frac{1}{3}\int_0^\infty \langle\boldsymbol{u}(x,t)\boldsymbol{u}(x,t-\tau)\rangle\,\mathrm{d}\tau. \tag{1.5}$$

From Eq. (1.4) the advection velocity of the mean field $\langle\theta\rangle$ is, therefore, $\chi_T\boldsymbol{G}$. The transport is obviously towards the higher density ("inwards") caused by the expansion of rising and the compression of sinking fluid elements.

One expects the same transport effect operating with vector fields. There is, indeed, a special realization of this idea, i.e. for 2D mixing. Drobyshevski (1977) has shown that 2D-turbulence transports a mean 2D magnetic field with an effective velocity of $\boldsymbol{U}_{\rm dens} = \eta_T\nabla\log\rho$.

On the other hand, Vainshtein (1978) demonstrated the "topological pumping" disappearing for 3D locally isotropic

Send offprint requests to: G. Rüdiger

★ On leave from the Astrophysical Institute Potsdam, Rosa-Luxemburg-Str. 17a, D/O-1591 Potsdam, Germany mixing. Magnetic field transport due to density stratification was found to be sensitive to the assumed anisotropy of the turbulence. It works only if a preferred direction of anisotropy close to the magnetic field direction is present. So, the significance of the transport effect would be essentially lowered if the sources of the required anisotropy are uncertain.

As a possible solution of the dilemma we demonstrate here that the magnetic field itself can produce the desired anisotropy by its influence on the inhomogeneous motion. Anisotropy is one of the essential effects of a large-scale magnetic field on turbulence (suppression is the other one). The ratio of the correlation lengths along and perpendicular to the magnetic field increases with the field strength (Rüdiger 1974; Rädler 1974). This means that the mean field changes the turbulence towards two-dimensionality – for which the considered transportation effect has been shown to exist.

Our derivations provide a magnetically non-linear field-advection effect due to the density stratification for which no linear counterpart exists. Similar to the buoyant rise of magnetic flux, this advection arises due to the influence of magnetic field on properties of the turbulent fluid. Although density fluctuations are not included we shall call it "turbulent buoyancy". After the above discussion it was expected to act downwards but the finding is that it does so only for sufficiently strong fields.

On this way, we have to rediscuss also the diamagnetic transport due to the inhomogeneity of the turbulence intensity taking into account the full non-linearities in the magnetic field. The diamagnetic effect decreases with increasing magnetic field and becomes almost negligible (B^{-3}) in the strong-field limit when the turbulence is close to two-dimensionality. Strong magnetic fields are, thus, minimally affected by the turbulent diamagnetism.

The present paper continues the detailed investigation of the influence of large-scale magnetism on anisotropic, but homogeneous turbulence (Rüdiger & Kichatinov 1990). The inclusion of inhomogeneity makes the derivations more complicate, but it is a necessary step towards a realistic modelling of stellar convection zones.

2. The basic equations

What we need is the mean electromotive force,

$$\mathscr{E} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle, \tag{2.1}$$

for an inhomogeneously turbulent fluid with fully non-linear account of the mean magnetic field, $\bar{B}$. The latter can here be considered as uniform. For simplicity, we assume the mean velocity, $\bar{u}$, to be zero and restrict the calculations to the first-order terms in the scale ratio $l_{\rm corr}/L$; $l_{\rm corr}$ and L being typical spatial scales of fluctuating and mean fields, respectively.

Working with Fourier transforms,

$$\boldsymbol{u}(x,t) = \int \hat{\boldsymbol{u}}(k,\omega) e^{i(\boldsymbol{k}\boldsymbol{x}-\omega\tau)} d\boldsymbol{k}\, d\omega, \tag{2.2}$$

we derive from the linearized induction equation (Rüdiger 1990) the relation

$$(-i\omega+\eta k^2)\hat{\boldsymbol{B}}(k,\omega) = ik_j \int \{\hat{\boldsymbol{m}}(\boldsymbol{k}-\boldsymbol{q},\omega)\hat{\rho}^{-1}(\boldsymbol{q})\bar{B}_j - \hat{m}_j(\boldsymbol{k}-q,\omega)\hat{\rho}^{-1}(\boldsymbol{q})\bar{\boldsymbol{B}}\}\, d\boldsymbol{q} \tag{2.3}$$

($B = \bar{B} + B'$), where the momentum density $\boldsymbol{m} = \rho \boldsymbol{u}'$ has been used instead of the random velocity u'. Equation (2.3) directly leads to

$$\rho\mathscr{E}_f = i\varepsilon_{fpi}\bar{B}_j \int \frac{k_j}{-i\omega+\eta k^2} \langle \hat{m}_p(\boldsymbol{k}',\omega')\hat{m}_i(\boldsymbol{k}-\boldsymbol{q},\omega)\rangle \times \hat{\rho}^{-1}(\boldsymbol{q})\, d\boldsymbol{q}\, d\boldsymbol{k}\, d\boldsymbol{k}'\, d\omega\, d\omega' + \varepsilon_{fpi}G_j\bar{B}_i\rho^{-1}\int \frac{\hat{M}^0_{jp}(k,\omega)}{-i\omega+\eta k^2} d\boldsymbol{k}\, d\omega, \tag{2.4}$$

where $\hat{M}^0$ is the momentum density spectral tensor for a given *homogeneous* fluid.

If $\hat{M}^0$ is a symmetric tensor, the first term in Eq. (2.4) can be simplified to

$$\rho\mathscr{E}_f = -i\rho^{-1}\bar{B}_j\varepsilon_{fip}\int \frac{k_j}{-i\omega+\eta k^2}\langle \hat{m}_p(k',\omega') \times \hat{m}_i(k,\omega)\rangle\, d\boldsymbol{k}\, d\boldsymbol{k}'\, d\omega\, d\omega' + \varepsilon_{fpi}G_j\bar{B}_i\rho^{-1}\int \frac{\hat{M}^0_{jp}(k,\omega)}{-i\omega+\eta k^2} d\boldsymbol{k}\, d\omega. \tag{2.5}$$

Now we turn to the momentum equation. It reads in its linearized form

$$\{-i\omega+\nu k^2+i\nu(\boldsymbol{G}\boldsymbol{k})\}\hat{\boldsymbol{m}} - \frac{1}{\mu} i(\boldsymbol{k}\bar{\boldsymbol{B}})\hat{\boldsymbol{B}} = \hat{\boldsymbol{f}}^s, \tag{2.6}$$

where f^s is the solenoidal part of the *fluctuating force* f driving the turbulence: $\hat{f}^s_i = (\delta_{ij} - k_i k_j/k^2)\hat{f}_j$. Density fluctuations as the source of "normal" buoyancy have been neglected in the present paper.

Note the derivation of $\mathscr{E}$ as linear in the scale ratio $l_{\rm corr}/L$. This is why G is considered as constant; allowance for spatial variations of G would involve higher orders of $l_{\rm corr}/L$. The magnetic field fluctuations can be replaced in Eq. (2.6) by using Eq. (2.3). This yields

$$\{-i\omega+\nu k^2+(\boldsymbol{k}\boldsymbol{V})^2/(-i\omega+\eta k^2)\}\hat{\boldsymbol{m}} + i\nu(\boldsymbol{G}\boldsymbol{k})\hat{\boldsymbol{m}} - G_j\frac{(\boldsymbol{k}\boldsymbol{V})^2}{(-i\omega+\eta k^2)}\frac{\partial\hat{\boldsymbol{m}}}{\partial k_j} - \frac{i(\boldsymbol{k}\boldsymbol{V})\,\boldsymbol{V}}{(-i\omega+\eta k^2)}(\boldsymbol{G}\hat{\boldsymbol{m}}) = \hat{\boldsymbol{f}}^s, \tag{2.7}$$

with $V = \bar{B}/(\mu\rho)^{1/2}$ as the Alfvén velocity. Equation (2.7) must be solved by a perturbation method with the scale ratio $l_{\rm corr}/L \approx G/k$ as a small parameter.

In the zeroth order one finds that

$$\hat{\boldsymbol{m}}(k,\omega) = \hat{\boldsymbol{m}}^0(k,\omega)/N(k,\omega), \tag{2.8}$$

with

$$N = 1 + (\boldsymbol{k}\boldsymbol{V})^2/((-i\omega+\eta k^2)(-i\omega+\nu k^2)), \tag{2.9}$$

in which

$$\hat{\boldsymbol{m}}^0 = \hat{\boldsymbol{f}}^s/(-i\omega+\nu k^2) \tag{2.10}$$

is thought of as the momentum density for an "original" turbulence existing without any magnetic field.

In the next step all the first-order terms in the scale ratio are collected:

$$\hat{\boldsymbol{m}} = \left\{1 - \frac{i\nu(\boldsymbol{G}\boldsymbol{k})}{N(-i\omega+\nu k^2)}\right\}\frac{\hat{\boldsymbol{m}}^0}{N} + \frac{i(\boldsymbol{k}\boldsymbol{V})(\boldsymbol{G}\hat{\boldsymbol{m}}^0)\,\boldsymbol{V}/N^2}{(-i\omega+\nu k^2)(-i\omega+\eta k^2)} + \frac{i(\boldsymbol{k}\boldsymbol{V})^2 G_j}{(-i\omega+\nu k^2)(-i\omega+\eta k^2)}\frac{1}{N}\frac{\partial}{\partial k_j}\frac{\hat{\boldsymbol{m}}^0}{N}. \tag{2.11}$$

With this relation the spectral tensor for the momentum density can be expressed in terms of the spectral tensor for $\hat{m}^0$ through a linear relation.

The original turbulence will be assumed to be statistically steady but *not homogeneous.* We adopt for the turbulence with quasi-isotropic spectral tensor (Kichatinov 1987), which certainly is the most simple representation for inhomogeneous turbulence:

$$\langle \hat{m}_i^0(z,\omega)\hat{m}_j^0(z',\omega')\rangle = \frac{\hat{E}(k,\omega,\kappa)}{16\pi k^2}\{\delta_{ij}-k_ik_j/k^2 + (\kappa_ik_j-\kappa_jk_i)/2k^2\}\delta(\omega+\omega'), \qquad (2.12)$$

with $\boldsymbol{k}=(z-z')/2$ and $\boldsymbol{\kappa}=z+z'$. We have only kept in Eq. (2.12) the terms up to the first order in the scale ratio $l_{\rm corr}/L$. The function $\hat{E}$ in Eq. (2.12) is the Fourier transform of the local spectrum E:

$$E(k,\omega,x)=\int \hat{E}(k,\omega,\kappa)e^{i(x\kappa)}\,d\boldsymbol{\kappa},$$

$$\langle m^{02}\rangle = \int_0^\infty E(k,\omega,\kappa)\,dk\,d\omega.$$

The random field defined by Eq. (2.12) is locally isotropic, i.e.

$$\langle m_i^0(x,t)m_j^0(x,t)\rangle = \tfrac{1}{3}\langle \boldsymbol{m}^{02}\rangle\delta_{ij}, \qquad (2.14)$$

but the gradient of $\langle \boldsymbol{m}^{02}\rangle$ does not vanish. These features are identical to that of the tensor given in Krause & Rädler (1980). Indeed, the tensor (2.12) describes exactly the situation of their "shear turbulence" model as a special case if

$$\hat{E}\approx m^2\delta(\kappa)+i\frac{\partial m^2}{\partial x_k}\frac{\partial\delta(\kappa)}{\partial\kappa_k}$$

is chosen.

Another big advance of the spectral tensor (2.12) is its factorization in the first-order approximation in the scale ratio,

$$E(k,\omega,x)=\rho^2(x)q(k,\omega,x), \qquad (2.15)$$

with q as the local velocity spectrum,

$$\langle \boldsymbol{u}^0(x)^2\rangle = \int_0^\infty q(k,\omega,x)\,dk\,d\omega. \qquad (2.16)$$

That is the property which allows the separation of effects which are due to the density gradient from those due to gradient of the turbulence intensity. In particular, for the calculation of the alpha-effect such a separation is of great importance.

The proof of statement (2.15) starts from Eq. (2.12), which can also be written as

$$E(k,\omega,x)\delta(\omega+\omega')=8\pi k^2\int\langle \hat{m}_i^0(\boldsymbol{\kappa}/2-\boldsymbol{k},\omega')\hat{m}_i^0(\boldsymbol{\kappa}/2+\boldsymbol{k},\omega)\rangle \times e^{i\boldsymbol{\kappa x}}\,d^3\kappa. \qquad (2.17)$$

Inverse transformation leads to

$$E(k,\omega,x)=\frac{8\pi k^2}{(2\pi)^4}\int \rho(\boldsymbol{x}-\xi/2)\rho(\boldsymbol{x}+\xi/2)\langle u_i'(\boldsymbol{x}-\xi/2,t) \times u_i'(\boldsymbol{x}+\xi/2,t+\tau)\rangle e^{i(\boldsymbol{k}\xi+\omega\tau)}\,d^3\xi\,d\tau, \qquad (2.18)$$

yielding Eq. (2.15), with property (2.16) in first order in the ratio of correlation length and density scale height.

For homogeneous turbulence we have $\hat{E}(k,\omega,\kappa)=E(k,\omega)\,\delta^3(\kappa)$. The spectral tensor for the magnetized fluid reads in this case

$$\hat{M}_{jp}^0(k,\omega)=\frac{E(k,\omega)(\delta_{jp}-k_jk_p/k^2)}{16\pi k^2 N(k,\omega)N^*(k,\omega)}. \qquad (2.19)$$

Equation (2.5) for the mean electromotive force contains, however, the spectral tensor taking into account the first-order terms in the scale ratio $l_{\rm corr}/L$. This derivation can be done easily using Eqs. (2.11) and (2.12) and presents no essential difficulties. The result is bulky, however, and we shall not write it in detail.

3. The effective-field advection

The information on the spectral properties of the turbulence supplied by Eqs. (2.12)–(2.16) suffices to reduce the expression for the mean EMF $\mathscr{E}$ to its traditional form where only integrations over wave number k and frequency ω are left. After such reductions we find

$$\mathscr{F}=(\boldsymbol{U}_{\rm dia}+\boldsymbol{U}_{\rm dens})\times\boldsymbol{B} \qquad (3.1)$$

with

$$\boldsymbol{U}_{\rm dens}=\boldsymbol{G}\int_0^\infty \mathscr{R}_{\rm dens}(k,\omega,B)\frac{\eta k^2 q(k,\omega,x)}{\omega^2+\eta^2k^4}\,dk\,d\omega, \qquad (3.2)$$

and

$$\boldsymbol{U}_{\rm dia}=-\nabla\int_0^\infty \mathscr{R}_{\rm dia}(k,\omega,B)\frac{\eta k^2 q(k,\omega,x)}{\omega^2+\eta^2k^4}\,dk\,d\omega. \qquad (3.3)$$

The effective velocities $U_{\rm dens}$ and $U_{\rm dia}$ are consequences of the non-uniformity of density and turbulence intensity, respectively, where the latter is attributed to the known diamagnetic pumping. The velocities (3.2) and (3.3) depend on the magnetic field through the kernels $\mathscr{R}_{\rm dens}$ and $\mathscr{R}_{\rm dia}$.

With the new parameters

$$\beta^2=\frac{(kV)^2}{[(\omega^2+\nu^2k^4)(\omega^2+\eta^2k^4)]^{1/2}} \qquad (3.4)$$

and

$$\cos\varphi=\frac{\nu\eta k^4-\omega^2}{[(\omega^2+\nu^2k^4)(\omega^2+\eta^2k^4)]^{1/2}} \qquad (3.5)$$

it follows that

$$\mathscr{R}_{\rm dens}=\frac{1}{16\beta^2}\left\{\frac{4\beta^2}{1+2\beta^2\cos\varphi+\beta^4}-\frac{9+\beta^2}{4\beta\sin(\varphi/2)}\times\log\left(\frac{1-2\beta\sin(\varphi/2)+\beta^2}{1+2\beta\sin(\varphi/2)+\beta^2}\right)-\frac{9-\beta^2}{4\beta\cos(\varphi/2)}\left[\arctan\left(\frac{\beta-\sin(\varphi/2)}{\cos(\varphi/2)}\right)+\arctan\left(\frac{\beta+\sin(\varphi/2)}{\cos(\varphi/2)}\right)\right]\right\}. \qquad (3.6)$$

and

$$\mathscr{R}_{\rm dia}=\frac{1}{16\beta^3}\left\{\frac{1}{\sin(\varphi/2)}\log\left(\frac{1-2\beta\sin(\varphi/2)+\beta^2}{1+2\beta\sin(\varphi/2)+\beta^2}\right)+\frac{2}{\cos(\varphi/2)}\left[\arctan\left(\frac{\beta-\sin(\varphi/2)}{\cos(\varphi/2)}\right)+\arctan\left(\frac{\beta+\sin(\varphi/2)}{\cos(\varphi/2)}\right)\right]\right\}. \qquad (3.7)$$

All our following results are derived from these expressions. We are particularly interested in the discussion of the limiting cases of weak and strong magnetic fields – and that for both the effects of the turbulent buoyancy as well as the diamagnetic pumping.

3.1. Non-linear diamagnetism

3.1.1. Weak fields

For small magnetic fields only second-order terms in the series expansion of the kernels after β are relevant:

$$\mathscr{R}_{\rm dia} = \frac{1}{6} - \frac{\beta^2}{5}\cos\varphi; \tag{3.8}$$

hence,

$$\boldsymbol{U}_{\rm dia} = -\nabla \int_0^\infty \frac{\eta k^2 q(k,\omega,x)}{\omega^2+\eta^2 k^4}\left\{\frac{1}{6} - \frac{\beta^2}{5}\cos\varphi\right\} {\rm d}k\,{\rm d}\omega. \tag{3.9}$$

For $\beta = 0$ the known linear expression for the velocity of diamagnetic pumping is reproduced:

$$\boldsymbol{U}_{\rm dia} = -\tfrac{1}{2}\nabla\eta_{\rm T} \tag{3.10}$$

(cf. Krause & Rädler 1980), with

$$\eta_{\rm T} = \frac{1}{3}\int_0^\infty \frac{\eta k^2 q(k,\omega)}{\omega^2+\eta^2 k^4}\,{\rm d}k\,{\rm d}\omega \tag{3.11}$$

as the linear magnetic diffusivity.

The second-order term in Eq. (3.9) shows that weak magnetic field quenches the diamagnetism except for the case where negative values of $\cos\varphi$ would dominate the integral. This case, however, is rather exotic because the dominance of negative $\cos\varphi$ would mean that the turbulence intensity for the weak-field limit is increased by the magnetic field action contrary to the usually expected quenching.

3.1.2. Strong fields

If the field is strong, we may keep in Eq. (3.7) the lowest-order terms in β^{-1} only. This yields

$$\mathscr{R}_{\rm dia} = \frac{\pi}{8\beta^3\cos(\varphi/2)}. \tag{3.12}$$

The effect of the diamagnetic pumping decreases with increasing magnetic field, i.e. with B^{-3}. That is a rather fast decrease; it is faster, in particular, than that for the turbulent velocities, $\langle u^2\rangle\beta B^{-1}$ (Rüdiger 1974).

The fast decrease of the diamagnetic transport velocity with increasing B is probably caused by the quasi-dimensionality of the turbulence in the strong-field limit. It may be easily shown that a 2D random flow does not produce diamagnetic transport of magnetic field parallel to that axis along which the flow does not vary. The only non-zero component of such a field obeys the same Eq (1.1) as scalar fields do and there is, thus, no diamagnetism for such fields.

3.2. Turbulent buoyancy

3.2.1. Weak fields

Series expansion of $\mathscr{R}_{\rm dens}$ in terms of β provides

$$\mathscr{R}_{\rm dens} = -\frac{2}{15}\beta^2\cos\varphi. \tag{3.13}$$

Substitution of this expression into Eq. (3.2) yields the weak-field representation for the transport velocity

$$\boldsymbol{U}_{\rm dens} = -\boldsymbol{G}\frac{2B^2}{15\mu\rho}\int_0^\infty \frac{\eta k^4(\nu\eta k^4-\omega^2)q(k,\omega,x)}{(\omega^2+\nu^2k^4)(\omega^2+\eta^2k^4)^2}\,{\rm d}k\,{\rm d}\omega. \tag{3.14}$$

As the integral in this expression is "almost always" positive, the turbulent buoyancy is directed towards the lower density, i.e. *upwards*, amplifying the "usual" buoyancy.

That is an important and unexpected result. Possibly the upward transport is due to the magnetic suppression of the turbulence. In any case, our result is that for sufficiently weak magnetic fields mean-field buoyancy and flux-tube buoyancy are acting parallel transporting the magnetic flux upwards. The questions remain whether that is true also for higher fields and what are the relative amplitudes of the effect.

3.2.2. Strong fields

In the strong-field limit, the turbulence approaches two-dimensionality. In accordance with Eq. (3.12) the diamagnetic pumping is weak, and thus the density effect may easily dominate. Indeed it is reduced only as

$$\mathscr{R}_{\rm dens} \cong \frac{\pi}{32\beta\cos(\varphi/2)}. \tag{3.15}$$

The positivity of this expression indicates that the transport is now *downwards*, in great contrast to the above result for weak fields.

The variation of $U_{\rm dens}$ with $1/B$ is quite similar to that of the turbulence intensity $\langle u'^2\rangle$ in the strong-field approach.

4. Comparison with flux-tube buoyancy

To simplify estimates of the velocities of the turbulent buoyancy, we adopt the simplest representation for the spectral function,

$$q(k,\omega,x) = 2\langle u'^2\rangle\delta(k-1/l)\delta(\omega) \tag{4.1}$$

and

$$\nu = \eta = l\sqrt{\langle u'^2\rangle}, \tag{4.2}$$

reflecting a turbulence model very close to the mixing length theory (Kichatinov 1991), with l being the mixing length. Equations (4.1) and (4.2) substituted into Eq. (3.14) gives the rise velocity

$$U_{\rm dens} = -\frac{2lV^2}{15\sqrt{\langle u'^2\rangle}}G. \tag{4.3}$$

On the other hand, the velocity of an Archimedic rise of a flux tube of radius R in a turbulent fluid is

$$U_{\rm buoy} = C\frac{R^2V^2}{Hl\sqrt{\langle u'^2\rangle}}, \tag{4.4}$$

(for $R < H$, Parker 1978), with C a constant of order unity and H the pressure scale-height. The mixing length is usually assumed to be of the order H; hence, the product Gl approaches unity. Under these conditions, the velocities (4.3) and (4.4) are roughly equal for not too small flux-tubes ($R \cong l$). For thinner tubes, the "turbulent buoyancy" transport [Eq. (4.3)] is more effective.

We find that the turbulent buoyancy of weak fields ($V < \sqrt{\langle u'^2 \rangle}$ acts in the same direction ("upwards") and that its strength is at least comparable to the usual buoyancy. For weak fields the problems for the dynamo are, thus, amplified.

For strong magnetic field our "turbulent" buoyancy acts *downwards*, i.e. in the *opposite direction* to the traditional magnetic buoyancy, but it decreases for increasing magnetic field. It is questionable whether the expression (4.4) is still valid in this regime. For an answer to this outstanding question it is necessary to repeat our calculations but with the Archimedic buoyancy term in the momentum equation involved.

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