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Structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere

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Abstract

The spatial structure of fast magnetosonic eigenoscillations of an axisymmetric magnetosphere is investigated theoretically. It is shown that transparent regions of the eigenmodes where the bulk of the fast magnetosonic oscillation energy is concentrated lie near the equatorial plane. The transparent regions have a narrow channel from the magnetosphere to the solar wind near the equatorial plane. This enables the magnetosonic waves to penetrate from the solar wind deep into the magnetosphere, as well as some of the energy of intramagnetospheric magnetosonic oscillations to escape to the solar wind. Some of the eigenmodes have an isolated portion of the transparent region above the plasmapause that forms a well-known magnetospheric cavity for fast magnetosonic waves.

1. Introduction

The intention of this paper is to carry out a theoretical investigation into the spatial structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere. The axial symmetry assumes that the geomagnetic field and plasma are inhomogeneous both along magnetic field lines and across magnetic shells. Geomagnetic field lines are curved and traverse the ionosphere in the Northern and Southern Hemispheres (the dipole magnetic field being a typical example). It is further assumed that the magnetopause is a sufficiently sharp transition layer, inside of which plasma parameters vary from values characteristic for the magnetosphere to those typical of the solar wind. Magnetosonic oscillations can overcome such a boundary and, with a certain effectiveness, penetrate from the solar wind deep into the magnetosphere and back. For the magnetospheric plasma a sufficiently realistic model is used in this paper, which assumes an abrupt change in plasma density at the plasmapause.

Three branches of MHD oscillations (Alfven waves, and fast and slow magnetosound) exist in the plasma embedded in a magnetic field. Oscillations of the three types are well understood theoretically in a homogeneous plasma. This paper is based on using a "cold" plasma approximation ($\beta = 0$). This means that slow magnetosonic oscillations are not assumed by the magnetospheric model under consideration. The only mode of magnetosonic oscillations in such a system is fast magnetosound. Therefore it is fast (compressional Alfven) waves that will be meant by the term magnetosonic oscillations in the discussion to follow. The term Alfven waves will be used to designate transverse (shear) Alfven waves. In an inhomogeneous plasma, only Alfven waves may be thought of as being known in sufficient detail. In an axisymmetric system any monochromatic oscillations can be expanded into eigenharmonics of the form $\exp(\mathrm{i}m\phi - \mathrm{i}\omega t)$, where ϕ is the azimuthal angle, m is the azimuthal wave number, and ω is the wave frequency. For Alfven waves with small azimuthal wave numbers ($m \sim 1$) the theory of field line resonance has been developed to date to describe the structure of the waves that are excited at resonance magnetic shells by fast magnetosound that enters the magnetosphere from outside.

Originally, this theory was developed for a one-dimensionally inhomogeneous model of the magnetosphere assuming the plasma to be inhomogeneous across straight geomagnetic field lines [*Radoski*, 1974; *Southwood*, 1974; *Chen and Hasegawa*, 1974]. Subsequently, the theory was generalized for magnetospheric models that assume the plasma to be inhomogeneous both in one of the transverse coordinates and in longitudinal coordinate. In papers of *Kivelson and Southwood* [1986] and *Southwood and Kivelson* [1986], a box model with straight field lines was used, while in papers of *Leonovich and Mazur* [1989], *Chen and Cowley* [1989], and *Wright* [1992], an axisymmetric model was employed with curved geomagnetic (dipole-type) field lines.

For oscillations with large azimuthal wave numbers ($m \gg 1$), there also is a theory describing their spatial structure in an axisymmetric model of the magnetosphere. Unlike the oscillations with small m, magnetosonic oscillations with $m \gg 1$ cannot effectively penetrate the magnetosphere from outside. In this connection, Alfven oscillations with $m \gg 1$ require that their source be located on the same field lines where they are generated. In the outer magnetosphere, which is filled with warm plasma, the possibility exists of slow magnetosound propagating along geomagnetic field lines. This provides a possibility for the resonance interaction of slow magnetosonic and Alfven oscillations [*Southwood and Saunders*, 1985; *Walker*, 1987]. *Leonovich and Mazur* [1993, 1997] used, as the source of such Alfven oscillations, external currents in the conducting ionospheric layer.

Magnetosonic oscillations in inhomogeneous plasmas are significantly more poorly understood when compared to Alfven oscillations, which is due to their more complicated dispersion. Noteworthy are papers of *Gul'elmi* [1970, 1972], who considered the possibility for magnetosonic oscillations to be trapped within the magnetospheric resonator formed by a non-monotonic plasma density gradient near the plasmapause. *Kivelson and Southwood* [1986] and *Southwood and Kivelson* [1986] investigated analytically, in terms of the one-dimensionally inhomogeneous magnetospheric model (box model), the structure of magnetosonic eigenoscillations of the magnetosphere. When passing to more realistic two-dimensionally inhomogeneous models of the magnetosphere with curved geomagnetic field lines, an analytical investigation of magnetosonic oscillations was not practicable because of emerging mathematical difficulties. In this connection, numerical simulation methods had to be used to investigate magnetosonic oscillations in terms of such models of the magnetosphere. In papers of *Lee and Lysak* [1989, 1991] and *Lee* [1996] on the basis of using a numerical simulation it was shown that the structure of magnetosonic oscillations in a dipole model of the magnetosphere differs essentially from that obtained in the box model with straight field lines. Specifically, it was shown that most of the energy of these oscillations is concentrated near the equatorial plane and decreases rapidly with distance from it. This inference is also confirmed by observational evidence [*Yumoto et al.*, 1985; *Cheng and Lin*, 1987; *Engebretson et al.*, 1988].

Yet a numerical simulation fails to answer all questions arising in the study of magnetosonic oscillations. This method makes it possible to construct a total field of MHD oscillations inside the magnetosphere from given initial and boundary conditions. Unfortunately, while it answers the question "What?", it does not answer the question "Why?". In other words, it gives no way of investigating the inner structure of separate magnetosonic eigenmodes that comprise a total field of magnetosonic oscillations of the magnetosphere. In this context, a numerical simulation should be regarded as a numerical experiment. However, research into the structure of separate modes of magnetosonic oscillations is of fundamental importance in gaining a more penetrating insight into the distribution properties of these oscillations in the magnetosphere and their interaction with other oscillation modes.

This paper is based on using a combined method (analytical + numerical investigation), enabling us to study the structure of individual oscillation modes. This enables us to appreciate results from previous work where numerical simulation methods were used. In addition, since we are using a sufficiently realistic axisymmetric model of the magnetosphere, we are able to carry out this investigation not only inside the magnetosphere but extend it to the solar wind region.

This paper is organized as follows. Section 2 presents the coordinate system, and equations are obtained, which describe monochromatic MHD oscillations of a three-dimensionally inhomogeneous magnetosphere. In section 3, we obtain the solutions for the equation describing magnetosonic oscillations of an axisymmetric magnetosphere within the WKB approximation. Sections 4 and 5 give an outline of the model of the medium and present a numerical investigation of the magnetosonic mode structure in longitudinal coordinate. The oscillation structure in radial coordinate is described in the WKB approximation in this case. Main results of this study are summarized in the conclusions.

2. The Coordinate System and Equations of MHD Oscillations

We use a curvilinear orthogonal coordinate system (x^1, x^2, x^3) tied to geomagnetic field lines (see Figure 1). In this coordinate system the surface $x^1 = \text{const}$ coincides with the magnetic shell, the coordinate x^2 specifies a field line on a given magnetic shell, and x^3 varies along the field line. In an axisymmetric magnetosphere the azimuthal angle ϕ and the radius of the magnetic shell are conveniently used as the coordinate x^2 and x^1, respectively. After that, the coordinate x^3 is fixed by requiring the orthogonality of the coordinate system. The square of a length element in this coordinate system has the form

$$\mathrm{d}s^2 = g_1(\mathrm{d}x^1)^2 + g_2(\mathrm{d}x^2)^2 + g_3(\mathrm{d}x^3)^2,$$

where $g_i (i = 1, 2, 3)$ are diagonal components of the metric tensor. When constructing the theory of MHD oscillations in the magnetosphere with curved geomagnetic field lines, including calculations within the WKB approximation, we take advantage of generalized curvilinear coordinates (x^1, x^2, x^3) without concretizing their selection. On the other hand, numerical calculations, performed in sections 4 and 5, use a dipole model of the geomagnetic field, and particular components are specified for the metric tensor components. In the perfect MHD approximation a closed system of equations describing MHD oscillations is of the form

$$\varrho_0 \nabla_t \mathbf{v} = [[\nabla \times \mathbf{B}] \times \mathbf{B}_0]/4\pi, \qquad (1a)$$

$$\nabla_t \mathbf{B} = [\nabla \times [\mathbf{v} \times \mathbf{B}_0]], \qquad (1b)$$

where $\nabla_t = \partial/\partial t$ and ϱ_0 and $\mathbf{B}_0$ are, respectively, the unperturbed plasma density and the undisturbed magnetic field. It is also assumed that the unperturbed plasma velocity $\mathbf{v}_0 = 0$. The perturbed electric field is related to the perturbed velocity by the relation

$$\mathbf{E} = -[\mathbf{v} \times \mathbf{B}_0]/c. \qquad (2)$$

For subsequent calculations it is convenient to use the covariant vector components v_i, B_i and E_i $(i = 1, 2, 3)$, which are related to the usual physical components of the vectors $\bar{v}_i, \bar{B}_i$, and $\bar{E}_i$ by the relations $v_i = \sqrt{g_i}\bar{v}_i$, $B_i = \sqrt{g_i}\bar{B}_i$, and $E_i = \sqrt{g_i}\bar{E}_i$. From (1a) and (2) it follows that $v_3 = 0$ and $E_3 = 0$. For the other two components of the perturbed velocity vector, from (2) we obtain

$$v_1 = \frac{cE_2}{B_0 p}, \qquad v_2 = -\frac{cE_1 p}{B_0}, \qquad (3)$$

where $p = \sqrt{g_2/g_1}$. In all subsequent calculations we will be concerned with monochromatic oscillations of the form $\exp(-\mathrm{i}\omega t)$, where ω is the oscillation frequency. Then, substitution of (3) into (1a) gives

$$B_1 = \mathrm{i}\frac{c}{\omega}\frac{p^{-1}}{\sqrt{g_3}}\nabla_3 E_2, \qquad B_2 = -\mathrm{i}\frac{c}{\omega}\frac{p}{\sqrt{g_3}}\nabla_3 E_1,$$

$$B_3 = -\mathrm{i}\frac{c}{\omega}\frac{g_3}{\sqrt{g}}\left[\nabla_1 E_2 - \nabla_2 E_1\right], \tag{4}$$

where $\nabla_i \equiv \partial/\partial x^i \quad (i = 1, 2, 3), \quad g = g_1 g_2 g_3$. Substituting the relations (3) and (4) into (1a) gives a system of two coupled equations:

$$\begin{aligned} \hat{L}_P E_2 &= -\frac{1}{\sqrt{g_3}}\nabla_1\frac{g_3}{\sqrt{g}}(\nabla_1 E_2 - \nabla_2 E_1), \\ \hat{L}_T E_1 &= \frac{1}{\sqrt{g_3}}\nabla_2\frac{g_3}{\sqrt{g}}(\nabla_1 E_2 - \nabla_2 E_1), \end{aligned} \tag{5}$$

where

$$\begin{aligned} \hat{L}_P &= \frac{1}{\sqrt{g_3}}\nabla_3\frac{p^{-1}}{\sqrt{g_3}}\nabla_3 + \frac{\omega^2}{pA^2}, \\ \hat{L}_T &= \frac{1}{\sqrt{g_3}}\nabla_3\frac{p}{\sqrt{g_3}}\nabla_3 + \frac{p\omega^2}{A^2}, \end{aligned}$$

are the poloidal and toroidal operators [see *Leonovich and Mazur*, 1993], with A being the Alfven velocity. Let the two-dimensional vector $\mathbf{E}_\perp = (E_1, E_2)$ be expanded in terms of the orthogonal vectors $\nabla_\perp\varphi$ and $[\nabla_\perp \times \boldsymbol{\psi}]$ (in this case the vector $\boldsymbol{\psi}$ has only one nonzero longitudinal component $\boldsymbol{\psi} = (0, 0, \psi)$)

$$\mathbf{E}_\perp = -\nabla_\perp\varphi + [\nabla_\perp \times \boldsymbol{\psi}].$$

Such an expansion in respect of MHD oscillations in the magnetosphere was used in papers of *Klimushkin* [1994] and *Fedorov et al.* [1998]. It is evident from this that φ represents a usual electric potential of the MHD oscillation field, and $\boldsymbol{\psi}$ is a vector potential of this field. They showed that the potential component φ describes the Alfven oscillation field, and the vortical component ψ describes the field of magnetosonic oscillations of the magnetosphere. Substitution of these relations into (5) gives the following system of coupled equations for φ and ψ:

$$\sqrt{g_3}\hat{L}_P(\nabla_2\varphi + \frac{g_2}{\sqrt{g}}\nabla_1\psi) =$$
$$-\nabla_1\frac{g_3}{\sqrt{g}}(\nabla_1\frac{g_2}{\sqrt{g}}\nabla_1\psi + \nabla_2\frac{g_1}{\sqrt{g}}\nabla_2\psi), \tag{6a}$$

$$\sqrt{g_3}\hat{L}_T(\nabla_1\varphi - \frac{g_1}{\sqrt{g}}\nabla_2\psi) =$$
$$\nabla_2\frac{g_3}{\sqrt{g}}(\nabla_1\frac{g_2}{\sqrt{g}}\nabla_1\psi + \nabla_2\frac{g_1}{\sqrt{g}}\nabla_2\psi). \tag{6b}$$

Upon differentiating (6a) with respect to x^2 and (6b) with respect to x^1 and combining the resulting equations, we obtain

$$\nabla_1\sqrt{g_3}\hat{L}_T\nabla_1\varphi + \nabla_2\sqrt{g_3}\hat{L}_P\nabla_2\varphi =$$
$$\nabla_1\sqrt{g_3}\hat{L}_T\frac{g_1}{\sqrt{g}}\nabla_2\psi - \nabla_2\sqrt{g_3}\hat{L}_P\frac{g_2}{\sqrt{g}}\nabla_1\psi. \tag{7}$$

Equations (6b) and (7) form a closed (with respect to φ and ψ) system of equations describing monochromatic MHD oscillations of a three-dimensionally inhomogeneous magnetosphere.

In this paper we shall restrict our consideration to the oscillations of an axisymmetric magnetosphere, where they can be resolved into azimuthal components of the form $\exp(\mathrm{i}k_2 x^2)$ (if $x^2 = \phi$ is the azimuthal angle, then $k_2 = m$ is the azimuthal wave number). In this case the system of equation for a separate azimuthal harmonic may be represented as

$$(\nabla_1\sqrt{g_3}\hat{L}_T\nabla_1 - k_2^2\sqrt{g_3}\hat{L}_P)\varphi =$$
$$\mathrm{i}k_2(\nabla_1\sqrt{g_3}\hat{L}_T\frac{g_1}{\sqrt{g}}\psi - \sqrt{g_3}\hat{L}_P\frac{g_2}{\sqrt{g}}\nabla_1\psi), \tag{8a}$$

$$\nabla_1\frac{g_2}{\sqrt{g}}\nabla_1\psi - k_2^2\frac{g_1}{\sqrt{g}}\psi + \sqrt{\frac{g}{g_3}}\hat{L}_T\frac{g_1}{\sqrt{g}}\psi =$$
$$-\frac{\mathrm{i}}{k_2}\sqrt{\frac{g}{g_3}}\hat{L}_T\nabla_1\varphi. \tag{8b}$$

The left-hand sides of (8a) and (8b) involve operators which in the case of a homogeneous plasma give, respectively, dispersion relations for Alfven ($\omega = k_\| A$) and fast magnetosonic ($\omega = kA$) waves. For Alfven waves we have ($\psi = 0, \varphi \neq 0$), and hence they represent purely potential-field oscillations. For magnetosonic waves, on the contrary, ($\psi \neq 0, \varphi = 0$), and they are purely vortical. In the case of an inhomogeneous plasma the Alfvenic and magnetosonic oscillations can be described, as before, in terms of the scalar potential φ and the vector potential $\boldsymbol{\psi}$, respectively. The interaction of these waves in a inhomogeneous plasma is described by the right-hand sides of these equations.

3. Structure of Magnetosonic Eigenmodes in the WKB Approximation

From the solution of the problem of Alfven oscillations of the magnetosphere it is known that their interaction with magnetosonic waves occurs in a narrow vicinity of resonance magnetic shells where the magnetosonic wave frequency coincides with the eigenfrequency of the Alfven oscillations of the magnetic shell. Far from these resonance shells, the mode interaction may be neglected, and the homogeneous equation (8b) can be used to investigate the structure of magnetosonic oscillations. This can be done if the Alfven oscillations are strongly damped ones. Furthermore, it will be assumed that the wavelength of the magnetosonic oscillations under consideration in the direction across the magnetic shells is much shorter than the characteristic inhomogeneity scale of magnetospheric plasma. Such a treatment is also qualitatively applicable for investigating oscillations whose wavelength is of the order of the magnetospheric inhomogeneity. Alternatively, oscillations with the opposite ratio of the wavelength to the inhomogeneity scale inside the magnetosphere are nonexistent. Such an approach makes it possible to seek the solution of (8b) in the form

$$\psi = H(x^1, x^3)\exp[\mathrm{i}(\tilde{\Phi}(x^1) + k_2x^2 - \omega t)], \qquad (9)$$

where $\tilde{\Phi}$ is a large quasi-classical phase and $H(x^1, x^3)$ is a function describing the oscillation structure in the direction along geomagnetic field lines.

Substitution of this equation into the homogeneous equation (8b) in the zero order of perturbation theory gives an equation for the function $H(x^1, x^3)$:

$$\nabla_3 \frac{p}{\sqrt{g_3}} \nabla_3 \frac{g_1}{\sqrt{g}} H + \left(\frac{\omega^2}{A^2} - k_\perp^2\right) H = 0, \qquad (10)$$

where $k_\perp^2 = (k_1^2/g_1 + k_2^2/g_2)$ is the square of the transverse wave vector, with $k_1 = \nabla_1\tilde{\Phi}$. Equation (10) should be complemented by boundary conditions on the ionosphere. As will be shown below, the ionosphere lies deep inside the opaque region for the magnetosonic waves under consideration. In this connection the ideal ionospheric conductivity assumption can be considered a sufficiently good approximation. Hence it will be assumed that on the ionosphere $E_1 = E_2 = 0$, which yields

$$H|_{x^3 = x^3_\pm} = 0, \qquad (11)$$

where $x^3_\pm$ represents points at which a field line intersects the ionosphere, respectively, in the Northern and Southern Hemispheres. The problem in (10), (11) is an eigenvalue problem for the square of the wave number k_1^2. The solution of this problem is a set of eigenfunctions H_n (where $n = 0, 1, 2, 3, ...,$ is the eigenmode number) and a corresponding set of eigenvalues k_{1n}^2. Any perturbation of the magnetosound field in the magnetosphere can be expended with respect to a full set of eigenfunctions.

To imagine qualitative the form of solutions for eigenmodes, we use the WKB approximation in longitudinal coordinate x^3; i.e., we let the solution to be represented as

$$H = \exp[\mathrm{i}\Theta(x^1, x^3)],$$

where $\Theta(x^1, x^3)$ is a quasi-classical phase. Substituting this solution into (10) gives the equation for the phase $\Theta(x^1, x^3)$:

$$-(\Theta')^2 + \mathrm{i}\Theta'' + \mathrm{i}\Theta'\left(2\nabla_3 \ln\frac{g_1}{\sqrt{g}} + \nabla_3 \ln\frac{g_2}{\sqrt{g}}\right) + g_3\nabla_3\frac{g_2}{\sqrt{g}}\nabla_3\frac{g_1}{\sqrt{g}} + k_3^2 = 0, \qquad (12)$$

with $k_3^2 = g_3\left(\omega^2/A^2 - k_\perp^2\right)$. In the zero order of perturbation theory we have $\Theta_0' = \pm k_3$; hence

$$\Theta_0 = \pm\int_{x^3_-}^{x^3} k_3 \mathrm{d}x^3.$$

In the first order we obtain

$$\Theta_1 = \frac{\mathrm{i}}{2}\ln\left(\frac{|k_3|g_1}{g_3\sqrt{g}}\right).$$

Thus a general WKB solution of (10) may be represented as

$$H = f[A\exp(\mathrm{i}\Theta_0) + B\exp(-\mathrm{i}\Theta_0)], \qquad (13)$$

where $f \equiv \sqrt{g_3\sqrt{g}/|k_3|g_1}$. It make sense to carry out a further investigation of the resulting equation by specifying a particular model of the medium.

4. Model of the Medium and Eigenmode Structure as a WKB Approximation

As is apparent from the general form of (13), its behavior is governed by the $k_3 = \pm\sqrt{g_3(\Phi - k_{1n}^2)/g_1}$, where

$$\Phi = g_1\left(\frac{\omega^2}{A^2} - \frac{m^2}{g_2}\right).$$

In those magnetospheric regions where $\Phi > k_{1n}^2$ the solution is a periodic function, and where $\Phi < k_{1n}^2$ it consists of the sum of the decreasing and increasing exponentials. Hence the form of the solution is determined by the behavior of the function Φ. To investigate this function, we specify a model of the medium as follows. In this and subsequent sections we are using a dipole model of the geomagnetic field. We put $x^1 = a$, where a is the equatorial radius of a field line, $x^2 = \phi$ is the azimuthal angle, and as an longitudinal coordinate x^3 we use the angle θ between a point on a given field line and the equatorial plane (see Figure 1). Of course, they are not orthogonal ones. The length element along the field line is given by

$$\mathrm{d}l = \sqrt{g_3}\mathrm{d}x^3 = a\beta(\theta)\cos\theta\mathrm{d}\theta,$$

where $\beta(\theta) = \sqrt{1+3\sin^2\theta}$ and a is the equatorial radius of the field line under consideration. Transverse components of the metric tensor in this coordinate system have the form

$$g_1 = \cos^6\theta/\beta(\theta), \qquad g_2 = a^2\cos^6\theta.$$

It is also assumed that $k_2 = m$. The latitude where the field line intersects the ionosphere is defined as

$$\theta^* = \arccos\sqrt{r_i/a},$$

where r_i is the radius of the ionosphere.

Another important element of the model of the medium is the Alfven velocity A. For this we use the following analytical model:

$$A = \frac{A_m + A_{sw}}{2} - \frac{A_m - A_{sw}}{2}\tanh\frac{a\cos^2\theta - a_m}{\Delta_m},$$

where A_{sw} is the value of the Alfven velocity in the solar wind and Δ_m is the thickness of the magnetopause. It will be assumed throughout this paper that the magnetopause represents a sphere of radius a_m, although the model presented allows for specifying the magnetopause in any axisymmetric form. The function $A_m(a,\theta)$ describes a two-dimensional distribution of the Alfven velocity inside the magnetosphere. For this we use the following representation:

$$\begin{aligned} A_m \;=\; & \left(\frac{\beta(\theta)}{\cos^6\theta}\right)^{\nu}\left[\frac{1}{2}\left(A_1\left(\frac{a_1}{a}\right)^{\mu_1} + A_2\left(\frac{a_2}{a}\right)^{\mu_2}\right)\right. \\ - \;& \left.\frac{1}{2}\left(A_1\left(\frac{a_1}{a}\right)^{\mu_1} - A_2\left(\frac{a_2}{a}\right)^{\mu_2}\right)\tanh\frac{a-a_p}{\Delta_p}\right], \end{aligned}$$

where A_1 and A_2, respectively, are the characteristic values of the Alfven velocity in the inner and outer magnetosphere, and a_p and Δ_p are the equatorial radius and the characteristic thickness of the plasmapause, respectively. The first cofactor in this relation describes the variation of the Alfven velocity along geomagnetic field lines. This model at the following values of the parameters: $A_1 = 250$ km/s, $A_2 = 500$ km/s, $A_{sw} = 50$ km/s, $a_m = 10\,R_E$ (R_E= 6370 km - radius of the Earth), $\Delta_m = 0.5\,R_E$, $a_p = 4\,R_E$, $\Delta_p = 0.5\,R_E$, $a_1 = 2.5\,R_E$, $a_2 = 5\,R_E$, $\mu_1 = 1.5$, $\mu_2 = 1$, $\nu = 0.25$ describes sufficiently well the Alfven velocity distribution in the dayside part of the moderately disturbed magnetosphere [see *Horwitz et al.*, 1986; *Comfort*, 1986]. Figure 2 presents the equatorial distribution of $A(a,0)$ across the magnetic shells. Figure 2 also plots the main period of Alfven eigenoscillations of the magnetosphere

$$t_A = 2\int_{x_-^3}^{x_+^3}\frac{\sqrt{g_3}\mathrm{d}x^3}{A(x^1,x^3)}$$

as a function of the magnetic shell parameter $L = a/R_E$ (McIlwain parameter).

Figure 3 presents the dependence of the function $\Phi(a,\theta)$ on the longitudinal coordinate x^3 (angle θ) for four different values of the parameter m on the magnetic shell $L = 6.6$. As follows from boundary conditions on the ionosphere (11), the solutions H_n contain at least one interval on the field line where $\Phi > k_{1n}^2$. Figure 3 portrays four possible variants of the difference $\Phi - k_{1n}^2$ (I, II, III, IV), corresponding to four different types of solution for eigenmodes. The simplest solution is of type I, for which $\Phi > k_{1n}^2$ along the entire field line. The solution in this case has the form

$$H_n = Cf\sin\left(\int_{x_-^3}^{x^3} k_3'\mathrm{d}x^{3\prime}\right), \tag{14}$$

where C is an arbitrary constant, $k_3' \equiv k_3(x^1, x^{3\prime})$, and the eigenvalue condition is satisfied:

$$\int_{x_-^3}^{x_+^3} k_3\mathrm{d}x^3 = \pi(n+1),$$

(n=0,1,2,...), which determines the magnitude of the eigenvalue of k_{1n}^2. Type II solutions in the WKB approximation have the form

$$H_n = Cf\begin{cases} \sinh\left(\int_{x_-^3}^{x^3}|k_3'|\mathrm{d}x^{3\prime}\right), & x_-^3 \le x^3 < x_1^3, \\ \mathrm{e}^{\bar\psi}\sin\left(\int_{x_1^3}^{x^3}k_3'\mathrm{d}x^{3\prime} + \frac{\pi}{4}\right), & x_1^3 < x^3 < x_2^3, \\ \mp\sinh\left(\int_{x_2^3}^{x^3}|k_3'|\mathrm{d}x^{3\prime} - \bar\psi\right), & x_2^3 < x^3 \le x_+^3, \end{cases} \tag{15}$$

where two different signs minus/plus correspond to even and odd n in the eigenvalue condition

$$\int_{x_1^3}^{x_2^3} k_3 \mathrm{d}x^3 = \pi(n + \frac{1}{2}).$$

The coordinates x_1^3, x_2^3 are turning points, in which $k_{1n}^2 = \Phi(x^1, x_{1,2}^3)$. Besides, the following designation is introduced:

$$\bar{\psi} = \int_{x_-^3}^{x_1^3} |k_3| \mathrm{d}x^3 = \int_{x_2^3}^{x_+^3} |k_3| \mathrm{d}x^3.$$

The expressions for type III and IV WKB solutions are more unwieldy and are not given here, although their qualitative behavior is sufficiently evident from the two examples given above.

5. Numerical Solution of the Longitudinal Problem and Discussion of Results

Consider results from a numerical integration of (10) for the model of the medium described in the section 4. Figure 4 presents four first eigenfunctions H_n on the magnetic shell $L = 6.6$, normalized by the condition

$$\oint H_n^2 \mathrm{d}x^3 \equiv 2 \int_{x_-^3}^{x_+^3} H_n^2 \mathrm{d}x^3 = 1.$$

These calculations used the following values of the wave parameters: $m = 1, f = \omega/2\pi = 0.025$ Hz is the frequency equal to the second harmonic frequency of Alfven eigenoscillations on the magnetic shell $L = 6.6$. In Figures 4-8 , the heavy line shows functions corresponding to the eigenmode $(n = 0, m = 1, f = f_2 = 0.025\,\mathrm{Hz})$ which we shall choose as a reference mode and with respect to which we shall consider all other modes. The solutions H_1, H_2 represent solutions of the form (15) (type II in Figure 3), i.e., are periodic functions near the equatorial plane and decrease exponentially in amplitude as one approaches the ionosphere. The solutions H_3, H_4 are solutions in (14) (type I in Figure 3), having the form of periodic functions along the entire field line.

Figure 5 presents the dependencies of eigenvalues of k_{1n}^2 on parameters of the magnetic shell L for the five first eigenmodes (n=0,1,2,3,4). Figure 5b presents the dependence $k_{1n}^2(L)$ over the entire range of the shells under consideration $1.5 \le L \le 15$, and Figure 5a shows the dependence $k_{1n}^2(L)$ inside the magnetosphere $1.5 \le L \le 10$. It is evident that for the reference harmonic $(n = 0, m = 1, f = f_2)$, $k_{10}^2 > 0$ when L >2.2 , and when L <2.2, - $k_{10}^2 < 0$. For the second harmonic the inequality $k_{11}^2 > 0$ holds in the outer magnetosphere when L >6.1 and under the plasmapause when 3.1< L <3.6. For harmonics $n > 1$, $k_{1n}^2 > 0$ in the outer magnetosphere only.

Let us determine the transparent region for the magnetosound modes under consideration by requiring that the three inequalities $k_{1n}^2 > 0, k_2^2 > 0$, and $k_{3n}^2 > 0$ hold at a time. This means that in the region under consideration, the magnetosound wave can propagate freely in any direction. Figure 6 presents the boundaries of transparent regions for the five eigenmodes considered above. The reference mode boundary is nearest the ionosphere. At the same time, this mode is most localized near the equatorial plane. For the second eigenmode $(n = 1, m = 1, f = f_2)$ the inner boundary of the transparent region lies farther away from the ionosphere, and this region itself consists of two unassociated regions: the outer region when L >6.1 and inner when 3.1< L <3.6. This second transparent region represents a cavity for magnetosonic oscillations that was originally described by *Gul'elmi* [1970, 1972] and investigated in later work [*Zhu and Kivelson*, 1989; *Lee*, 1996; *Fedorov et al.*, 1998].

For n=2,3,4 harmonics the inner boundaries of transparent regions recede farther away from the ionosphere while the lateral boundaries move away from the equator. It is evident that all oscillation modes have a narrow channel for the escape from the magnetosphere to the solar wind. This permits the magnetosonic oscillations to penetrate from the solar wind region deep into the magnetosphere. The source for such oscillations can be provided by, for example, an instability of solar wind protons reflected from the bow shock wave front [*Potapov*, 1974; *Gul'elmi*, 1974]. Note that the form of transparent regions outside the magnetosphere can differ from that obtained in this paper because of the presence of the solar wind plasma in motion. It may be anticipated, however, that this difference at a not very large distance from the magnetosphere is not too large, because it is near the equator where the stagnation point lies, where the solar wind velocity $v_{sw} = 0$. The size of transparent regions in the magnetosphere decrease with increasing n. Hence it can be said with confidence that only a few fundamental eigenmodes of magnetosonic oscillations can penetrate the magnetosphere at fixed values of m and ω.

Figure 7 presents the boundaries of transparent regions of the mode $(n = 0, m = 1)$ for different values of their frequency $f = f_1 = f_2/2; f = f_2; f = f_3 = 3f_2/2$. Figure 8 shows the boundaries of transparent regions for the eigenmode $(n = 0, f = f_2)$ at three different azimuthal wave numbers m=1,2,3. The distance of these boundaries from the equator in the three cases is virtually the same, and the inner boundary approaches the magnetopause with increasing m. For the $m = 2$ harmonic the transparent region consists of two parts: the inner part forming the cavity under the plasmapause, and the outer part that is open to the solar wind. Such a configuration of the transparent regions agrees nicely with results of a numerical simulation reported by *Lee and Lysak* [1994]. Figures 4 and 5 from the cited reference present the mean amplitude distribution of monochromatic magnetosonic oscillations in the dipole magnetosphere. It is evident from Figures 4 and 5 that the shape of the regions corresponding to a maximum oscillation amplitude almost faithfully patterns after the transparent regions calculated in this paper. Furthermore, there is ample observational evidence [*Yumoto et al.*, 1985; *Engebretson et al.*, 1987, 1988] acquired by geostationary satellites, suggesting that the magnetosonic oscillations are localized in the magnetosphere near the geomagnetic equator. This is also consistent with the conclusions of the theory outlined in this paper that the transparent regions of magnetosonic oscillations lie near the equatorial surface.

6. Conclusions

In this paper we have solved the problem of the spatial structure of magnetosonic eigenoscillations of the axisymmetric magnetosphere. The eigenmode structure in the direction along geomagnetic field lines and the value of the radial component of the wave vector k_{1n} to the WKB approximation have been determined. Main results of this study may be summarized as follows.

1. The system of equations (6b) and (7) were obtained to describe MHD oscillations of a three-dimensionally inhomogeneous magnetosphere in the perfect MHD approximation.

2. The spatial structure of magnetosonic eigenmodes of an axisymmetric magnetosphere was qualitatively investigated in the WKB approximation.

3. The problem of the longitudinal structure of magnetosonic oscillations was solved numerically in terms of a sufficiently realistic axisymmetric model of the magnetosphere with a dipole geomagnetic field and Alfven velocity distribution, taking into account abrupt changes of its value on the plasmapause and magnetopause.

4. We have investigated the boundary configuration of transparent regions of eigenmodes depending on the frequency of the wave ω, the longitudinal harmonic number n, and azimuthal wave number m. It has been shown that these transparent regions are open into the solar wind near the geomagnetic equator. This makes it possible for the penetration of magnetosound waves from the solar wind into the magnetosphere and, conversely, for the escape of some of the energy of intramagnetospheric magnetosonic oscillations to the solar wind.

5. Under certain conditions the transparent region for magnetosonic eigenmodes produces under the plasmapause a closed surface bounding the well-known (from previous works) cavity for magnetosound waves.

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References

Chen, L., and S.C. Cowley, On field line resonances of hydromagnetic Alfven waves in a dipole magnetic field, *Geophys. Res. Lett., 16,* 895-897, 1989.

Chen, L., and A. Hasegawa, A theory of long period magnetic pulsation, 1, Steady state excitation of field line resonances, *J. Geophys. Res., 79,* 1024-1032, 1974.

Cheng, C.Z., and C.S. Lin, Eigenmode analysis of compressional waves in the magnetosphere, *Geophys. Res. Lett., 14,* 884-887, 1987.

Comfort, R.H., Plasmasphere thermal structure as measured by ISEE-1 and DE-1, *Adv. Space Res., 6(3),* 31-40, 1986.

Engebretson, M.J., L.J. Zanetti, T.A. Potemra, W. Baumjohann, H. Lühr, M.H. Acuna, Simultaneous observation of Pc3-4 pulsations in the solar wind and in the Earth's magnetosphere, *J. Geophys. Res., 92,* 10,053-10,062, 1987.

Engebretson, M.J., L.J. Zanetti, T.A. Potemra, D.M. Klumpar, R.J. Strangeway, and M.N. Acuna, Observations of intense ULF pulsation activity near the geomagnetic equator during quiet times, *J. Geophys. Res. 93,* 12,795-12,804, 1988.

Fedorov, E., M. Mazur, and V. Pilipenko, MHD wave conversion in plasma waveguides, *J. Geophys. Res., 103,* 26,595-26,605, 1998.

Gul'elmi, A.V., A ring trap for low-frequency waves in the Earth's magnetosphere, (in Russian), *Pisma Zh. Eksp. Teor. Fiz., 12,* 35-38, 1970.

Gul'elmi, A.V., The magnetosound channel under the plasmapause, (in Russian), *Geomagn. Aeron., 12,* 147-154, 1972.

Gul'elmi, A.V., Diagnostics of the magnetosphere and interplanetary medium by means of pulsations, *Space Sci. Rev., 16,* 331-358, 1974.

Horwitz, J.L., R.H. Comfort, and C.R. Chappell, Plasmasphere and plasmapause region characteristics as measured by DE-1, *Adv. Space Res., 6(3),* 21-29, 1986.

Kivelson, M.G., and D.J. Southwood, Coupling of global magnetospheric MHD eigenmodes to field line resonances, *J. Geophys. Res., 91,* 4345-4351, 1986.

Klimushkin, D.Y., A method of describing the Alfven and magnetosound branches of inhomogeneous plasma oscillations, (in Russian), *Fiz. Plazmy, 20,* 309-315, 1994.

Lee, D.-H., Dynamics of MHD wave propagation in the low latitude magnetosphere, *J. Geophys. Res., 101,* 15,371-15,386, 1996.

Lee, L.D., and R.L. Lysak, Monochromatic ULF wave coupling in the dipole model: The impulsive excitation, *J. Geophys. Res., 94,* 17,097-17,109, 1989.

Lee, L.D., and R.L. Lysak, Monochromatic ULF wave excitation coupling in the dipole magnetosphere, *J. Geophys. Res., 96,* 5811-5823, 1991.

Lee, D.-H., and R.L. Lysak, Numerical studies on ULF wave structures in the dipole model, in *Solar Wind Sources of Ultra-Low-Frequency Waves,* Geophys. Monogr. Ser., vol. 81, edited by M.J. Engebretson et al., pp. 293-297, AGU, D.C., Washington, 1994.

Leonovich, A.S., and V.A. Mazur, Resonance excitation of standing Alfven waves in an axisymmetric magnetosphere (monochromatic oscillations), *Planet. Space Sci., 37,* 1095-1108, 1989.

Leonovich, A.S., and V.A. Mazur, A theory of transverse small-scale standing Alfven waves in an axially symmetric magnetosphere, *Planet. Space Sci., 41,* 697-717, 1993.

Leonovich, A.S., and V.A. Mazur, A model equation for monochromatic standing Alfven waves in the axially symmetric magnetosphere, *J. Geophys. Res., 102,* 11,443- 11,456, 1997.

Potapov, A.S., The excitation of geomagnetic Pc3 pulsations ahead of the bow shock by the beam of reflected protons, (in Russian), *Issled. Geomagn. Aeron. Fiz. Solntsa, 46,* 152-157, 1974.

Radoski, H.R., A theory of latitude dependent geomagnetic micropulsations: The asymptotic fields, *J. Geophys. Res., 79,* 595-604, 1974.

Southwood, D.J., Some features of field line resonances in the magnetosphere, *Planet. Space Sci., 22,* 483-492, 1974.

Southwood, D.J., and M.G. Kivelson, The effect of parallel inhomogeneity of magnetospheric hydromagnetic wave coupling, *J. Geophys. Res., 91,* 6871-6877, 1986.

Southwood, D.J., and M.A. Saunders, Curvature coupling of slow and Alfven MHD waves in a magnetotail field configuration, *Planet. Space Sci., 33,* 127-134, 1985.

Walker, A.D.M., Theory of magnetospheric standing hydromagnetic waves with large azimuthal wave number, 1, Coupled magnetosonic and Alfven waves, *J. Geophys. Res., 92,* 10,039-10,045, 1987.

Wright, A.N., Coupling of fast and Alfven modes in realistic magnetospheric geometries, *J. Geophys. Res., 97,* 6429-6438, 1992.

Yumoto, R., T. Saito, S.-I. Akasofu, B.T. Tsurutani, and E.J. Smith, Propagating mechanism of daytime Pc3-4 pulsations observed at synchronous orbit and multiple ground based stations, *J. Geophys. Res., 90,* 6439-6450, 1985.

Zhu, X., and M.G. Kivelson, Global mode ULF pulsations in a magnetosphere with a nonmonotonic Alfven velocity profile, *J. Geophys. Res., 94,* 1479-1485, 1989.

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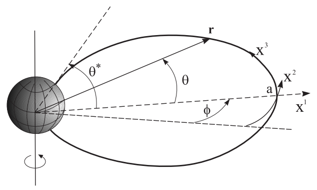

Figure 1. Curvilinear orthogonal coordinate system (x^1, x^2, x^3) and dipole-type coordinate system (a, ϕ, θ).

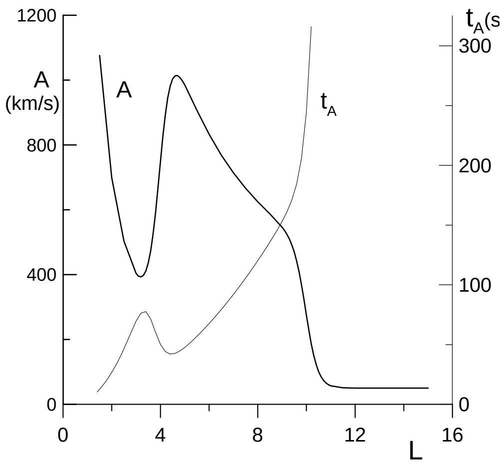

Figure 2. Equatorial dependence of the Alfven velocity model $A(L, 0)$ used in this paper, on the parameter of the magnetic shell L, and the corresponding dependence of the main period of Alfven eigenoscillations of the magnetosphere $t_A(L)$.

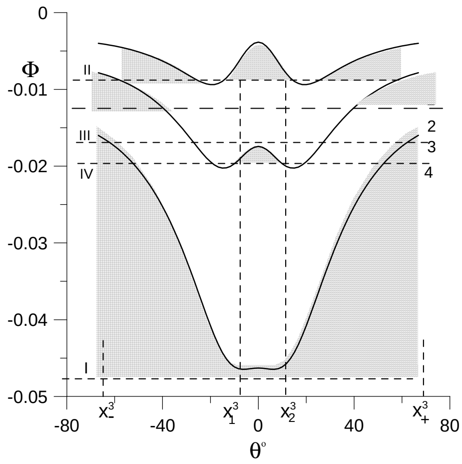

Figure 3. Dependence of the function $\Phi(L, \theta)$ on geomagnetic latitude and azimuthal wave number m on the magnetic shell $L = 6.6$. Curve 1 corresponds to m=1; curve 2, - m=5; curve 3, - m=7; and curve 4, - m=10. Also shown are four possible differences $\Phi(a, \theta) - k_{1n}^2$ corresponding to four different variants of the longitudinal structure of the magnetosound eigenmode.

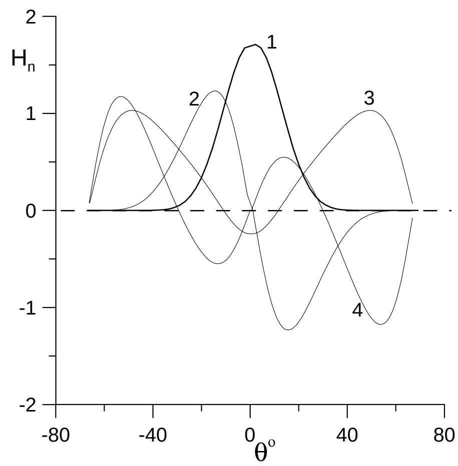

Figure 4. Longitudinal structure of the first four magnetosound eigenmodes of the axisymmetric magnetosphere on the magnetic shell $L = 6.6$.

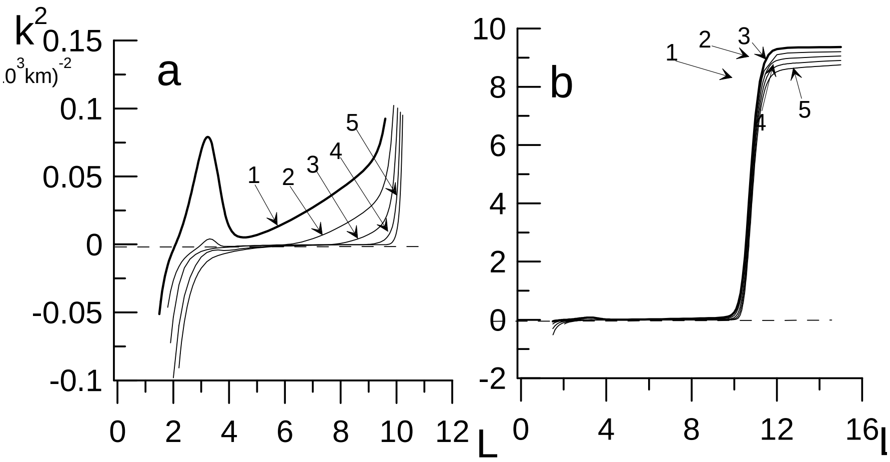

Figure 5. Dependence of the square of the quasi-classical wave vector k_{1n}^2 for the first five magnetosound eigenmodes (n=0,1,2,3,4), on the parameter of the magnetic shell L: (a) $k_{1n}^2(L)$ distribution inside the magnetosphere, and (b) $k_{1n}^2(L)$ distribution in the range of magnetic shells 1.5< L <15, including the solar wind region.

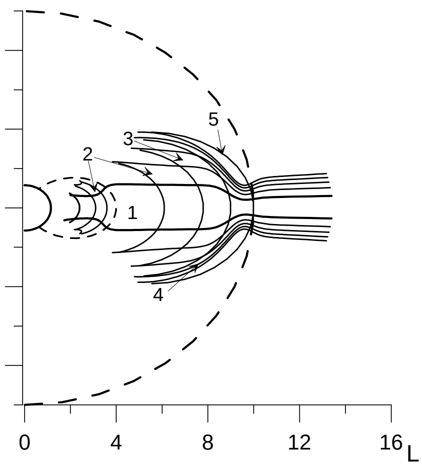

Figure 6. Boundaries of transparent regions in the meridional plane of the axisymmetric magnetosphere for the first five magnetosound eigenmodes (n=0,1,2,3,4) at a fixed frequency $\omega = 2\pi f_2$ and azimuthal wave number m=1. Dashed lines show arbitrary boundaries of the plasmapause and magnetopause.

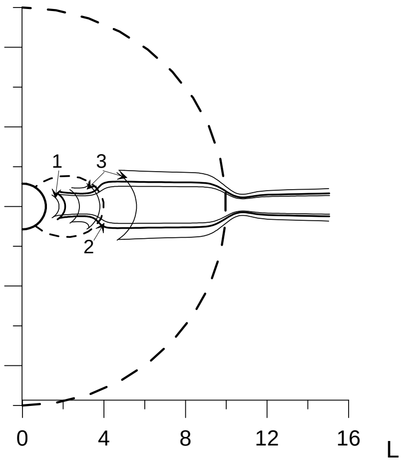

Figure 7. Boundaries of transparent regons in the meridional plane of the axisymmetric magnetosphere for the magnetosound eigenmode ($n = 0, m = 1$) at different values of the frequency (1) $f = f_1$, (2) $f = f_2$, and (3) $f = f_3$. Dashed lines show arbitrary boundaries of the plasmapause and magnetopause.

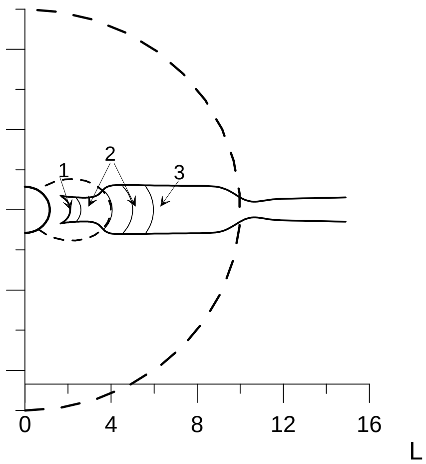

Figure 8. Boundaries of transparent regions in the meridional plane of the axisymmetric magnetosphere for the magnetosound eigenmode ($n = 0, f = f_2$) at different values of the azimuthal wave number (1) m=1, (2) m=2, and (3) m=3. Dashed lines show arbitrary boundaries of the plasmapause and magnetopause.