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Natural Ultra-Low-Frequency Magnetosonic Oscillations of the Near Plasma Sheet

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Abstract—A critical analysis of existing theories of the magnetospheric resonator for fast magnetosonic waves is performed. A new variant of the theory is suggested, according to which the near-Earth part of the plasma sheet plays the role of the resonator. It is shown that the magnetosonic wave is choked within this region over its entire boundaries. The eigen frequencies of resonator modes are in a good agreement with observed values (0.8, 1.3, 1.9, 2.6, 3.4, … MHz), both when estimated in the order of magnitude and when calculated exactly in the context of a simple model.

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1. INTRODUCTION

The hypothesis that the Earth's magnetosphere can play the role of a natural resonator for fast magnetosonic (FMS) waves came into being rather long ago [1–3]. The eigenmodes of such a resonator referred to in [3] as global modes should represent geomagnetic oscillations with the largest scale (and, hence, with the lowest frequencies) that encompass the entire magnetosphere. The near-Earth part of the magnetosphere is beyond the reach for these oscillations, being a region of opacity. In this connection, stationary Alfvenic waves can be a ground manifestation of such a resonator. These waves are excited by the field line resonance mechanism on the field lines passing through the resonator region, and they should have eigen frequencies close to the frequencies of the resonator. The frequency of these oscillations is independent of the point of their detection on the Earth's surface, and this is a specific feature of the oscillations.

Such ultra-low-frequency oscillations were detected both in observations with HF radars [4, 5] and by the network of ground-based magnetometers [6, 7]. There are well pronounced maxima at frequencies 1.3, 1.9, 2.6, and 3.4 mHz in the spectra of these oscillations. The frequencies are almost invariable from event to event, and within a separate event as well. As a rule, these oscillations are detected in the midnight–morning sector of the magnetosphere at latitudes from 60° to 80°.

However, it would be premature to consider these observations as a confirmation to the theory of the global magnetospheric resonance. The critical analysis presented in the next section shows that available variants of the theory do not describe basic characteristics of observed oscillations. Moreover, they are even unable to give a reasonable explanation of the very possibility of existence of the magnetospheric resonance.

In [8], a new concept is suggested for ultra-low-frequency MHD oscillations with a discrete frequency spectrum observed in the magnetosphere. It is based on a possibility of choking such waves inside the resonator in the part of the plasma sheet close to the Earth. It is shown that choking is possible in this region for magnetosonic waves with a frequency of basic harmonics ~1 mHz, which is well consistent with observations. This paper develops the ideas of paper [8].

2. PROBLEMS OF EXISTING THEORY

Before going over to an analysis of existing variants of the theory, let us give some general information from the theory of resonators for magnetosonic waves. This knowledge will be necessary for us in further consideration. The frequency of a fast magnetosonic wave in homogeneous cold plasma with uniform magnetic field is determined by the dispersion equation

$$\omega = A\sqrt{k_x^2 + k_y^2 + k_z^2}, \tag{1}$$

where $A = B/\sqrt{4\pi\rho}$ is the Alfven velocity, and k_x, k_y, k_z are the wave vector components. If the magnetosonic wave is localized in a rectangular resonator with reflecting walls whose sides are l_x, l_y, and l_z, respectively, then the wave vector components are quantized

$$k_x = \frac{\pi n_x}{l_x}; \quad k_y = \frac{\pi n_y}{l_y}; \quad k_z = \frac{\pi n_z}{l_z}, \tag{2}$$

where n_x, n_y, and n_z are quantum numbers that can take values of the natural number sequence. It follows from

(1) and (2) that the order of magnitude of the frequency of the resonator's fundamental tone is

$$f = \frac{\omega}{2\pi} \sim \frac{A}{l}, \quad (3)$$

where l is the minimum of dimensions of the resonator box. Approximate formula (3) remains valid also for resonators of arbitrary form with inhomogeneous plasma and magnetic field. In this case A should be understood as a characteristic value of the Alfven velocity, while l is the least of its three mutually perpendicular dimensions. One may specify that l is the minimum distance between parallel planes within which the resonator is placed.

For natural resonators, including magnetospheric, irregularities of the medium play a part of reflecting walls. One can isolate two substantially different cases. In the first case, reflection occurs from the points (more exactly, surfaces) when a wave turns around. This concept is defined rigorously, if the WKB approximation is applicable. Let the resonator considered above be nonhomogeneous in one of coordinates, for example in x, and the heterogeneity is such that the WKB approximation is applicable. Then, from (1) one can find the quasi-classic wave vector

$$k_x^2(x) = \frac{\omega^2}{A^2(x)} - k_y^2 - k_z^2. \quad (4)$$

Here it is assumed that ω, k_y, and k_z are fixed, while the Alfven velocity is a function of x due to its non-homogeneity. The wave propagates in the region of transparency, where $k_x^2 > 0$, and it cannot propagate in the region of opacity, where $k_x^2 < 0$. One can easily see that the Alfven velocity has lower values in the region of transparency in comparison with the region of opacity. The surfaces on which $k_x^2 = 0$ act as reflecting walls. Reflection can be partial or total, depending on the width and height of the potential barrier formed by the region of opacity. A resonator originates when the region of transparency in coordinates x is limited by turning points. In the three-dimensional case the existence of a resonator is possible, if the region of transparency, being located in the region of relatively small values of the Alfven velocity, is limited by a turning surface from all sides.

The second variant of reflecting walls of a resonator corresponds to sharp boundaries separating jump-like changes in the medium parameters (the Alfven velocity in our case). The WKB approximation is inapplicable on such a boundary. A wave experiences partial reflection (it is the stronger the larger is the jump in the Alfven velocity), even if the boundary separates two regions of transparency. Two variants of a resonator with different types of reflecting walls discussed above do not exclude each other. The situation, when one part of the resonator boundaries is represented by turning surfaces, and another part has sharp boundaries, is quite possible.

Let us now go over from general remarks to consideration of different variants of the theory of magnetospheric resonance. In all these variants the magnetopause—the sharp boundary of the magnetosphere—is the outer boundary of the resonator [2, 9–11], and in some variants the bow shock front is the boundary [5]. Below the magnetopause, where the resonator is located, there is a region of transparency for magnetosonic waves. The space beyond the magnetopause, where the Alfven velocity is substantially lower, is also the region of transparency for such a wave. Therefore, the magnetopause cannot be a usual surface of reflection, but it is capable of playing the role of a partially reflecting boundary because of a sharp jump of the Alfven velocity taking place on it.

Up to the present time, quite a few theoretical papers have been published, where calculations of characteristics of the global modes are performed, including calculations of the spectra of their possible frequencies. In [2, 3] it has been done for a model of the magnetosphere in the form of a rectangular box, paper [12] used a model in the form of a half-cylinder, and papers [10, 11, 13, 14] worked with an axially symmetrical model with a dipole geomagnetic field. One can estimate some of these models as more realistic and other models as less realistic, but all of them ignore the most important element of the magnetosphere, the geomagnetic tail. Therefore, these papers cannot be considered suitable for description of the global magnetospheric resonator, and, more likely, their results have some interest only as an illustration of method. Noteworthy are two results obtained using an axially symmetrical model of the magnetosphere with a dipole magnetic field and such plasma distribution in the meridional plane which approximately correspond to the dayside of the magnetosphere [14, 15].

The eigenmodes of the resonator are confined between the magnetopause (axially symmetrical in this model) and the inner turning surface that separates the region of transparency in the outer magnetosphere from the region of opacity in the inner magnetosphere. The existence of the region of opacity is due to a considerable increase of the Alfven velocity with an approach to the Earth.

The eigen frequencies calculated in this model ($f > 5$ mHz) turn out to be much higher than observed (~1 mHz). One could expect this result already from estimate (3). The typical values for the magnetosphere are $A \sim 10^3$ km/s and $l \sim 10^5$ km, which gives $f \sim 10$ mHz. Of course, the result of exact calculation can differ from estimation by a factor of 2–3, but hardly by an order of magnitude. The value $f > 5$ mHz obtained in the papers mentioned above confirms this.

However, the problems with values of eigen frequencies can be considered subordinate in comparison with the circumstance of principal importance discussed above. The existence of the magnetotail calls in question the entire concept of magnetosphere resonance. It may be thought that the magnetosonic wave considered above can propagate freely into the open tail. Due to this reason, in [5, 16, 17] it was suggested to interpret the observed ultra-low-frequency oscillations with discrete spectrum as eigenmodes of a waveguide in the geomagnetic tail. However, one can hardly consider this interpretation as successful. It is a difficult problem to explain in this case the discrete spectrum of oscillations. The authors use the model of a waveguide in the form of a rectangular channel, designating transverse coordinates as y and z, and channel-aligned coordinate going along the waveguide into the geomagnetic tail is designated as x. Quantization of k_y and k_z follows from geometrical considerations, but the component k_x can assume a continuous series of values, as well as frequency ω. The hypothesis of the authors that observed oscillations correspond to values $k_x \ll k_y$, k_z (in this case, the particular value of k_x is insignificant, when eigen frequency is determined) is not supported by arguments whatsoever. It is impossible to understand why the mechanism exciting the waveguide generates a wave with k_y, $k_z \sim 1/l$ (l is the transverse scale of the magnetosphere) and cannot generate a wave with $k_x \sim 1/l$, but only the waves with $k_x \ll 1/l$? Taking into account that there is an irregularity along coordinate x of approximately the same scale l, the assumption about existence of a wave with $k_x \ll 1/l$ is unlawful altogether. The wavelength cannot be much larger than the scale of the irregularity.

In the remote part of the tail, where the scale of irregularity along the tail is much lager than the transverse scale, the waves with $k_x \ll k_y$, k_z are possible. But even in this case the problem of a mechanism exciting the oscillations remains unsolved. In addition, the difficulties of bringing theoretical eigen frequencies of oscillations into better agreement with observed values increase. At characteristic transverse size of the waveguide $l \sim (1–3) \cdot 10^5$ km the Alfven velocity in the tail lobes $A \sim (3–10) \cdot 10^3$ km/s, which gives for the fundamental harmonic a frequency of $f \approx 30$ mHz, too far from the frequencies of observed oscillations. Finally, there is a possibility for a waveguide to exist in the neutral plasma sheet [18]. In particular, for a frequency of the fundamental harmonic at $l \sim 10^4$ km and $A \sim 10^2$ km/s we have $f \sim 10$ mHz.

Thus, one has to ascertain that the concept of magnetospheric global modes is inadequate at the moment. The region of localization of eigenmodes, the causes of their choking inside the magnetosphere, and their spatial structure are unknown. The difference between theoretical and observed values of the eigen frequencies of oscillations is too high.

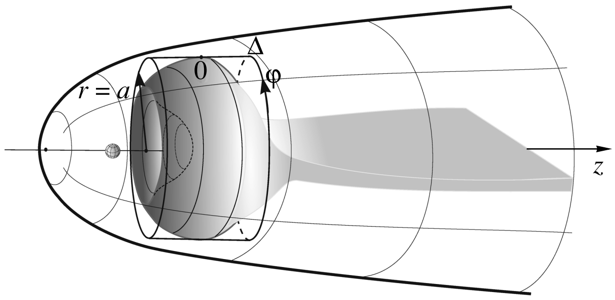

Fig. 1. Schematic picture of the plasma sheet and cylindrical model of the resonator. The region where the Alfven velocity has relatively small values (conventionally $A < 250$ km/s) is shown in gray color. Structurally, it is divided in lenticular near-Earth part of the plasma sheet and the plane neutral sheet.

3. RESONATOR IN THE NEAR-EARTH PART OF THE PLASMA SHEET

The near-Earth part of the plasma sheet (NEPS) is an important structural element of the magnetosphere. It represents a lenticular region filled with sufficiently dense plasma. In combination with the small strength of the geomagnetic field this results in the typical values of Alfven velocities minimum for the magnetosphere, $A \sim 10^2$ km/s. One cannot even exclude the values lower by a factor of 2–3 [19]. The near-Earth part of the plasma sheet (NEPS) is continuously transformed into a plane neutral sheet separating the lobes of the magnetotail (see Fig. 1). Of importance for fast magnetosonic waves is the spatial distribution of their propagation velocity which is virtually equal to the Alfven velocity almost over the entire magnetosphere (excluding a narrow plasma layer in the middle and remote parts of the magnetotail). Therefore, in the subsequent calculations we use the equation for magnetosonic waves derived in the approximation of cold plasma. The velocity of propagation of the FMS waves in this approximation is equal to the Alfven velocity.

The boundary of the plasma sheet shown in Fig. 1 is a surface separating the magnetosphere regions with high and low values of the Alfven velocity. Therefore, it does not reach the Earth: when approaching it the geomagnetic field strength and the Alfven velocity value increase. The figure does not depict the region of maximum values of the Alfven velocities ($A \sim 100–200$ km/s) in the dayside magnetosphere. It can be represented as a half toroid located near the geomagnetic equator and connected with NEPS in the evening and morning sectors. At a strong compression of the dayside magnetosphere by the solar wind, ex-equatorial minima of the magnetic field also appear [20, 21] and the above toroid can disintegrate into two lying on each side of the geomagnetic equator. In any case, the cross section of the toroid does not exceed $10^4 10^4$ km. Hence, the eigen frequency of such a resonator is ~10 mHz and, as it will be demonstrated below, for oscillations with frequencies ~1 mHz considered by us it represents an opacity region.

The typical size (radius) of the near-Earth part of the plasma sheet across the magnetosphere is approximately the same as the radius of the tail in this part of the magnetosphere, ~10^5 km. The region under discussion in the anti-solar direction is limited from one side by the inner edge of the plasma sheet located at a distance of (5–10)R_E. From the other side it is limited by the outer boundary of the plasma sheet at distances of (20–30)R_E. Thus, the characteristic dimension of the near-Earth part of the plasma sheet in this direction is ~10^5 km. The presented values of l and A give the value $f \sim 1$ mHz for a possible frequency of the fundamental harmonic of natural magnetosonic oscillations localized in the near-Earth plasma sheet. This frequency coincides in the order of magnitude with the frequencies of observed oscillations.

Let us now assure ourselves in the possibility of existence of eigenmodes in this region of the plasma sheet. First we consider the causes of choking in it of magnetosonic waves. The magnetopause is a lateral boundary of the region under consideration, and reflection from it was already discussed in detail. The turning surfaces play the role of reflecting walls in the directions to the Earth and away from it. Their existence is caused by a considerable increase of the Alfven velocity from a value of ~10^2 km/s in the plasma sheet up to values of (2–5) · 10^3 km/s near the Earth and (3–10) · 10^3 km/s in the tail lobes. Apparently, these turning surfaces virtually coincide with the inner and outer edges of the near-Earth part of the plasma sheet.

However, there is one more circumstance. The near-Earth plasma sheet is continuously transformed into the neutral plasma sheet. So, the question arises: whether the resonator eigenmode could propagate into the remote tale. Simple considerations show that this should not take place. The neutral sheet can be considered as a plane waveguide with thickness $\Delta \sim 10^4$ km. Let us use the coordinate system in which axis x coincides with the anti-solar direction, and axes z and y are directed across the waveguide and in azimuth, respectively. Then, $k_z = \pi/\Delta$, $k_y = \pi/l \ll k_z$ (recall that l is the transverse dimension of the magnetosphere). The frequency of the resonator mode considered by us is $\omega = 2\pi/f \sim \pi A/l$. For estimations one can assume that the Alfven velocity is identical both in the near-Earth part of the plasma sheet and in the neutral sheet, which gives $\omega^2/A^2 \ll k_z^2$. Then from (4) we have inside the waveguide $k_x^2 \approx -k_z^2 = -\pi^2/\Delta^2$. This means that the neutral sheet is opaque in the anti-solar direction for the waves under consideration. In other words, the eigenmode of the resonator penetrates into the waveguide in the neutral sheet no deeper than to a distance of order $\Delta \sim 10^4$ km. Using the same arguments one can proof opacity for oscillations under consideration of above-mentioned plasma toroids in the dayside magnetosphere.

Thus, the near-Earth part of the plasma sheet can serve as a resonator for magnetosonic waves, since it is surrounded from all sides by a boundary reflecting these waves. The characteristic frequency of eigenmodes of such a resonator $f \sim 1$ mHz, which corresponds to observed oscillations. As has already been noted in the Introduction, the modes of a magnetosonic resonator should reveal themselves on the ground as stationary Alfven waves excited through the field line resonance mechanism. The field lines passing through the near-Earth part of the plasma sheet intersect the ground surface at latitudes from 60° to 80°, which is in agreement with latitude localization of the observed oscillations. One can also explain a certain asymmetry of their longitude localization. As is known, the convective flow of magnetospheric plasma (from the nightside through the morning sector to the dayside) shifts NEPS from its symmetric midnight position to the morning sector [19]. As a result, the resonator for low-frequency FMS oscillations turns out to be displaced to the same sector in close agreement with observations.

4. EIGENMODES OF RESONATOR IN THE MODEL OF CIRCULAR CYLINDER

We present now the results of calculating the spectrum of eigenmodes of a resonator whose simple model represents a right circular cylinder (see Fig. 1). Using such a mode, one can obtain the exact analytical solution to the problem. In order to describe fast magnetosonic oscillations, we take the following equation

$$\Delta\Psi + \frac{\omega^2}{A^2}\Psi = 0. \qquad (5)$$

Here, Ψ is a certain field quantity describing the amplitude of oscillation (disturbed electric field, displacement of plasma, etc.). Equation (5) should be considered as a model equation. It is exact for homogeneous plasma and leads to dispersion equation (1). In the WKB approximation, when the wavelength of oscillations is much less than the characteristic scale of plasma irregularities, this equation also remains valid. Qualitatively, it correctly describes oscillations even near the limit of applicability of the WKB approximation, when the wavelength is of the order of the irregularity scale.

In cylindrical coordinates (ρ, z, φ) equation (5) takes the form

$$\frac{\partial^2\Psi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Psi}{\partial\rho} + \frac{\partial^2\Psi}{\partial z^2} + \frac{1}{\rho^2}\frac{\partial\Psi}{\partial\varphi^2} + \frac{\omega^2}{A^2}\Psi = 0. \qquad (6)$$

In the context of this equation the resonator model is determined by the choice of a particular function $A = A(\rho, z, \varphi)$. We assume that A does not depend on φ

(the case of axial symmetry), while dependence on z and ρ is chosen in the form

$$\frac{1}{A^2(\rho, z)} = f(z) + g(\rho). \quad (7)$$

Such dependence allows one to separate variables in Eq. (6). For functions f and g we choose the following forms

$$f(z) = \begin{cases} 1/A_1^2, & z < -\Delta \\ 1/A_0^2, & |z| < \Delta \\ 1/A_2^2, & z > \Delta, \end{cases}$$

and

$$g(\rho) = \begin{cases} 0, & \rho < a \\ 1/A_w^2, & \rho > a. \end{cases}$$

The piece of the cylinder $\rho < a$, $|z| < \Delta$ models the inner part of the near-Earth plasma sheet. Inside it $A = A_0$. The region $\rho < a$, $z < -\Delta$ simulates the bow-shock part of the magnetosphere, where $A = A_1$, and the region $\rho < a$, $|z| > \Delta$ models the geomagnetic tail lobes, where $A = A_2$. We assume that $A_1, A_2 \gg A_0$. The region $\rho > a$ corresponds to the solar wind. It can be separated in three subregions in which

$$\frac{1}{A^2} = \frac{1}{A_w^2} + \frac{1}{A_1^2}, \quad z < -\Delta,$$

$$\frac{1}{A^2} = \frac{1}{A_w^2} + \frac{1}{A_0^2}, \quad |z| < \Delta,$$

$$\frac{1}{A^2} = \frac{1}{A_w^2} + \frac{1}{A_2^2}, \quad z > \Delta.$$

We assume that $A_w \ll A_0$, A_1, A_2, while in all three regions $A \approx A_w$, where A_w is the Alfven velocity in the solar wind.

One can seek solution to (6) for Alfven velocity model (7) in the form $\Psi(\rho, z, \varphi) = u(\rho)v(z)e^{im\varphi}$, where m is the azimuth wave number. The variables are separable, and for u and v we get the ordinary differential equations

$$u''(\rho) + \frac{u'(\rho)}{\rho} + \left[\omega^2 g(\rho) - \frac{m^2}{\rho^2} + q\right] u(\rho) = 0, \quad (8)$$

$$v''(z) + [\omega^2 f(z) - q]v(z) = 0, \quad (9)$$

where q is the separation constant.

Equation (8) at $\rho < a$ has the form

$$u'' + \frac{u'}{\rho} + \left[q - \frac{m^2}{\rho^2}\right] u = 0, \quad (10)$$

As it will be made certain below, for existence of an eigenmode it is necessary that $q > 0$. Therefore, we designate $q = k_\perp^2$. Later it will become clear that this constant can be treated as a square of the transverse wave vector $k_\perp^2 = k_x^2 + k_y^2$. The solution to Eq. (10), regular at $\rho = 0$, represents the Bessel function of order m:

$$u = C_1 J_m(k_\perp \rho).$$

It should be joined at point $\rho = a$ with a solution in the region $\rho > a$, where the equation has the following form

$$u'' + \frac{u'}{\rho} + \left[\kappa^2 - \frac{m^2}{\rho^2}\right] u = 0.$$

Here we designated

$$\kappa^2 = \frac{\omega^2}{A_w^2} + k_\perp^2.$$

Hence one can see that, if the region $\rho < a$ is a region of transparency, then the region $\rho > a$ is also a region of transparency, since $\kappa^2 > q \equiv k_\perp^2$. Therefore, the general solution in the region $\rho > a$ represents a superposition of two waves: one running away to infinity (at $\rho \longrightarrow \infty$) and another arriving from infinity. Let us set at $\rho \longrightarrow \infty$ the condition of running the wave away. Physically, this means that the wave is generated in the region $\rho < a$, and therefore, at infinity it can be only running away wave. Thus, at $\rho > a$

$$u = C_2 H_m^{(1)}(\kappa\rho), \quad (11)$$

where $H_m^{(1)}$ is the modified Bessel function. Its asymptotic behavior at $\kappa\rho \gg 1$ looks like

$$u = C_2 \sqrt{\frac{2}{\pi\kappa\rho}} \exp\left[i\left(\kappa\rho - \frac{\pi m}{2} - \frac{\pi}{4}\right)\right].$$

Since $\omega \gg k_\perp A_0$ (see (20) below), then $\omega^2/A_w^2 \gg k_\perp^2(A_0^2/A_w^2) \gg k_\perp^2$, hence, $\kappa \approx \omega/A_w$. As we will see $k_\perp a \sim 1$, which means $\kappa_\perp a \gg 1$, and one can use for function (11) in the condition of joining at point $\rho = a$ its asymptotic expression. Taking the said above into account, the condition of joining has the form

$$J_m(k_\perp a) = -i\frac{k_\perp}{\kappa} J'_m(k_\perp a). \quad (12)$$

The ratio $k_\perp/\kappa \leq A_0/A_w$ is a small parameter of the problem, for which we have in the principal order

$$J_m(k_\perp a) = 0. \quad (13)$$

In this order there is no escape of oscillations from the resonator, and the eigenmode frequency is real. When the right-hand side of (12) is taken into account, a small escape of oscillations from the resonator comes into existence in the problem, and the eigenmode frequency

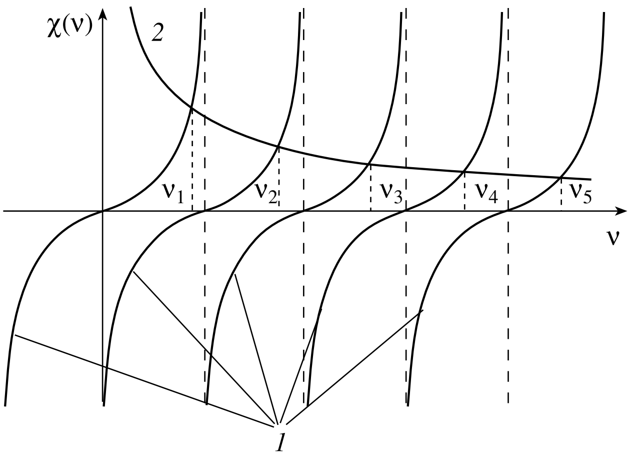

Fig. 2. Graphical solution to Eq. (19). The plots of two-valued function $\chi(\nu)$ and function C/ν ($C = \mu_{ml}\Delta/a$) are presented. Roots of Eq. (19) are the abscissas of intersection points ν_{nml}.

attains an imaginary part, decrement of damping. Let us designate as μ_{ml} the l th (l = 1, 2, 3, …) zero of the Bessel function of order m, i.e., $J_m(\mu_{ml}) = 0$. The zeros of Bessel functions are tabulated in many mathematical handbooks on special functions. The solution to Eq. (13) can be written in the form $k_\perp = \mu_{ml}/a$.

Equation (9) in our model has the form

$$
\begin{aligned}
&v'' + \left(\frac{\omega^2}{A_1^2} - k_\perp^2\right)v = 0, \quad z < -\Delta,\\
&v'' + \left(\frac{\omega^2}{A_0^2} - k_\perp^2\right)v = 0, \quad |z| < \Delta,\\
&v'' + \left(\frac{\omega^2}{A_2^2} - k_\perp^2\right)v = 0, \quad z > -\Delta.
\end{aligned}
\tag{14}
$$

One can see from these equations that a mode is choked in the region $|z| < \Delta$, if $\omega^2/A_0^2 > k_\perp^2$ and

$$\frac{\omega^2}{A_1^2}, \frac{\omega^2}{A_2^2} < k_\perp^2. \tag{15}$$

Eigen frequencies f_{mnl} (mHz) of MHD-resonator in the near-Earth plasma sheet

	$m = 0$			$m = 1$			$m = 2$		
n/l	0	1	2	0	1	2	0	1	2
0	0.80	1.54	2.29	1.14	1.90	2.66	1.45	2.24	3.01
1	1.40	2.01	2.67	1.67	2.32	3.01	1.93	2.62	3.34
2	1.68	2.68	3.24	2.42	2.94	3.54	2.62	3.20	3.83

From inequalities (15) follows the above statement that $q = k_\perp^2 > 0$. Taking into account that $A_1, A_2 \gg A_0$, we assume that inequalities (15) are strong:

$$\frac{\omega^2}{A_1^2}, \frac{\omega^2}{A_2^2} \ll k_\perp^2 \tag{16}$$

and neglect the left-hand sides of these inequalities in comparison with the right-hand sides. Then, Eqs. (14) can be rewritten in the form

$$v'' - k_\perp^2 v = 0, \quad |z| > \Delta, \tag{17a}$$

$$v'' - k_z^2 v = 0, \quad |z| < \Delta. \tag{17б}$$

Here we designated

$$k_z^2 = \frac{\omega^2}{A_0^2} - k_\perp^2. \tag{18}$$

The solution to Eq. (17b) has the form

$$v = \begin{cases} C_1 \cos k_z z, & \text{for even modes,} \\ C_1 \sin k_z z, & \text{for add modes,} \end{cases}$$

and the solution to Eq. (17a) looks like

$$v = \begin{cases} C_- e^{k_\perp z}, & \text{for } z < -\Delta, \\ C_+ e^{-k_\perp z}, & \text{for } z > \Delta. \end{cases}$$

For even modes $C_- = C_+$, while for odd modes $C_- = -C_+$. Joining at point $z = \Delta$ (or at $z = -\Delta$) gives the following equation for the parameter $\nu = k_z\Delta$:

$$\chi(\nu) = \mu_{ml}\frac{\Delta}{a}\frac{1}{\nu}. \tag{19}$$

Here, $\chi(\nu)$ is a two-valued function (see Fig. 2)

$$\chi(\nu) = \begin{cases} \tan\nu, & \text{for even modes}, \\ -\cot\nu, & \text{for odd modes}. \end{cases}$$

Let us designate as ν_{nml} the nth (n = 1, 2, 3, …) solution to Eq. (19). It is clear from Fig. 2 that

$$\frac{\pi(n-1)}{2} < \nu_{nml} < \frac{\pi n}{2}.$$

Notice also that the values n = 1, 3, 5, … correspond to even modes in z, while n = 2, 4, 6, … correspond to add modes. From definition of parameter ν it follows that $k_z = \nu_{nml}/\Delta$, and from relationship (18) we get the frequency spectrum

$$\omega \equiv \omega_{nml} = \sqrt{k_z^2 + k_\perp^2}A_0 \equiv \sqrt{\frac{\nu_{nml}^2}{\Delta^2} + \frac{\mu_{ml}^2}{a^2}}A_0. \tag{20}$$

One can see that the eigen frequency of the resonator is determined by three quantum numbers: azimuth m, radial l, and longitudinal n. The fundamental mode has the quantum numbers $m = 0m = 0$ and $n = l = 1$. Table presents the frequencies of eigemodes of the resonator for first three values of each wave number calculated for the following parameter values: $a = \Delta = 10R_E \approx 6.4 \cdot 10^4$ km and $A_0 = 2 \cdot 10^2$ km/s.

In the approximations used by us A_1, A_2, and A_w do not influence the values of eigen frequencies. In order to render concrete the imposed constraints, we assume the following values for these quantities: $A_1 = 2 \cdot 10^3$ km/s, $A_2 = 5 \cdot 10^3$ km/s, and $A_w = 40$ km/s. Inequalities (16) with allowance made for (20) have the following form

$$\frac{k_z}{k_\perp} \ll \frac{A_1}{A_0}, \frac{A_2}{A_0} \sim 10.$$

This means that for specified values of m and l wave number n should not be too large. It has the order of magnitude $n \ll \frac{A_1}{A_0}(m+l) \approx 10(m+l)$.

This condition is certainly valid for the first harmonics of resonator oscillations in which we are interested.

As has already been said, when the finiteness of ratio A_w/A_0 is taken into account, this leads to the appearance of an imaginary part of frequency—decrement of natural oscillations: $\omega \longrightarrow \omega - i\gamma$. It is associated with their escape through the lateral surface of the resonator. Omitting the details of simple but rather awkward calculations, we give here only the final result:

$$\gamma = \frac{A_w}{a} \frac{k_\perp^2 + k_\perp/\Delta}{k_\perp^2 + k_z^2 + k_\perp/\Delta}.$$

In the order of magnitude $\gamma/\omega \sim A_w/A_0$. For the values given above $\gamma/\omega \sim 0.2$, i.e., the Q-factor of the resonator is equal to $Q = \omega/\gamma \sim 5$. It should be noted that our model can underestimate the Q-factor value. In the real magnetosphere the Alfven velocity value increases from the center of the resonance region to its periphery, including the magnetopause near which it can reach values $A_0 \sim 300$–500 km/s. The coefficient of reflection from a sharp boundary is determined, obviously, by local properties of this boundary, i.e., by the ratio of Alfven velocity values on both sides of the boundary. This means that the Q-factor can be larger by a factor of 2–3 than its estimate presented above, thus reaching a typical value of $Q \sim 10$.

CONCLUSIONS

Let us list the basic results of this paper.

1. It is demonstrated that the near-Earth part of the plasma sheet can play the role of a resonator for fast magnetosonic waves. In this region there exist magnetosonic oscillations that are choked over the entire boundaries surrounding it. On the lateral surface, represented by a part of the magnetopause, reflection occurs due to a sharp jump of the Alfven velocity. Reflection on the inner and outer edges of the plasma sheet is caused by the fact that behind them in the near-Earth region and in the tail lobes of the magnetosphere the region of opacity for such oscillations is located. The large-scale oscillations under consideration cannot penetrate into the neutral plasma sheet because of its small thickness.

2. The frequency of the fundamental mode of the resonator in the near-Earth part of the plasma sheet is ≈1 mHz in the order of magnitude. We calculated the spectrum of eigen frequencies of this resonator. A simple model of the near-Earth part of the plasma sheet in the form of a segment of a right circular cylinder filled with homogeneous plasma in uniform magnetic field is used, and an analytical formula describing the frequency spectrum of the resonator is derived. The frequency of each eigenmode is determined by three wave numbers: azimuth number ($m = 0, 1, 2, \ldots$), and radial ($l = 1, 2, 3, \ldots$) and longitudinal ($n = 1, 2, 3 \ldots$) numbers. A table of numerical values of eigen frequencies for first three values of each wave number is presented. Damping decrements γ of eigenmodes are determined to be caused by mode escape through the magnetopause and solar wind. The corresponding values of the $Q = \omega/\gamma$ lie within the interval 3–10.

3. The results formulated in preceding items allow us to treat the observed ultra-low-frequency oscillations with discrete spectrum ($f \approx 0.8, 1.3, 1.9, 2.6, 3.4, \ldots$ mHz) as eigenmodes of the resonator in the near-Earth part of the plasma sheet. A good agreement of the observed and theoretical values of the frequencies prevails.

4. Natural oscillations of the resonator under discussion cannot immediately reach the Earth's surface: the near-Earth part of the magnetosphere is an opacity region for them. On the ground one observes stationary Alfven waves excited by the field line resonance mechanism on the field lines passing through the resonator region in the near-Earth plasma sheet. This explains in a natural way the localization of observed oscillations in latitude from 60° to 80° and in longitude within the midnight–morning sector of the magnetosphere. A certain displacement of the near-Earth plasma sheet to the morning sector is explained by convection motion of the magnetosphere plasma from the night-side through the morning sector to the dayside.

In conclusion of this paragraph let us discuss in brief amazing stability of the frequencies of observed oscillations being reproduced from one event to another. Unfortunately, using our hypothesis we cannot suggest a clear and convincing explanation to this property of the oscillations. However, this difficulty is common for all variants of theory of the magnetospheric resonator. The Earth's magnetosphere is not a stable formation, and its parameters are variable within broad limits. This is fully correct for the near-Earth part of the plasma sheet. It well may be that one will be able to find explanation of stability of the observed frequencies going after the following line of reasoning. Eigenmodes of the resonator are extremely long-periodic oscillations ($T \sim 10^3$ s). In order that they would appear the magnetosphere should keep a steady state over time intervals of 1 h and longer, i.e., the state of small disturbance of the magnetosphere is necessary. Indeed, the oscillations with discrete spectrum are observed at relatively small values $K_p \lesssim 3$. At such a disturbance the parameters of the magnetosphere (including the near-Earth plasma sheet) lie within much narrower limits than their extreme possible values. In addition, one can suppose that such integral characteristic as eigen frequency (let us remind that this is A/l in the order of magnitude) is more stable quantity than individual parameters of the resonator. Finally, it is admissible to suppose that the mechanism of excitation of the resonator eigenmodes which we do not discuss here is operative only in a narrow range of states of the resonator.

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