Nonlinear dynamo effects for an inhomogeneously turbulent rotating fluid

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The mean electromotive force and the turbulent stress tensor are derived for an inhomogeneously turbulent conducting fluid under global rotation. The influence of the mean magnetic field on microscale turbulence is taken into account. As a result, the coefficients of magnetic field generation and sources of differential rotation are now functions of the magnetic field; i.e. nonlinear effects are involved. Only nonlinear corrections quadratic in the magnetic field are included (weak nonlinearity). These corrections reduce the α-effect of the $\alpha\Omega$-dynamo for both slow and rapid rotation. Some astrophysical implications are discussed. It is suggested that observed torsional oscillations of the Sun may be brought about by cyclic variations of the magnetic corrections to the Λ-effect of differential rotation over the solar activity cycle.

Im Rahmen der Magnetohydrodynamik der mittleren Felder werden die turbulenzbedingte elektromotorische Kraft und der turbulenzbedingte Spannungstensor für eine leitende Flüssigkeit mit inhomogener Turbulenz auf einem rotierenden Körper berechnet. Dabei wird der Einfluß des Magnetfeldes auf der Turbulenz berücksichtigt. So erscheinen nichtlineare Effekte bei den Vorgängen, die für das Vorhandensein von Magnetfeld und differentieller Rotation verantwortlich sind; die dafür maßgebenden Koeffizienten sind Funktionen des Magnetfeldes. Die Nichtlinearitäten werden bis zur zweiten Ordnung im mittleren Magnetfeld (d. h. als schwache Nichtlinearitäten) erfaßt. Der α-Effekt beim $\alpha\Omega$-Dynamo erleidet sowohl bei langsamer als auch bei rascher Rotation eine Schwächung durch das Magnetfeld. Es werden einige astrophysikalische Anwendungen der Ergebnisse erörtert. Insbesondere wird auf die Möglichkeit hingewiesen, daß die bei der Sonne beobachteten Torsionsschwingungen als Folge des im Aktivitätszyklus schwankenden magnetischen Einflusses auf den für die differentielle Rotation maßgebenden Λ-Effekt erscheinen.

Key words: magnetic field — differential rotation — nonlinearity

1. Introduction

The theory of the hydromagnetic dynamo is being developed mainly in the linear approximation. In other words, the hydromagnetic motions are supposed given and independent of the magnetic field. Such a kinematic approach is quite justifiable in the case where the magnetic field is small enough, and Lorentz forces do not substantially affect the flow of the conducting fluid. However, if dynamo amplification of the magnetic field occurs, the field grows exponentially and a nonlinear regime is rapidly reached. Hence, nonlinear effects must play a role in natural conditions, in particular on the Sun.

The kinematic theory, though with some substantial problems unsolved, is regarded now as being understood in its principal features, which was indicated by the appearance of a series of monographs (MOFFATT 1978; PARKER 1979; KRAUSE and RÄDLER 1980; VAINSHTEIN et al. 1980) cataloging its achievements. The natural next step is to attack nonlinear problems. Some progress was also made in this field (c.f. ROBERTS and SOWARD 1975; POUQUET et al. 1976; KLEEORIN and RUZMAIKIN 1982).

The most substantial ingradient of the theory of turbulent dynamo is the so-called α-effect (STEENBECK et al. 1966) — generation of a mean electromotive force proportional to the mean magnetic field. That is probably why nonlinear treatments were concentrated on the derivation of magnetic corrections to the α-effect. ROBERTS and SOWARD (1975) considered the nonlinear α-effect for inhomogeneously turbulent slowly rotating fluid. It is believed that it is the rotational influence on turbulence which brings about the α-effect under natural conditions. The nonlinear α-effect for arbitrary rotational velocities will be derived in the present paper.

Apart from the α-effect, there is still another nondissipative (i.e. not proportional to the spatial derivatives of the mean magnetic field) contribution to the mean electromotive force for the case of inhomogeneous turbulence. This is the diamagnetic pumping (ZELDOVICH 1956; RÄDLER 1968). The velocity of diamagnetic transport of mean field will be derived below by taking into account global rotation and nonlinearity.

The nonlinear effects to be considered here are brought about by the feedback of the magnetic field on motion. Hence it seems natural to supplement the analysis of the mean induction equation by deriving the equation for the mean velocity field. It is necessary to derive the turbulent stress tensor to close this latter equation. Within the framework of a nonlinear approach, this tensor is contributed by both Reynolds stresses and Maxwellian stresses of the fluctuating magnetic fields. The equation of motion will be averaged below with allowance for the magnetic corrections. Thus, an attempt is made to

obtain the complete, weakly nonlinear system of mean-field MHD equations. Such an approach permits, in particular, a unified treatment of the differential rotation and magnetic field generation which are usually considered separately.

Of course, the general solution of the full nonlinear problem is impossible to get. A number of assumptions and simplifying approximations will be accepted below. All calculations will be made for the case of large viscosity and low conductivity that seems to impose severe restrictions on the range of validity of the results obtained. It may be hoped, however, that these results can be applied to astrophysical bodies characterized by the opposite situation of small viscosity and high conductivity. Indeed, the most interesting effects of generation of magnetic field (α-effect) and differential rotation (Λ-effect) are induced by the rotational influence on turbulence. This influence is most pronounced for the largest long-lived eddies. Eddies of smaller scales may be included formally through effective viscosity and magnetic diffusivity. Hence, it may be anticipated that the results of the present paper will hold qualitatively in the case of large Reynolds numbers if the coefficients ν and η (viscosity and magnetic diffusivity, respectively) are considered to be of turbulent origin. In addition, we assume, when deriving the turbulent stress tensor, that the main contribution to the mean magnetic field is made by its toroidal component and the dominant component of the mean flow is (differential) rotation. Such a situation is typical of the rather popular $\alpha\Omega$-dynamo. The nonlinear corrections will be taken into account only when calculating the nondissipative effects, i.e. α-effect, pumping and Λ-effect of differential rotation. The eddy diffusivities and viscosities vill be derived in the linear approximation.

The approximations and assumptions made are discussed in more detail in Section 2. In this section we also write down the starting equations and explain the scheme of calculations adopted. The mean EMF and properties of different contributions to it are considered in Section 3. The turbulent stress tensor is derived and discussed in Section 4. In Section 5 we write down the weakly nonlinear system of equations of the $\alpha\Omega$-dynamo and discuss the astrophysical implications of the results obtained.

2.   Starting equations and the approximations used

We start from the MHD equations for magnetic field H and velocity v,

$$\partial H/\partial t = \mathrm{curl}\,(v \times H) + \eta\,\Delta H \tag{1}$$

$$\frac{\partial v}{\partial t} + (v \cdot V)\,v - \frac{1}{\mu\varrho}(H \cdot V)\,H + \frac{1}{\varrho}V\left(P + \frac{H^2}{2\mu}\right) - \nu\,\Delta v + V\psi = 0\,, \tag{2}$$

where ψ is the gravitational potential and μ is the magnetic permeability. The incompressibility condition, div $v = 0$, will be used. If turbulence takes place, the fields v and H have random fluctuations u and h with a relatively small scale l against a background of mean components V and B of spatial scale L, $l \ll L$,

$$v = V + u\,, \qquad \langle v \rangle = V\,,$$
$$H = B + h\,, \qquad \langle H \rangle = B\,.$$

The problem is to derive equations for mean fields, V and B. On averaging Eqs. (1) and (2) we find

$$\partial B/\partial t - \mathrm{curl}\,(V \times B) - \eta\,\Delta B = \mathrm{curl}\,\langle (u \times h) \rangle\,,$$

$$\partial V_i/\partial t + V_j\,\nabla_j V_i - \frac{1}{\mu\varrho}B_j\,\nabla_j B_i + \frac{1}{\varrho}\,\nabla_i\left(\langle P \rangle + \frac{B^2}{2\mu}\right) + \nabla_i\psi - \nu\,\Delta V_i = -\nabla_j(\langle u_i u_j \rangle + \langle \delta_{ij}h^2/2 - h_i h_j \rangle/\mu\varrho)\,. \tag{3}$$

Repetition of subscripts signifies summation. It is necessary to express the mean EMF,

$$\boldsymbol{\varepsilon} = \langle (u \times h) \rangle\,, \tag{4}$$

and the turbulent stress tensor,

$$T_{ij} = \varrho\langle u_i u_j \rangle + \frac{1}{\mu}\left\langle \frac{1}{2}\,\delta_{ij}h^2 - h_i h_j \right\rangle\,, \tag{5}$$

in terms of relatively simple parameters of the turbulence (such as $\langle u^2 \rangle$) and mean fields for taking advantages of Eqs. (3).

Next, we assume that the main mean flow is rotation which may be inhomogeneous (differential). There may also be mean motions different from rotation but their velocities are relatively small. It is convenient to calculate the correlations (4) and (5) for a point whose angular velocity of global rotation is in the corotating coordinate system; the origin of the system is placed in this point. With relativistic effects neglected, the induction equation (1) remains unaltered, whilst the equation of motion now reads

$$\partial v/\partial t + (v \cdot \nabla)\,v - \frac{1}{\mu\varrho}(H \cdot \nabla)\,H + 2(\Omega \times v) + \frac{1}{\varrho}\nabla(P + H^2/2\mu) + \nabla\psi' = \nu\,\Delta v\,, \tag{2'}$$

where ψ' is the sum of the gravitational and centrifugal potentials.

The method of perturbation in mean-field inhomogeneity (i.e. the decomposition in powers of the parameter l/L) will be applied below. So, the field B is written as $B = B_0 + \delta B$, where B_0 is the magnetic field in the origin of the coordinates, and

δB appears because of the inhomogeneity. On averaging eqs. (1) and (2') and subtracting the result from these very equations, we obtain the equations for the fluctuations:

$$\partial h_i/\partial t - \nabla_j\{u_i h_j - u_j h_i - \langle u_i h_j\rangle + \langle u_j h_i\rangle\} - \eta\,\Delta h_i - (\mathbf{B}_0\cdot\nabla)\,u_i = \nabla_j(u_i\delta B_j - u_j\delta B_i + V_i h_i - V_j h_i)\,,$$

$$\partial u_i/\partial t + \nabla_j\left\{u_i u_j - \langle u_i u_j\rangle - \frac{1}{\mu\varrho}(h_i h_j - \langle h_i h_j\rangle)\right\} - \frac{1}{\mu\varrho}(\mathbf{B}_0\cdot\nabla)\,h_i + 2\varepsilon_{ijk}\Omega_j u_{k_r} + \frac{1}{\varrho}\nabla_i P' - \nu\,\Delta u_i - f_i \qquad (6)$$

$$= -\nabla_j(u_i V_j + u_j V_i) + \frac{1}{\mu\varrho}\nabla_j(h_i\delta B_j + h_j\delta B_i)\,,$$

where P' is the fluctuating pressure that includes the magnetic term; f is the random force driving the turbulence. The widely used second order correlation approximation will be applied. In other words, the nonlinear terms singled out by curly brackets in (6) will be omitted. This approximation is believed to be valid for two cases: (a) short-correlated random process when $\tau_c \ll l/u$ (τ_c is the correlation time) and (b) small Reynolds numbers, $R = lu/\nu \ll 1$ and $R_m = lu/\eta \ll 1$. For simplicity, we restrict our consideration to the case of small Reynolds numbers and assume the typical frequences ω of the fluctuations to be small, i.e. $\omega \sim u/l$, $\omega \ll \eta/l^2$ and $\omega \ll \nu/l^2$. In this case, the time derivatives can also be neglected in (6) as compared with the dissipative terms.

It will be convenient for the sequel to Fourier-transform eqs. (6) in spatial variables; in the approximation just made we obtain

$$\nu k^2\hat u_i(k) + 2(\hat{\mathbf{k}}\cdot\Omega)\,\varepsilon_{imp}\hat k_m\hat u_p(k) - \frac{i}{\mu\varrho}(\mathbf{B}_0\cdot k)\,\hat h_i(k) - \pi_{im}(k)\hat f_m(k) = -ik_j\pi_{im}(k)\times$$

$$\times\int[\hat u_m(k-q)\,\hat V_j(q) + u_j(k-q)\,\hat V_m(q) - \hat h_m(k-q)\,\delta\hat B_j(q)/\mu\varrho - \hat h_j(k-q)\,\delta\hat B_m(q)/\mu\varrho]\,\mathrm{d}q\,, \qquad (7)$$

$$\eta k^2\hat h_i(k) - i(\mathbf{B}_0\cdot k)\,\hat u_i(k) = ik_j\int[\hat u_i(k-q)\,\delta\hat B_j(q) - \hat u_j(k-q)\,\delta\hat B_i(q) +$$

$$+ \hat h_j(k-q)\,\hat V_i(q) - \hat h_i(k-q)\,\hat V_j(q)]\,\mathrm{d}q$$

where a circumflex indicates Fourier transforms, e.g.

$$\hat u(k) = \int\exp(-ik\cdot r)\,u(r)\,\mathrm{d}r/(2\pi)^3\,,$$

$\hat k = k/k$ is a unit vector in the direction of wave vector k; $\pi_{im}(k) = \delta_{im} - \hat k_i\hat k_m$ is the projection operator onto the plane normal to vector k.

In the first approximation we neglect the right-hand sides of eqs. (7). These means ignoring the mean-field inhomogeneity and yields:

$$\hat h(k) = i(\mathbf{B}_0\cdot k)\,\hat u(k)/(\eta k^2)\,, \qquad (8\,\mathrm{a})$$

$$\nu k^2[\hat u - \hat u^0] + 2(\hat{\mathbf{k}}\cdot\Omega)(\hat{\mathbf{k}}\times\hat u) - \frac{i}{\mu\varrho}(\mathbf{B}_0\cdot k)\,\hat h = 0\,, \qquad (8\,\mathrm{b})$$

where $\hat u_i^0(k) = \pi_{im}(k)\hat f_m(k)/(\nu k^2)$ is the Fourier-transform of a fluctuating velocity generated by force f when there is neither rotation nor magnetic field. Such a turbulence in the absence of a magnetic field and rotation will be referred to as "original" turbulence". Using eqs. (8), a formula can be found that expresses velocity fluctuations in terms of relevant fluctuations for the original turbulence:

$$\hat u_i(k) = D_{ij}(k)\,\hat u_j^0(k)\,, \qquad (9)$$

$$D_{ij}(k) = [(1 + \gamma_*^2(\hat{\mathbf{k}}\cdot b)^2)\,\delta_{ij} + \gamma(\hat{\mathbf{k}}\cdot e)\,\varepsilon_{ijp}\hat k_p]/[(1 + \gamma_*^2(\hat{\mathbf{k}}\cdot b)^2)^2 + \gamma^2(\hat{\mathbf{k}}\cdot e)^2]\,, \qquad (10)$$

where $e = \Omega/\Omega$ and $b = \mathbf{B}_0/B_0$ are unit vectors which have the same sence of direction as angular velocity, Ω, and magnetic field, $\mathbf{B}_0$, respectively; γ and γ_* are the square root of the local (k-dependent) Taylor number and local Hartmann number, respectively:

$$\gamma = 2\Omega/\nu k^2\,, \qquad \gamma_* = B/k\sqrt{\mu\varrho\nu\eta}\,. \qquad (11)$$

To shorten notations, the dependence of γ and γ_* on wave number k is not shown explicitly in (11) and subsequently. The dimensionless parameters γ and γ_* can be considered to be the measures of perturbations of turbulent fluctuations by rotation and magnetic field, respectively.

Eqs. (9) and (10) describe the influence of the Coriolis and Lorentz forces on the turbulence. The linear relation (9) will be used repeatedly in the derivations to follow. However, the use of eq. (10) for arbitrary magnetic fields meets some principal difficulties and below we shall apply the approximate expression,

$$D_{ij}(k) = \frac{\delta_{ij} + \gamma(\hat{\mathbf{k}}\cdot e)\,\varepsilon_{ijp}\hat k_p}{1 + \gamma^2(\hat{\mathbf{k}}\cdot e)^2} - \gamma_*^2(\hat{\mathbf{k}}\cdot b)^2\,\frac{(1 - \gamma^2(\hat{\mathbf{k}}\cdot e))\,\delta_{ij} + 2\gamma(\hat{\mathbf{k}}\cdot e)\,\varepsilon_{ijp}\hat k_p}{(1 + \gamma^2(\hat{\mathbf{k}}\cdot e)^2)^2}\,, \qquad (12)$$

which is correct for the case where the local Hartmann number is small for the main portion of the power spectrum of the fluctuations (weak magnetic field).

The original turbulence will be considered as given. The inhomogeneity of the turbulence will be taken into account, which is essential for the α-effect. We also assume that the original turbulence does not show any preferred directions except for the gradient of the turbulence intensity. The simplest turbulence having these properties was called "quasi-isotropic" by VAIN-SHTEIN (1968) and has the following spectral tensor (cf. KICHATINOV 1987):

$$\langle \hat{u}_i^0(z)\hat{u}_j^0(z')\rangle = F_{ij}(k = (z - z')/2 \;;$$

$$\varkappa = z + z') = \frac{\hat{E}(k, \varkappa)}{8\pi k^2} [\delta_{ij} - (1 + \varkappa^2/4k^2)\,\hat{k}_i\hat{k}_j + (\varkappa_i k_j - \varkappa_j k_i)/2k^2 + \varkappa_i\varkappa_j/4k^2]\,,$$

(13)

where $\hat{E}(k, \varkappa)$ is the Fourier-transform of the local power spectrum, i.e. $E(k, r) = \int \hat{E}(k, \varkappa) \exp(i\varkappa \cdot r)\,d\varkappa$ is the local power spectrum: $\langle u^2\rangle^\circ = \int_0^\infty E(k, r)\,dk$. The typical value of $\varkappa$ is $2\pi/L$. Only the terms up to second order in the parameter $\varkappa/k \sim l/L$ are retained in (13). The tensor (13) satisfies (again up to order l^2/L^2) the incompressibility conditions

$$z_i F_{ij} = (k_i + \varkappa_i/2)\,F_{ij} = 0\,,$$
$$z_j' F_{ij} = (-K_j + \varkappa_j/2)\,F_{ij} = 0\,.$$

In the marginal limit of homogeneous turbulence, we have $\hat{E}(k, \varkappa) = E_0(k)\,\delta(\varkappa)$, and (13) turns into the well-known spectral tensor for homogeneous, isotropic and mirror-invariant solenoidal field:

$$F_{ij}(k, \varkappa) = \delta(\varkappa)\frac{E_0(k)}{8\pi k^2}\,(\delta_{ij} - \hat{k}_i\hat{k}_j)\,.$$

(14)

The spectral tensor (13) differs from that usually used for the derivation of the α-effect (KRAUSE and RÄDLER 1980). However, the α-effect to be derived in what follows agrees with the results by KRAUSE and RÄDLER (1980).

The quasi-isotropic turbulence (13) being perturbed by rotation yields the nondissipative fluxes of the angular momentum (Λ-effect), the sources of differential rotation (KICHATINOV 1986b, 1987). The Λ-effect is usually believed to be the result of the turbulence anisotropy. The quasi-isotropic turbulence possesses some anisotropy. Indeed, on integrating (13) over k and performing a reciprocal Fourier transform over $\varkappa$ we get

$$\langle u_i^\circ u_j^\circ\rangle = \frac{\langle u^2\rangle^\circ}{3}\,\delta_{ij} - \frac{1}{8}\left(\nabla_i\nabla_j - \frac{1}{3}\,\delta_{ij}\,\Delta\right)\langle u^2\rangle^\circ\,l^2\,,$$

(15)

where

$$l^2 = \int_0^\infty E(k, r)\,k^{-2}\,dk \Big/ \int_0^\infty E(k, r)\,dk$$

is the typical spatial scale of the fluctuations. The anisotropy brought about by inhomogeneity is present in (15). However, this anisotropy cannot be the sole source of the Λ-effect derived below. This is indicated in particular by the fact that in the limit of rapid rotation the Λ-effect, found in Section 3 of this paper, tends to constant value, whereas the Λ-effect, caused by the anisotropy, should decrease in this limit as Ω^{-2} (RÜDIGER, 1983). In addition, the "compensating anisotropy" can be introduced in (13) by multiplying the spectrum $\hat{E}(k, \varkappa)$ by the factor $1 + 15((\hat{k} \cdot \varkappa)^2 - \varkappa^2/3)/8k^2$. This procedure results in local isotropy; i.e. we now have $\langle u_i^0 u_j^0\rangle = \delta_{ij}\langle u^2\rangle^0/3$ rather then (15). Nevertheless, the new spectral tensor thus obtained yields the same results, in particular the same Λ-effect, as the tensor (13).

Using (8a), (9), and (12), the desired correlations (4) and (5) can be derived from the given spectral tensor (13) for the original turbulence. The corresponding spectral tensors are firstly constructed and then integrated over wave vectors, which is equivalent to the reciprocal Fourier transformation for the origin of coordinates. This yields the general result because of the origin of the rotating coordinate system used is an arbitrary point.

We neglect above in the first approximation the right-hand sides of eqs. (7). In the same approximation, the original turbulence should be considered homogeneous; i.e. eq. (14) should be used. It may be shown that this leads to the α-effect and the sources of differential rotation fall to zero. It is necessary to carry out all derivations in the next approximation by taking inhomogeneity into account. On using the spectral tensor (13) and eqs. (8a), (9) and (12), the nondissipative contributions to the mean EMF and turbulent stress tensor (α-effect, pumping and Λ-effect) with regard for the nonlinear corrections can be derived. It is also necessary to take into account the perturbations u' and h' of the fluctuations u and h by the inhomogeneity of mean fields, i.e. to allow for the right-hand sides of eqs. (7). The fluctuations u and h, calculated for a homogeneous case, can be placed in the right-hand sides of (7) when deriving these perturbations because the large-scale inhomogeneity of the mean fields is always present in these terms. The allowance for the perturbations u' and h' in correlations (4) and (5) yields effective viscosity and magnetic diffusivity. The back reaction of the magnetic field on the turbulence will be neglected

in derivations of the dissipative effects and the following expressions will be used,

$$u'_i(k) = -\frac{ik_j}{\nu k^2} D^\circ_{in}(k)\, \pi_{nm}(k) \int \left[\hat{u}_m(k-q)\,\hat{V}_j(q) + \hat{u}_j(k-q)\,V_m(q)\right] \mathrm{d}q \,,$$

$$\tilde{h}'_i(k) = \frac{ik_j}{\eta k^2} \int \left[\hat{u}_i(k-q)\,\delta B_j(q) - \hat{u}_j(k-q)\,\delta B_i(q)\right] \mathrm{d}q \,,$$

where

$$D^\circ_{ij} = D_{ij}|_{\gamma_* = 0} = \left[\delta_{ij} + \gamma(\hat{k}\cdot e)\,\varepsilon_{ijp}\hat{k}_p\right]/\left[1 + \gamma^2(\hat{k}\cdot e)^2\right] \,.$$

Certainly, the neglect of nonlinearities in calculations of dissipative effects makes the analysis incomplete and rules out the possibility of analysing the whole family of nonlinearities. However, this restriction seems reasonable in two respects. First, the nonlinear derivations of the eddy viscosities and magnetic diffusivities are too involved and seem to be the subject of a separate paper. Second, the main consequences of the nonlinearity, that probably are nonlinear stabilization of the generation of magnetic fields and magnetically induced variations of differential rotation, should manifest themselves through the magnetic corrections to the nondissipative effects even if these corrections are neglected in the eddy transport coefficients.

Let us drop rather cumbersome intermediate calculations, that are made according to the perturbation scheme presented above, and proceed to the consideration of the final results.

3.   The mean EMF

The derivations, whose sequence was explained in the preceding section, yield the expression for the mean EMF (4). Three groups of terms are convenient to distinguish in this expression that differ in both structure and physical sense:

$$\langle(u \times h)\rangle_i = \alpha_{ij}B_j + (\varepsilon_{imj}U_m + \varepsilon_{imn}U'_m e_n e_j)\,B_j - (D\varepsilon_{imj} + D_{//}\varepsilon_{inj}e_n e_m)\,\nabla_m B_j \,. \tag{16}$$

α_{ij} is the symmetric pseudotensor, odd with respect to the change of the sense of rotation ($\Omega \to -\Omega$). It represents the α-effect of magnetic field generation in the rotating turbulent conducting fluid.

The next two terms between round brackets correspond to the effect of magnetic field transport by inhomogeneous turbulence. The velocities u and u' are even functions of the angular velocity.

The last two terms in (16) describe the turbulend diffusion of the mean magnetic fields.

Naturally, there is no RÄDLER's (1969) $\Omega \times j$ effect under the approximations adopted.

Let us consider each of the above-mentioned effects separately.

3.1.   The α-effect

The structure of the tensor α_{ij} is

$$\alpha_{ij} = \alpha_1\delta_{ij} + \alpha_2(g_i e_j + g_j e_i) + \alpha_3 e_i e_j + \alpha_4 b_i b_j + \alpha_5(e_i b_j + e_j b_i) + \alpha_6(b_i g_j + b_j g_i) \,,$$

$$g = (\nabla\langle u^2\rangle^0)/|\nabla\langle u^2\rangle^0| \,. \tag{17}$$

The expressions for the coefficients α_n of (17) are:

$$\alpha_1 = -\int_0^\infty E'(k,r)\,\gamma\{(e\cdot g)\left[f_1(\gamma) - \gamma_*^2\tilde{f}_1(\gamma) + \gamma_*^2(e\cdot b)^2\,\tilde{f}_3(\gamma)\right] + (g\cdot b)(b\cdot e)\,\gamma_*^2\tilde{f}_4(\gamma)\}\,\frac{\mathrm{d}k}{\eta k^2} \,,$$

$$\alpha_2 = \int_0^\infty E'(k,r)\,\gamma\left[f_2(\gamma) - \gamma_*^2\tilde{f}_2(\gamma) + \gamma_*^2(e\cdot b)^2\,\tilde{f}_5(\gamma)\right]\frac{\mathrm{d}k}{\eta k^2} \,, \tag{18}$$

$$\alpha_3 = \int_0^\infty E'(k,r)\,\gamma\{(e\cdot g)\left[f_3(\gamma) - \gamma_*^2\tilde{f}_3(\gamma) + \gamma_*^2(e\cdot b)^2\,\tilde{f}_6(\gamma)\right] + (g\cdot b)(b\cdot e)\,\gamma_*^2\tilde{f}_7(\gamma)\}\,\frac{\mathrm{d}k}{\eta k^2} \,,$$

$$\alpha_4 = 2(e\cdot g)\int_0^\infty E'(k,r)\,\gamma\gamma_*^2\tilde{f}_1(\gamma)\,\frac{\mathrm{d}k}{\eta k^2} \,,$$

$$\alpha_5 = -\int_0^\infty E'(k,r)\,\gamma\gamma_*^2\left[(g\cdot b)\,f_4(\gamma) + 2(g\cdot e)(e\cdot b)\,\tilde{f}_3(\gamma)\right]\frac{\mathrm{d}k}{\eta k^2} \,,$$

$$\alpha_6 = -2(e\cdot b)\int_0^\infty E'(k,r)\,\gamma\gamma_*^2\tilde{f}_2(\gamma)\,\frac{\mathrm{d}k}{\eta k^2} \,,$$

where $E'(k, r) = |\nabla E(k, r)|$, i.e. it is assumed that the direction g of the turbulence intensity gradient does not depend on k; otherwise gE' should be replaced by ∇E; the k-dependent parameters γ and γ_* are defined by (11); the functions $f_n(\gamma)$ and $\hat{f}_n(\gamma)$ are written down in Appendix A where asymptotic expressions for these functions for the cases $\gamma \ll 1$ and $\gamma \gg 1$ are also given.

In the slow rotation limit, when the major portion of the power spectrum of the fluctuations is locsliser in the range of the wave numbers, for which $\gamma \ll 1$, we can use eqs. (A1) to find

$$\alpha_1 = -\frac{2}{15} \int_0^\infty E'(k, r)\, \gamma[(1 - 4\gamma_*^2/7)\,(e \cdot g) + (g \cdot b)\,(b \cdot e)\, 6\gamma_*^2/7]\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_2 = \frac{1}{15} \int_0^\infty E'(k, r)\, \gamma[3 - 13\gamma_*^2/7]\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_3 = 0\,,$$

$$\alpha_4 = \frac{16}{105}\,(e \cdot g) \int_0^\infty E'(k, r)\, \gamma\gamma_*^2\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_5 = -\frac{4}{35}\,(g \cdot b) \int_0^\infty E'(k, r)\, \gamma\gamma_*^2\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_6 = -\frac{26}{105}\,(e \cdot b) \int_0^\infty E'(k, r)\, \gamma\gamma_*^2\,\frac{dk}{\eta k^2}\,.$$

(19)

The opposite limit of rapid rotation also somewhat simplifies eqs. (18):

$$\alpha_1 = -\frac{\pi}{8}\,(e \cdot g) \int_0^\infty E'(k, r)\,\{1 - \gamma_*^2[1 - (e \cdot b)^2]/2\}\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_3 = \frac{\pi}{8}\,(e \cdot g) \int_0^\infty E'(k, r)\,\{1 - \gamma_*^2[1 - 3(e \cdot b)^2]/2\}\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_4 = \frac{\pi}{8}\,(e \cdot g) \int_0^\infty E'(k, r)\,\gamma_*^2\,\frac{dk}{\eta k^2}\,,$$

$$\alpha_5 = -\frac{\pi}{8}\,(g \cdot e)\,(e \cdot b) \int_0^\infty E'(k, r)\,\gamma_*^2\,\frac{dk}{\eta k^2}\,, \qquad \alpha_2 = \alpha_6 = 0\,.$$

(20)

The asymptotic expressions (A2) have been used to obtain (20).

The linear part of eqs. (19) agrees with known results obtained for the case of slow rotation (STEENBECK et al., 1966; see also KRAUSE and RÄDLER, 1980). In the comparison, it is necessary to take into consideration that the approximation of low conductivity and large viscosity is accepted in this paper. The nonlinear corrections in (19) are similar to that found by ROBERTS and SOWARD (1975) for the case of slow rotation. There is no complete agreement because of the distinction between the approximations adopted.

The case of rapid rotation in the linear approximation was considered by RÜDIGER (1978). The linear part of eqs. (20) coincides with RÜDIGER's results if these latter are considered for the case of low frequences ($\omega \ll \nu k^2$, $\omega \ll \eta k^2$). (Note that RÜDIGER used the spectrum $\hat{q}(k)$ that differs from E of this paper, and $E'(k, r) = \frac{4}{3} \pi k^4 W \hat{q}(k)$ in his notations.)

In the rapid rotation limit, the coefficients (20) of the tensor α_{ij} do not depend on the angular velocity and tend to constant asymptotic values. For the linear α-effect, this has been demonstrated by RÜDIGER (1978). As can be seen from (20), the nonlinear corrections to the tensor α_{ij} also have this property.

It seems worthwhile to note that the nonlinear part of the tensor α_{ij} falls to zero for the case of rapid rotation whenever magnetic field is parallel to the rotation axis ($b = e$). This is easy to see if we put $b = e$ in eqs. (20) and (17). In addition, the linear part of α_{ij} is proportional in this case to the tensor of projection onto the plane, normal to the axis of rotation, $\alpha_{ij} \sim \delta_{ij} - e_i e_j$, in agreement with MOFFATT (1970) and RÜDIGER (1978), and yields zero in convolution with vector B, $\alpha_{ij} B_j = 0$. In other words, there is no α-effect for the case of rapid rotation if $B \| \Omega$. This can be probably explained as follows.

When rotation is rapid ($2\Omega/vk^2 \gg 1$), the typical spatial scale of the fluctuations in the direction of the rotation axis is large against the scale normal to it. This means that the turbulence tends to be two-dimensional, though the velocities in the direction of the rotation axis are present and are not small. The magnetic field does not affect this turbulence if it has the direction e in which the velocities of the turbulent motions do not vary (Rüdiger, 1974; see also Rädler, 1974). Hence, such a field cannot yield any nonlinear corrections to the α-effect. In addition, this field behaves like a passive scalar field and obeys the same equation, $\partial B/\partial t + \mathrm{div}\, uB = \eta\, \Delta B\,(B = eB)$. There is no effect of turbulent generation for scalar fields; thus, the linear α-effect also vanishes (Zeldovich, 1956).

Eq. (17) with coefficients (18) is difficult to analyse for a general case. The nonlinearity induces fundamentally new structures in the tensor α_{ij}. Hence, the usually anticipated nonlinear reduction of α-effect is, at least, not obvious (through natural on physical grounds). However, this reduction really occurs for the widely applied mechanism of $\alpha\Omega$-dynamo. The role of the α-effect in this mechanism is to generate a poloidal field from a toroidal field (the axi-symmetric magnetic field is considered in the usually used spherical polar coordinates r, θ and φ with the polar angle θ measured from the axis of rotation). Only the azimuthal component of the mean EMF is important for this process. Hence, only the component $\alpha_{\varphi\varphi}$ of the tensor α_{ij} plays a role,

$$\alpha_{\varphi\varphi} = \alpha - B^2 \tilde{\alpha} \tag{21}$$

where

$$\alpha = -\cos\theta \int_0^\infty \frac{\partial E(k,r)}{\partial r} \gamma f_1(\gamma) \frac{\mathrm{d}k}{\eta k^2}, \tag{22}$$

$$\tilde{\alpha} = -\frac{3\cos\theta}{\mu\varrho\eta^2 v} \int_0^\infty \frac{\partial E(k,r)}{\partial r} \gamma \tilde{f}_1(\gamma) \frac{\mathrm{d}k}{k^4}. \tag{23}$$

It was assumed when deriving (22) and (23) that the turbulence inhomogeneity is radial. It was also taken into account that the dominant component of the magnetic field for the $\alpha\Omega$-dynamo is a toroidal one, and we may consider b as an azimuthal unit vector for which $(b \cdot g) = (b \cdot e) = 0$. Certainly, there must be both moments of time and positions when and where the toroidal field is not large against the poloidal field. However, the net magnetic field is relatively small in these cases and nonlinear corrections to the α-effect can be neglected. Growth of the magnetic field reduces the $\alpha_{\varphi\varphi}$-coefficient (21).

3.2. Pumping

It is customary to refer to the effects of the mean field transport in inhomogeneously turbulent fluids, that is not the result of the mean flow as pumpings (Drobyshevski and Yuferev, 1974; Krause and Rädler, 1980). The expressions for the velocities of mean field transport appearing in the mean EMF (16) are

$$U = -g \int_0^\infty E'(k,r) \left[\varphi_1(\gamma) - \gamma_*^2\tilde{\varphi}_1(\gamma) + \gamma_*^2(e \cdot b)^2\, \tilde{\varphi}_2(\gamma)\right] \frac{\mathrm{d}k}{\eta k^2}, \tag{24}$$

$$U' = g \int_0^\infty E'(k,r) \left[\varphi_2(\gamma) - \gamma_*^2\tilde{\varphi}_2(\gamma) + \gamma_*^2(e \cdot b)^2\, \tilde{\varphi}_3(\gamma)\right] \frac{\mathrm{d}k}{\eta k^2}. \tag{25}$$

The functions $\varphi_n(\gamma)$ and $\hat{\varphi}_n(\gamma)$ are written down in Appendix A. Vector U' is the additional velocity of transport of the mean field component parallel to the rotation axis.

In the slow rotation limit, when the parameter $\gamma = 2\Omega/vk^2$ is small for the main portion of the power spectrum, we find the velocity of diamagnetic field transport not perturbed by rotation,

$$U = -g \int_0^\infty E'(k,r)\,(1/6 - \gamma_*^2/5) \frac{\mathrm{d}k}{\eta k^2}, \tag{26}$$

$$U' = 0.$$

Eqs. (A1) have been used to obtain (26). The linear part of (26) represents the usual diamagnetism: the mean field is expelled into the regions with relatively low intensity of the turbulence (Zeldovich, 1956; Rädler, 1968). The velocity (26) decreases with an increase of the magnetic field.

In the opposite limit of rapid rotation, the asymptotic expressions (A2) can be used to yield

$$U = -U' = -g\, \frac{\pi}{8} \int_0^\infty E'(k,r)\,\{1 - 3\,[1 - (e \cdot b)^2]/4\} \frac{\mathrm{d}k}{\eta k^2}. \tag{27}$$

The absolute value of the velocity (27) is also the decreasing function of the magnetic field. The general expressions (24) and (25) are rather complicated; however, as with the α-effect, the nonlinear reduction of the diamagnetic pumping can be found from these expressions for the particular case of a predominantly toroidal magnetic field.

It may also be noted that the nonlinear corrections to the velocities (27) go to zero when the magnetic field is parallel to the axis of rotation ($b = e$). Moreover, the linear part of the total velocity of transport of the field $B\|\Omega$ is zero as well for this case of rapid rotation, because the velocities U and U' (27) have equal absolute values but opposite directions. The reason for this has been discussed in the proceeding section, it probably is the tendency for the turbulence to become two-dimensional under rapid global rotation.

3.3.   Eddy diffusivities of the magnetic field

As mentioned in section 2, we neglect the back reaction of the magnetic field on the turbulence when deriving the dissipative effects.

The calculations, whose sequence has been explained in Section 2, yield the following eddy diffusivities D and $D_{\|}$ involved in (16),

$$D = \int_0^\infty E(k, r)\, \varphi_3(\gamma)\, \frac{\mathrm{d}k}{\eta k^2} \,,$$

$$D_{\|} = \int_0^\infty E(k, r)\, \varphi_2(\gamma)\, \frac{\mathrm{d}k}{\eta k^2} \,. \tag{28}$$

The functions $\varphi_n(\gamma)$ are given in Appendix A. The quantity D is the isotropic part of the eddy diffusivity of the magnetic field. The rotation induces some anisotropy in the turbulence. This results in the additional diffusivity, $D_{\|}$, parallel to the axis of rotation. Using eqs. (A1) and (A2), we find the coefficients D and $D_{\|}$ for two cases: with no rotation present,

$$D = \int_0^\infty \frac{E(k, r)}{3\eta k^2}\, \mathrm{d}k \,, \qquad D_{\|} = 0 \,,$$

and with rapid rotation,

$$D = D_{\|} = \frac{\pi}{8} \int_0^\infty \frac{E(k, r)}{\gamma\eta k^2}\, \mathrm{d}k = \frac{\pi\nu}{16\Omega\eta}\, \langle u^2 \rangle^0 \,.$$

In the latter case, the effective diffusivity parallel to the axis of rotation (the sum of D and $D_{\|}$) is twice the diffusivity normal to this axis.

4.   Turbulent stress tensor

The dissipative and nondissipative parts can be distinguished in the turbulent stress tensor (5),

$$T_{ij} = \Pi_{ij} + \Lambda_{ij} - N_{ijpf}\, \nabla_f V_p \,. \tag{29}$$

The last term on the right describes the turbulent damping of the large scale motions. The first two terms do not depend on the spatial derivatives of the mean velocities and represent the nondissipative part of T_{ij}. These two terms stand for the even (Π_{ij}) and odd (Λ_{ij}) contributions relative to the inversion of the sense of rotation.

The derivation of the nondissipative part of the tensor T_{ij} with allowance for global rotation is of interest to the investigations of the differential rotation of turbulent shells. Only the off-diagonal components, $T_{r\varphi}$ and $T_{\theta\varphi}$ (in spherical coordinates), are relevant to the differential rotation problem; these components are the radial and meridional Fluxes of Angular Momentum (FAM), respectively. The presence of nondissipative parts, $\Lambda_{r\varphi}$ and $\Lambda_{\theta\varphi}$, in the FAMs was named Λ-effect (RÜDIGER, 1980). These nondissipative off-diagonal elements are odd functions of the angular velocity. We shall avoid cluttering up the paper with calculation of the turbulent pressure, Π_{ij}, and shell confine our attention only to the derivation of the Λ-effect and eddy viscosities.

4.1. Λ-effect

To simplify matters we shall assume the direction b of the magnetic field to be normal to both the rotation axis and the direction g of inhomogeneity of the original turbulence. If the inhomogeneity is radial, as is usually adopted, our assumption means that the toroidal component of the mean magnetic field is dominant. This situation is typical of the $\alpha\Omega$-dynamo.

Though the assumption made above simplify the derivations considerably, the final result for the tensor Λ_{ij} of eq. (29) is somewhat cumbersome:

$$\Lambda_{ij} = \frac{1}{8} \varrho \int\limits_0^\infty [(\psi_1(\gamma) - \gamma_*^2 \tilde{\psi}_1(\gamma)) (\varepsilon_{imp} \nabla_j + \varepsilon_{jmp} \nabla_i) e_m \nabla_p - (\psi_2(\gamma) - \gamma_*^2 \tilde{\psi}_2(\gamma)) (e_j\varepsilon_{imp} + e_i\varepsilon_{jmp}) \times$$

$$\times e_m(\boldsymbol{e} \cdot \nabla) \nabla_p + \gamma_*^2\tilde{\psi}_3(\gamma) (e_j\varepsilon_{imp} + e_i\varepsilon_{jmp}) b_m(\boldsymbol{b} \cdot \nabla) \nabla_p + \gamma_*^2\tilde{\psi}_3(\gamma) (b_j\varepsilon_{imp} + b_i\varepsilon_{jmp}) e_m(\boldsymbol{b} \cdot \nabla) \nabla_p -$$

$$- \gamma_*^2\tilde{\psi}_4(\gamma) (b_j\varepsilon_{imp} + b_i\varepsilon_{jmp}) b_m(\boldsymbol{e} \cdot \nabla) \nabla_p] E(k, r) \gamma \frac{dk}{k^2}, \tag{30}$$

where the operators ∇ act upon the local spectrum $E(k, \boldsymbol{r})$; the functions $\psi_n(\gamma)$ and $\tilde{\psi}_n(\gamma)$ of the parameter $\gamma = 2\Omega/\nu k^2$ are written down in Appendix B together with the asymptotic expressions for these functions. Eq. (30) involves both Reynolds and Maxwellian stresses. These contributions are easy to distinguish. The point here is that the functions $\tilde{\psi}_n(\gamma)$ listed in Appendix B involve the magnetic Prandtl number, $P_m = \nu/\eta$. This number appears through the contribution of the fluctuating magnetic fields. Therefore, we find the contribution of the Maxwellian stresses to eq. (30) if $\psi_n(\gamma)$ is put equal to zero and $\tilde{\psi}_n(\gamma)$ retains only the terms containing P_m.

Let us consider the most popular particular case of the rotating spherical turbulent layer with purely radial inhomogeneity of the original turbulence in the usual spherical coordinates r, θ and φ. There are only two nonzero and mutually independent components, $\Lambda_{r\varphi}$ and $\Lambda_{\theta\varphi}$, of the tensor Λ_{ij}. Other components of the nondissipative part of the stress tensor T_{ij} are even functions of the angular velocity.

We shall distinguish in the radial, $\Lambda_{r\varphi}$, and meridional, $\Lambda_{\theta\varphi}$, nondissipative FAMs the linear parts, Λ_r and Λ_θ, and magnetic corrections, Λ_r^m and Λ_θ^m, respectively,

$$\Lambda_{r\varphi} = \Lambda_r - B^2\Lambda_r^m, \qquad \Lambda_{\theta\varphi} = \Lambda_\theta - B^2\Lambda_\theta^m, \tag{31}$$

where Λ_i and Λ_i^m are independent of the magnetic field. The explicit expressions for the coefficients of eqs. (31) are:

$$\Lambda_r = \frac{1}{8} \varrho \sin \theta \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) [\psi_1(\gamma) - \cos^2 \theta\psi_2(\gamma)] \gamma \frac{dk}{k^2},$$

$$\Lambda_\theta = \frac{1}{8} \varrho \sin^2 \theta \cos \theta \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \psi_2(\gamma) \gamma \frac{dk}{k^2},$$

$$\Lambda_r^m = \frac{\sin \theta}{8\mu\nu\eta} \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) [\tilde{\psi}_1(\gamma) - \cos^2 \theta\tilde{\psi}_2(\gamma)] \gamma \frac{dk}{k^4} + \frac{\sin \theta}{8\mu\nu\eta} \int\limits_0^\infty \left(\frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) [\tilde{\psi}_3(\gamma) + \tilde{\psi}_4(\gamma)] \gamma \frac{dk}{k^4},$$
$$\tag{32}$$

$$\Lambda_\theta^m = \frac{\sin^2 \theta \cos \theta}{8\mu\nu\eta} \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \tilde{\psi}_2(\gamma) \gamma \frac{dk}{k^4} + \frac{\cos \theta}{8\mu\nu\eta} \int\limits_0^\infty \left(\frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \tilde{\psi}_3(\gamma) \gamma \frac{dk}{k^4} +$$

$$+ \frac{\cos \theta}{8\mu\nu\eta} \int\limits_0^\infty \frac{\partial^2 E(k, r)}{\partial r^2} \tilde{\psi}_4(\gamma) \gamma \frac{dk}{k^4}.$$

The nondissipative FAMs for the case of slow rotation may be found by using eqs. (B1). Up to linear terms in angular velocity (in the parameter γ) we find,

$$\Lambda_r = \frac{1}{60} \varrho \sin \theta \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \gamma \frac{dk}{k^2},$$

$$\Lambda_\theta = 0,$$

$$\Lambda_r^m = \frac{\sin \theta}{\mu\nu\eta} \left[\frac{1}{28} (1/3 + P_m/5) \int\limits_0^\infty \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \gamma \frac{dk}{k^4} + \frac{1}{15} (1 + P_m/2) \int\limits_0^\infty \left(\frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \gamma \frac{dk}{k^4} \right], \tag{33}$$

$$\Lambda_\theta^m = \frac{\cos \theta}{7\mu\nu\eta} \left[(3/10 + 2P_m/15) \int\limits_0^\infty \left(\frac{1}{r} \frac{\partial E(k, r)}{\partial r} \right) \gamma \frac{dk}{k^4} + (1/6 + P_m/10) \int\limits_0^\infty \frac{\partial^2 E(k, r)}{\partial r^2} \gamma \frac{dk}{k^4} \right].$$

It is known that in the linear approximation in the angular velocity, the turbulence has to possess an anisotropy in the horizontal plane for meridional nondissipative FAM to arise (RÜDIGER, 1980). There is no such anisotropy for the radially inhomogeneous original turbulence with the spectral tensor (13). This is why Λ_θ is zero in (33). However, the magnetic correction Λ_θ^m differs from zero. This may be explained by the horizontal anisotropy brought about by the influence of the horizontal magnetic field on the turbulent motions, which gives rise to the meridional nondissipative FAM, $\Lambda_{\theta\varphi} = -B^2\Lambda_\theta^m$, even for slow rotation.

In the opposite case of rapid rotation ($\gamma \gg 1$) eq. (B2) may be used to yield,

$$\Lambda_r = -\frac{\pi\varrho}{32}\sin\theta\cos^2\theta\int_0^\infty\left(r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial E(k,r)}{\partial r}\right)\frac{dk}{k^2},$$

$$\Lambda_\theta = -\mathrm{tg}\,\theta\Lambda_r,$$

$$\Lambda_r^m = \frac{\pi\sin\theta}{64\mu\nu\eta}(1+P_m/2)\left[2\int_0^\infty\left(\frac{1}{r}\frac{\partial E(k,r)}{\partial r}\right)\frac{dk}{k^4} - \cos^2\theta\int_0^\infty\left(r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial E(k,r)}{\partial r}\right)\frac{dk}{k^4}\right], \qquad (34)$$

$$\Lambda_\theta^m = \frac{\pi\cos\theta}{64\mu\nu\eta}(1+P_m/2)\left[2\int_0^\infty\frac{\partial^2 E(k,r)}{\partial r^2}\frac{dk}{k^4} + \sin^2\theta\int_0^\infty\left(r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial E(k,r)}{\partial r}\right)\frac{dk}{k^4}\right].$$

Eqs. (34) show that both linear and magnetic nondissipative FAMs tend to constant values independent of the angular velocity in the rapid rotation limit. Such behaviour is typical of the Λ-effect caused by the inhomogeneity of the original turbulence (KICHATINOV, 1986b, 1987) and, consequently, this inhomogeneity is important for the nondissipative FAMs derived. If these FAMs were caused solely by the anisotropy present in the spectral tensor (13), the Λ-effect would decrease as the reciprocal of the angular velocity squared and approach zero in the limit of rapid rotation, as has been shown by RÜDIGER (1983).

The obtained Λ-effect (30)—(32) is of second order in the scales ratio, l/L, of fluctuating and mean fields. The adopted procedure of calculations can be reliably justified only when this ratio is small, and the Λ-effect is small, too. However, in natural conditions, in particular in the solar convection zone, $l/L \lesssim 1$. If the derivations made remain qualitatively valid for this case, which is usually anticipated, the Λ-effect of eqs. (30)—(32) is no longer small.

4.2.   Eddy viscosities

As in the derivation of the eddy diffusivities of the magnetic fields, we neglect the nonlinear (magnetic) contributions to the effective viscouos tensor N_{ijpf} present in (29). The derivations, whose sequence was explained in Section 2, yield:

$$\varrho^{-1}N_{ijpf} = \nu_1(\delta_{ip}\delta_{jf} + \delta_{if}\delta_{jp}) + \nu_2(\delta_{if}e_ie_p + \delta_{if}e_je_p + \delta_{jp}e_ie_f + \delta_{ip}e_je_f) - \nu_3\delta_{ij}e_pe_f + \nu_4e_ie_je_pe_f +$$

$$+ \nu_5(\varepsilon_{ipm}e_me_je_f + \varepsilon_{jpm}e_me_ie_f + \varepsilon_{imf}e_je_me_p + \varepsilon_{jmf}e_ie_me_p) + \nu_6(\varepsilon_{ipm}e_m\delta_{jf} + \varepsilon_{jpm}e_m\delta_{if}) -$$

$$- \nu_7\left(\varepsilon_{ipf}e_j + \varepsilon_{jpf}e_i + \frac{1}{2}\varepsilon_{imf}e_m\delta_{jp} + \frac{1}{2}\varepsilon_{jmf}e_m\delta_{ip}\right), \qquad (35)$$

where

$$\nu_n = \int_0^\infty E(k,r)\,\zeta_n(\gamma)\,\frac{dk}{\nu k^2}, \qquad (36)$$

the functions $\zeta_n(\gamma)$ are written down in Appendix B where the values of these functions for $\gamma = 0$ and $\gamma \gg 1$ are also given. The terms containing δ_{pf} are omitted in (35) because when convoluted with $\nabla_f V_p$ they yield zero as a result of the incompressibility condition and do not contribute to the stress tensor (29).

The presence of the preferred direction e in the tensor of effective viscosities (35) results from allowance for global rotation when deriving this tensor. If rotation is absent, eqs. (B1) can be used to find the known result (KRAUSE and RÜDIGER, 1974):

$$N_{ijpf}|_{\Omega=0} = \frac{4\varrho}{15\nu}\int_0^\infty E(k,r)\,k^{-2}\,dk(\delta_{ip}\delta_{jf} + \delta_{if}\delta_{jp}).$$

As may be seen from (B2) and (36), all coefficients ν except for the pseudoscalars ν_5, ν_6 and ν_7 differ from zero in the rapid rotation limit in the first order in the parameter γ^{-1}.

Let us consider the components $N_{r\varphi pf}\nabla_f V_p$ and $N_{\theta\varphi pf}\nabla_f V_p$ of the dissipative part of the turbulent stress tensor, that are important for the differential rotation problem, for a particular case of purely azimuthal mean flow,

$$V = e_\varphi\Omega(r,\theta)\,r\sin\theta,$$

where e_φ is the azimuthal unit vector; r, θ and φ are spherical polar coordinates. Using (35), we find:

$$N_{r\varphi pf} \nabla_f V_p = \varrho v_1 r \sin\theta \frac{\partial\Omega}{\partial r} + \varrho v_2 \sin\theta \cos\theta \left(r \cos\theta \frac{\partial\Omega}{\partial r} - \sin\theta \frac{\partial\Omega}{\partial\theta} \right),$$

$$N_{\theta\varphi pf} \nabla_f V_p = \varrho v_1 \sin\theta \frac{\partial\Omega}{\partial\theta} + \varrho v_2 \sin^2\theta \left(\sin\theta \frac{\partial\Omega}{\partial\theta} - r \cos\theta \frac{\partial\Omega}{\partial r} \right).$$

(37)

The dissipative FAMs (37) compensate for the nondissipative ones (31) in the stationary case. Eqs. (37) coincide in their structure with the dissipative FAMs found (KICHATINOV, 1986a) for the case of large Reynolds numbers in the mixing length approximation.

5.   Discussion

The derivations made of the mean EMF and the turbulent stress tensor allow one to infer the weakly nonlinear system of $\alpha\Omega$-dynamo equations,

$$\frac{\partial B}{\partial t} = \frac{1}{r} \frac{\dot{\partial}(\Omega, Ar \sin\theta)}{\partial(r, \theta)} - \frac{1}{r} \frac{\partial}{\partial r} rUB + \frac{1}{r} \frac{\partial}{\partial r} (D + \eta) \frac{\partial}{\partial r} rB +$$

$$+ \frac{1}{r^2} \frac{\partial}{\partial\theta} \frac{D+\eta}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta B + \left(\cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial\theta} \right) D_{\|} \left(\cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial\theta} \right) B,$$

$$\frac{\partial A}{\partial t} = (\alpha - B^2 \tilde{\alpha}) B - \frac{U}{r} \frac{\partial}{\partial r} Ar - U' \left(A/r + \sin^2\theta \frac{\partial A}{\partial r} + \sin\theta \cos\theta \frac{1}{r} \frac{\partial A}{\partial\theta} \right) + \frac{(D+\eta)}{r} \frac{\partial^2}{\partial r^2} rA$$

$$+ \frac{(D+\eta)}{r^2} \frac{\partial}{\partial\theta} \frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta A + D_{\|} \left(\cos\theta \frac{\partial}{\partial r} - \sin\theta \frac{1}{r} \frac{\partial}{\partial\theta} \right)^2 A,$$

(38)

$$\varrho r \sin\theta \frac{\partial\Omega}{\partial t} = \frac{1}{\mu r^3 \sin^2\theta} \frac{\partial(Br \sin\theta, Ar \sin\theta)}{\partial(r, \theta)}$$

$$+ \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \left[\varrho(v_1 + v) r \sin\theta \frac{\partial\Omega}{\partial r} + \varrho v_2 \cos\theta \sin\theta \left(r \cos\theta \frac{\partial\Omega}{\partial r} - \sin\theta \frac{\partial\Omega}{\partial\theta} \right) - \Lambda_r + B^2 \Lambda_r^m \right]$$

$$+ \frac{1}{r \sin^2\theta} \frac{\partial}{\partial\theta} \sin^2\theta \left[\varrho(v_1 + v) \sin\theta \frac{\partial\Omega}{\partial\theta} + \varrho v_2 \sin^2\theta \left(\sin\theta \frac{\partial\Omega}{\partial\theta} - r \cos\theta \frac{\partial\Omega}{\partial r} \right) - \Lambda_\theta + B^2 \Lambda_\theta^m \right],$$

where B is the toroidal magnetic field; A is the toroidal vector potential for the poloidal magnetic field B^p, $B^p = \text{curl}\, (e_\varphi A)$; and symbol $\partial(X, Y)/\partial(r, \theta) = (\partial X/\partial r)(\partial Y/\partial\theta) - (\partial Y/\partial r)(\partial X/\partial\theta)$ means Jacobian. A number of the parameters calculated above are used in (38); these are: the coefficient α (22) of magnetic field generation and magnetic correction $\tilde{\alpha}$ (23) to it, the velocities U (24) and U' (25) of diamagnetic pumping, the eddy diffusivities D and $D_{\|}$ (28) of the magnetic field, the nondissipative FAMs Λ_r and Λ_θ and magnetic corrections to them Λ_r^m and Λ_θ^m (32) and the eddy viscosities v_1 and v_2 (36). The mean meridional flow is neglected in (38).

Although the system of equations (38) involves a variety of effects of the $\alpha\Omega$-dynamo including nonlinearities, it is hardly applicable to describe the real mean fields and flows of the solar convection zone. The point here is not that the approximation of low conductivity and high viscosity has been used. As pointed out in the introduction, it is reasonable to hope that our findings will hold qualitatively in the case of large Reynolds numbers if the coefficients v and η are considered to be of turbulent origin; i.e. if the estimate $v \sim \eta \sim lu$ is used. (Note that other approximations used in mean field magnetohydrodynamics can also guarantee only an order-of-magnitude accuracy under natural astrophysical conditions.) More substantial restrictions seem to be imposed by the neglect of the compressibility. The density stratification of the solar convection zone is of vital importance for α-effect (PARKER, 1955), for turbulent transport of magnetic fields (DROBYSHEVSKI, 1977) as well as for generation of differential rotation (KICHATINOV, 1986b, 1987). However, the allowance for compressibility greatly complicates the problem and is neglected for simplicity in this paper.

In spite of the restrictions mentioned, the system (38) contains new nonlinear effects that deserve some discussion. These effects should also appear in more general theory taking the compressibility into account.

Let us consider first the magnetic corrections to the nondissipative FAMs (31) and (32). These corrections, $B^2 \Lambda_r^m$ and $B^2 \Lambda_\theta^m$, must yield the dependence of differential rotation on the magnetic field (a similar effect results from the mean fields Lorentz force discussed below). Only the nonlinear corrections to the α-effect are usually considered in nonlinear models of

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the $\alpha\Omega$-dynamo (see e.g. KLEEORIN and RUZMAIKIN, 1984). However, the Ω-effect (differential rotation) is not less essential for the magnetic field generation by the $\alpha\Omega$-dynamo mechanism than α-effect. In addition, the relative values of magnetic corrections to the α- and Λ-effects coincide in order of magnitude. Hence, the complete nonlinear model of the $\alpha\Omega$-dynamo should incorporate the magnetically perturbed Λ-effect together with the nonlinear α-effect.

The relative values of the magnetic corrections to the effects considered is γ_*^2 (11). The estimates of the measures of the convection perturbation by rotation (γ) and magnetic field (γ_*) for the case of large Reynolds numbers may be found if we put in (11) $\eta \sim \nu \sim lu$ and $k \sim 1/l$; this yields,

$$\gamma \sim l\Omega/u , \qquad \gamma_* \sim B/(\sqrt{\mu\varrho}\, u) . \qquad (39)$$

As one would expect, γ is the reciprocal of the Rossby number and γ_*^2 is the ratio of magnetic energy to the energy of convective motions in this case. The toroidal magnetic field in the solar convection zone is believed to reach values $\sim 10^3$ G. The value of γ_* (39) can be estimated from the models of the solar convection zone (BAKER and TEMESVARY, 1966) as $\gamma_* \sim \sim 0.1-1$. In other words, the perturbation of the convection by magnetic field and resulting nonlinear corrections to the α- and Λ-effects should be essential. Proceeding in the inverse order, we may state that, if $\alpha\Omega$-dynamo is indeed stabilized by nonlinear effects, the magnetic corrections to the α- and Λ-effects should not be too small and, consequently, the magnetic fields must be of the order $\sim 10^3$ G.

When magnetic fields change periodically over the solar cycles, the magnetic corrections to the nondissipative FAMs will also undergo cyclic variations. This will lead to cyclic changes of the angular velocity profile because the FAMs are the sources of differential rotation. Such periodic variations of solar rotation are indeed observed in the form of so-called torsional oscillations (HOWARD and LABONTE, 1980; LABONTE and HOWARD, 1982). The torsional oscillations are usually interpreted as being a result of the action of the mean field Lorentz force, $F_L = \dfrac{1}{\mu}(\text{curl } B) \times B$ (SCHÜSSLER, 1981; YOSHIMURA, 1981). This force is represented in eqs. (38) by the first term on the right-hand side of the last equation of the system. The estimate of the Lorentz force reads,

$$F_L \sim BB^p/\mu L .$$

Explaining the torsional oscillations in terms of the force F_L is complicated by the relatively small values of the poloidal magnetic field, B^p, which is two to three orders of magnitude smaller than the toroidal field. This makes the force F_L be small as well and, probably, was the reason why the torsional oscillations driven by the Lorentz force were attributed to the surface layer with the thickness $L \sim 10^4$ km and relatively low density. However, deeper and denser layers of the Sun seem also to take part in the torsional oscillations (HOWARD, 1984). The nonlinear corrections in (30)—(32) may be expressed in terms of the equivalent force, F_N, to yield the estimate,

$$F_N \sim \left(\frac{l}{L}\right)^2 \gamma\tilde{\psi}\,(\gamma)\, B^2/\mu L ,$$

where $\tilde{\psi}$ symbolises the functions $\tilde{\psi}_n(\gamma)$ present in (30); and γ stands for the reciprocal of the Rossby number (39). The force F_N results from a perturbation of the convection by a magnetic field and thus induced changes of the Reynolds stresses. The scale l of the solar convection coincides in order of magnitude with the depth of the convection zone, and the reciprocal of the Rossby number is of order unity; hence

$$F_N/F_L \sim 0.1B/B^p \sim 10 .$$

In other words, perturbation of the sources of differential rotation by magnetic fields seem to be a more likely cause of the torsional oscillations as compared with the Lorentz force of mean fields. Surely, the order-of-magnitude estimates can not play any decisive role. Nevertheless, if the dynamo-amplification of the magnetic field is indeed stabilized by nonlinearities, the nonlinear corrections to the sources of differential rotation (that have the same relative values as the corrections to the α-effect) should be of some importance.

Another way the nonlinearities are involved in the system (38) is through the velocities U (24) and U' (25) of the diamagnetic pumping, that greatly influences the distribution of magnetic fields in the solar convection zone (KRIVODUBSKIJ, 1984).

We finally note that all derivations of this paper were made for arbitrary velocities of global rotation. This adds to the system (38) another strong nonlinearity, namely, the coefficients of this system are the nonlinear functions of the angular velocity. This, in principle, allows the $\alpha\Omega$-dynamo to be analysed for arbitrary rotational velocities. Note that in the solar convection zone the case of moderate rotation (neither slow nor rapid rotation) occurs.

Appendix A

The functions $f_n(\gamma)$, $\tilde{f}_n(\gamma)$, $\varphi_n(\gamma)$ and $\tilde{\varphi}_n(\gamma)$ of the parameter $\gamma = 2\Omega/vk^2$ used in Section 3 in the expressions for various contributions to the mean EMF are:

$$f_1(\gamma) = \frac{1}{4\gamma^4}\left[\gamma^2 + 9 - \frac{9 + 4\gamma^2 - \gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$f_2(\gamma) = \frac{3}{4\gamma^4}\left[\frac{\gamma^2 + 3}{\gamma}\tan^{-1}(\gamma) - 3\right],$$

$$f_3(\gamma) = \frac{1}{4\gamma^4}\left[-3\gamma^2 + 45 + \frac{4\gamma^4}{1 + \gamma^2} - \frac{45 + 12\gamma^2 - \gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\varphi}_1(\gamma) = \frac{1}{4\gamma^2}\left[\frac{\gamma^2 + 1}{\gamma}\tan^{-1}(\gamma) - 1\right], \qquad \varphi_2(\gamma) = \gamma^2 f_2(\gamma)/3,$$

$$\varphi_3(\gamma) = \frac{1}{4\gamma^2}\left[\frac{\gamma^2 - 1}{\gamma}\tan^{-1}(\gamma) + 1\right],$$

$$\tilde{f}_1(\gamma) = \frac{1}{8\gamma^6}\left[\gamma^2 - 7\gamma^2/3 - 20 + \frac{\gamma^6 - 2\gamma^4 + 9\gamma^2 + 20}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{f}_2(\gamma) = \frac{1}{4\gamma^6}\left[10 - \gamma^2/3 + \frac{\gamma^4 - 3\gamma^2 - 10}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{f}_3(\gamma) = \frac{1}{8\gamma^6}\left[5\gamma^4/3 - 7\gamma^4 - 140 + \frac{8\gamma^6}{1 + \gamma^2} + \frac{140 + 45\gamma^2 - 6\gamma^4 + \gamma^6}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{f}_4(\gamma) = \frac{1}{\gamma^6}\left[4\gamma^2/3 + 5 - \frac{5 + 3\gamma^2}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{f}_5(\gamma) = \frac{1}{4\gamma^6}\left[70 - 25\gamma^2/3 + \frac{6\gamma^4}{1 + \gamma^2} + \frac{3\gamma^4 - 15\gamma^2 - 70}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{f}_6(\gamma) = \frac{3}{8\gamma^6}\left[-420 + 35\gamma^2 - 39\gamma^4 + \frac{40\gamma^6}{1 + \gamma^2} + \frac{\gamma^6 - 10\gamma^4 + 105\gamma^2 + 420}{\gamma}\tan^{-1}(\gamma)\right] - \frac{2}{(1 + \gamma^2)^2},$$

$$\tilde{f}_7(\gamma) = \frac{1}{\gamma^6}\left[35 + 10\gamma^2/3 + \frac{2\gamma^4}{1 + \gamma^2} - \frac{15\gamma^2 + 35}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\varphi}_1(\gamma) = \frac{3}{16\gamma^4}\left[3 + \gamma^2 - \frac{3 + 2\gamma^2 - \gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\varphi}_2(\gamma) = \frac{3}{16\gamma^4}\left[15 + \gamma^2 - \frac{15 + 6\gamma^2 - \gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\varphi}_3(\gamma) = \frac{1}{16\gamma^4}\left[105 - 5\gamma^2 + \frac{8\gamma^4}{1 + \gamma^2} - \frac{105 + 30\gamma^2 - 3\gamma^4}{\gamma}\tan^{-1}(\gamma)\right].$$

In the limit $\gamma \to 0$, these functions tend to the following values,

$$f_1 = 2/15, \qquad f_2 = 3/15, \qquad \varphi_1 = 1/6, \qquad \varphi_3 = 1/3, \qquad f_3 = \varphi_2 = 0,$$

$$\tilde{f}_1 = 8/105, \qquad \tilde{f}_2 = 13/105, \qquad \tilde{f}_4 = 4/35, \qquad \tilde{\varphi}_1 = 1/5, \tag{A1}$$

$$\tilde{f}_3 = \tilde{f}_5 = \tilde{f}_6 = \tilde{f}_7 = \tilde{\varphi}_2 = \tilde{\varphi}_3 = 0.$$

In the opposite limit ($\gamma \gg 1$), by retaining the terms of the order $0(\gamma^{-1})$, we find

$$f_1 = f_3 = \varphi_1 = \varphi_2 = \varphi_3 = \pi/8\gamma, \qquad f_2 = 0,$$

$$\tilde{\varphi}_1 = \tilde{\varphi}_2 = \tilde{\varphi}_3 = 3\pi/32\gamma, \qquad \tilde{f}_1 = \tilde{f}_3 = \pi/16\gamma, \qquad \tilde{f}_6 = 3\pi/16\gamma, \tag{A2}$$

$$\tilde{f}_2 = \tilde{f}_4 = \tilde{f}_5 = \tilde{f}_7 = 0.$$

Appendix B

The functions $\psi_n(\gamma)$, $\tilde{\psi}_n(\gamma)$ and $\zeta_n(\gamma)$ used in Section 4 in the expressions for the turbulent stress tensor are:

$$\psi_1(\gamma) = \frac{1}{\gamma^4}\left[5 + \frac{1}{1+\gamma^2} - \frac{6+\gamma^2}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\psi_2(\gamma) = \frac{1}{2\gamma^4}\left[60 - 5\gamma^2 + \frac{6\gamma^4}{1+\gamma^2} - \frac{60+15\gamma^2-\gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\psi}_1(\gamma) = \frac{1}{2\gamma^6}\left[5\gamma^2 - 15 - \frac{2\gamma^4}{1+\gamma^2} + \frac{15-\gamma^4}{\gamma}\tan^{-1}(\gamma)\right] + \frac{P_m}{2\gamma^6}\left[5 + 4\gamma^2/3 - \frac{5+3\gamma^2}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\psi}_2(\gamma) = \frac{1}{4\gamma^6}\left[\gamma^4 - 210 - 5\gamma^2 - \frac{12\gamma^4}{1+\gamma^2} + \frac{210+75\gamma^2-6\gamma^4+\gamma^6}{\gamma}\tan^{-1}(\gamma)\right] +$$
$$+ \frac{P_m}{8\gamma^6}\left[140 + 265\gamma^2/3 + \gamma^4 - \frac{140+135\gamma^2+18\gamma^4-\gamma^6}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\tilde{\psi}_3(\gamma) = \frac{1}{\gamma^6}\left[15 + 4\gamma^2 - \frac{15+9\gamma^2}{\gamma}\tan^{-1}(\gamma)\right] + \frac{P_m}{\gamma^6}\left[\frac{5+6\gamma^2+\gamma^4}{\gamma}\tan^{-1}(\gamma) - 5 - 13\gamma^2/3\right],$$

$$\tilde{\psi}_4(\gamma) = \frac{1}{2\gamma^6}\left[\gamma^4 - 30 - 5\gamma^2 + \frac{30+15\gamma^2-2\gamma^4+\gamma^6}{\gamma}\tan^{-1}(\gamma)\right] +$$
$$+ \frac{P_m}{4\gamma^6}\left[20 + 61\gamma^2/3 + \gamma^4 - \frac{20+27\gamma^2+6\gamma^4-\gamma^6}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\zeta_1(\gamma) = \frac{1}{32\gamma^4}\left[\gamma^2 - 21 - \frac{8\gamma^2}{1+\gamma^2} + \frac{21+14\gamma^2+\gamma^4}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\zeta_3(\gamma) = \frac{1}{32\gamma^4}\left[-105 + 77\gamma^2 + \frac{32\gamma^4}{(1+\gamma^2)^2} - \frac{72\gamma^4}{1+\gamma^2} + \frac{5\gamma^4-42\gamma^2+105}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\zeta_4(\gamma) = \frac{1}{32\gamma^4}\left[-735 + 35\gamma^2 + \frac{120\gamma^2}{1+\gamma^2} - \frac{32\gamma^6}{(1+\gamma^2)^2} + \frac{3\gamma^4+90\gamma^2+735}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\zeta_5(\gamma) = \frac{1}{8\gamma^3}\left[\frac{2\gamma^2}{1+\gamma^2} - 15 + \frac{3\gamma^2+15}{\gamma}\tan^{-1}(\gamma)\right],$$

$$\zeta_6(\gamma) = \frac{1}{8\gamma^3}\left[3 - \frac{2\gamma^2}{1+\gamma^2} + \frac{\gamma^2-3}{\gamma}\tan^{-1}(\gamma)\right], \qquad \zeta_2(\gamma) = \tilde{\varphi}_3(\gamma)/2, \ \ \zeta_7(\gamma) = \gamma f_2(\gamma)/3,$$

where $\tilde{\varphi}_3(\gamma)$ and $f_2(\gamma)$ are given in Appendix A, and $P_m = \nu/\eta$ is the magnetic Prandtl number. The functions given above are equal to the following marginal values when γ equals zero:

$$\psi_1 = 2/15, \qquad \psi_2 = 0, \qquad \tilde{\psi}_4 = 2\tilde{\psi}_1 = 4/21 + 4P_m/35, \qquad \tilde{\psi}_2 = 0,$$
$$\tilde{\psi}_3 = 12/35 + 16P_m/105, \qquad \zeta_1 = 4/15, \tag{B1}$$
$$\zeta_2 = \zeta_3 = \zeta_4 = \zeta_5 = \zeta_6 = \zeta_7 = 0.$$

In the opposite case of large γ, by retaining the terms of the order $0(\gamma^{-1})$, we find

$$\psi_2 = \pi/4\gamma, \qquad \psi_1 = 0, \qquad \tilde{\psi}_4 = 2\tilde{\psi}_2 = \pi(1 + P_m/2)/4\gamma,$$
$$\tilde{\psi}_1 = \tilde{\psi}_3 = 0, \qquad \zeta_2 = \zeta_4 = 3\pi/64\gamma, \qquad \zeta_1 = \pi/64\gamma, \tag{B2}$$
$$\zeta_3 = 5\pi/64\gamma, \qquad \zeta_5 = \zeta_6 = \zeta_7 = 0.$$

References

BAKER, N., and TEMESVARY, S.: 1966, Tables of Convective Stellar Envelope Models, 2nd ed. N.Y.: NASA.

DROBYSHEVSKI, E. M.: 1977, Astrophys. Space Sci. **46**, 41.

DROBYSHEVSKI, E. M., and YUFEREV, V. S.: 1974, J. Fluid Mech. **65**, 33.

HOWARD, R.: 1984, Ann. Rev. Astron. Astrophys. **22**, 131.
HOWARD, R., and LaBONTE, B. J.: 1980, Astrophys. J. **239**, L33.
KICHATINOV, L. L.: 1986a, Geophys. Astrophys. Fluid Dyn. **35**, 93.
KICHATINOV, L. L.: 1986b, Sov. Astron. Lett. **12**, 172.
KICHATINOV, L. L.: 1987, Geophys. Astrophys. Fluid Dyn. **38**, 273.
KLEEORIN, N. I., and RUZMAIKIN, A. A.: 1982, Magnitnaja Gidrodinamika **2**, 17.
KLEEORIN, N. I., and RUZMAIKIN, A. A.: 1984, Astron. Nachr. **305**, 265.
KRAUSE, F., and RÜDIGER, G.: 1974, Astron. Nachr. **295**, 185.
KRAUSE, F., and RÄDLER, K.-H.: 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory, Akademie-Verlag, Berlin.
KRIVODUBSKIJ, V. N.: 1984, Astron. Zh. **61**, 354.
LaBONTE, B. J., and HOWARD, R.: 1982, Solar Phys. **75**, 161.
MOFFATT, H. K.: 1970, J. Fluid Mech. **44**, 705.
MOFFATT, H. K.: 1978, Magnetic Field Generation in Electrically Conducting Fluids, Cambridge Univ. Press, Cambrige.
PARKER, E. N.: 1955, Astrophys. J. **122**, 293.
PARKER, E. N.: 1979, Cosmical Magnetic Fields, Clarendon Press, Oxford.
POUQUET, A., FRISCH, U., and LÉORAT, J.: 1976, J. Fluid Mech. **77**, 321.
RÄDLER, K.-H.: 1968, Z. Naturforsch. **23a**, 1851.
RÄDLER, K.-H.: 1969, Monatsber. dtsch. Acad. Wiss. Berlin **11**, 272.
RÄDLER, K.-H.: 1974, Astron. Nachr. **295**, 265.
ROBERTS, P. H., and SOWARD, A. M.: 1975, Astron. Nachr. **296**, 49.
RÜDIGER, G.: 1974, Astron. Nachr. **295**, 275.
RÜDIGER, G.: 1978, Astron. Nachr. **299**, 217.
RÜDIGER, G.: 1980, Geophys. Astrophys. Fluid Dyn. **16**, 239.
RÜDIGER, G.: 1983, Geophys. Astrophys. Fluid Dyn. **25**, 213.
SCHÜSSLER, M.: 1981, Astron. Astrophys. **94**, L17.
STEENBECK, M., KRAUSE, F., and RÄDLER, K.-H.: 1966, Z. Naturforsch. **21a**, 369.
VAINSHTEIN, S. I.: 1978, Magnitnaja Gidrodinamika **1**, 45.
VAINSHTEIN, S. I., ZELDOVICH, YA. B., and RUZMAIKIN, A. A.: 1980, Turbulent Dynamo in Astrophysics (in Russian), Nauka, Moskow.
YOSHIMURA, H.: 1981, Astrophys. J. **247**, 1102.
ZELDOVICH, YA. B.: 1956, Zh. Ehksp. Teor. Fiz. **31**, 154.

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