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SLOW CHANGES OF SOLAR IRRADIANCE AND ENERGETICS OF ACTIVE REGIONS

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Abstract. A new numerical technique is applied to study long-term variations of total solar irradiance. The background solar flux is estimated not from, e.g., a running mean but as the mode on a moving short time interval. Statistical properties of short-term variations with respect to the running mode are studied. The probability distribution function describing the data from Nimbus-7 is asymmetric and departs from a Gaussian.

The ratio of time-integrated short-term negative and positive deviations shows that the energy re-radiated from faculae makes up about 40% of the energy blocked by sunspots. The amplitude and phase relations are studied between deviations which decrease and increase the irradiance. They characterize the mechanism of energy transformation with frequency. The cross-covariance analysis reveals that some parts of the energy blocked by sunspots come to the surface of the Sun after long delays.

1. Introduction

It is common practice to smooth time series of solar irradiance measurements using traditional techniques without a knowledge of whether these procedures are adequate to the nature of irradiance variability. This approach is supported by a great variety of readily available subroutine packages for data processing. Smoothing is, e.g., carried out by the least-squares fitting of polynomials with increasing degree. Also low-pass filtering is used, given the cut-off frequency that divides a signal into low- and high-frequency components. The simplest linear filtering is carried out by a running average over, e.g., an 81-day interval. After filtering, the high-frequency component represents short-term variations related to the influence of separate active regions. The low-frequency component includes variations with periods of 4–9 months caused by large-scale features of solar activity and a modulation during an 11-yr cycle (Donnelly, 1987; Foukal and Lean, 1988; Willson and Hudson, 1991). It is important to remove a trend and to smooth data correctly, because these are the first steps for further processing.

Smoothing for estimating the background solar flux with respect to which short-term oscillations occur is of fundamental importance for understanding sunspot blocking and the overall energy balance in active regions. The relationship between sunspot blocked energy and that radiated from faculae and the magnetic network has been estimated in previous work (e.g., Schatten *et al.*, 1985; Foukal and Lean, 1988; and references therein). The results differ from each other significantly. This is because of large uncertainties in measured sunspot and facular areas, but it is

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also due to the uncertainty of the average level associated with the background irradiance of the Sun, which changes with time. Irradiance deviations which decrease with respect to the background characterize a deficit of the total irradiance, and those which increase characterize excessive irradiance. Active sunspot groups and faculae are the main factors which control total solar irradiance on short time scales, but there are other factors which contribute to the irradiance variations (Fröhlich and Pap, 1989).

2. Smoothing as the Running Mode Estimation

Time series of solar indices have a non-steady behaviour. The mean value and the dispersion of a running time interval vary over the 11-yr cycle. The total irradiance variations have a specific kind of non-stationarity related to the fact that statistical properties of variations which decrease or increase with respect to an averaged level differ fundamentally. The decreasing irradiance changes are often observed as prolonged dips, whereas similar increasing changes are not observed. This results from the fact that positive and negative excursions are due to different physical mechanisms.

It is well known that the mean and variance are the best estimates of the 'central tendency' and its width, if we are dealing with a random variable described by a gaussian distribution. In the case of irradiance data it is necessary to look for an alternative approach to the problem because positive and negative variations have different statistical properties and the probability distribution function (PDF) is essentially asymmetric. Moreover, departure from the gaussian PDF no longer allows least-squares fitting. Therefore, the running average or least-square polynomial smoothing are not adequate methods to find the 'central tendency'.

Other descriptions of the 'central tendency' are the median or the mode. A median characterizes the level with respect to which positive and negative deviations are equal in probability. Robust statistics often use a median as an estimate of a mean value (Huber, 1981) because of its resistance to outliers, but still it relies on a symmetric PDF.

The mode of a PDF is the value where it becomes a maximum. Thus the mode describes the central state for any PDF by the condition that within some time interval the undisturbed state is occupied most frequently. The condition of the existence of such a time interval may be regarded as an analogy to the quasi-stationarity. This assumption is reasonable for a wide class of time series and holds at least for those which are useful to study long-term changes. An important property of a median or mode estimation is robustness.

The smoothing procedure can be carried out as filtering if one estimates the running mode and ascribes the value obtained to the middle of the interval:

$$S_i^{\text{mode}} = \text{mode}\{S_{i-k},\ S_{i-k+1},\ S_{i-k+2},\ \ldots,\ S_{i+k}\}\ , \tag{1}$$

where S_i, $i = 1, 2, \ldots, N$ are measured values of irradiance with a sampling interval Δt, and S_i^{mode} is an instantaneous mode. Clearly, this procedure represents nonlinear filtering, because it involves nonlinear operations of comparing and choosing the most frequent values. The time interval is then shifted by one sampling step, and all the computations are repeated.

The length of a running time interval for which the mode is estimated, $R = 2k\Delta t$, must be longer than the minimum time period during which the undisturbed state dominates. For studying long-term variations of solar indices, the duration of the interval should be at least greater than the solar rotation period. It is difficult to formulate an exact quantitative criterion for the selection of R. R can be regarded as the period corresponding to the cut-off frequency in low-pass filtering, and the result includes only spectral components with periods longer than R. Thus, R could be estimated under the condition that it exceeds the duration of the longest short-term fluctuation. From the irradiance curve it is evident that dips related to large complexes of activity have a duration of about 2–3 months, and to smooth out these dips, R should be about 3 months or 3 solar rotations. With $R = 81$ d the smoothed component includes variations with periods 4–9 months and longer.

For the calculation of the mode the maximum of PDF is estimated based upon a simplified algorithm. Within the interval R, values of S_i, are ordered. The bin with the maximum density is then found where a certain percentage of measurements is concentrated. The amount of 5–15% of S_i within this bin provides reasonable estimates for the existing noise level. The width of this bin characterizes the error bar of the mode estimation at the chosen significance level. There are sometimes outliers in the mode estimate because of noisy data, and because of imperfections of the algorithm. Therefore it is reasonable to smooth the mode using a low-pass filter. Low-pass filtering should be consistent with the procedure of mode estimation. Thus the same R should be used.

The solar irradiance measurements from Nimbus-7 (processed according to the algorithm by Hoyt *et al.* (1992)) are presented in Figure 1(a). The dotted line shows the running mode estimated with the method described. The smoothed mode runs, as a rule, on a higher level with respect to a running mean and the difference is especially large during long dips due to large activity complexes. Physically, this means that the undisturbed state is associated with the background, enhanced due to a bright component related largely to the magnetic network. The difference between observed irradiance and the mode is plotted in Figure 1(b). The behaviour of the negative part of this graph resembles the PSI function (Willson *et al.*, 1981), but the amplitude is greater due to the contribution of a persistent bright component.

3. Statistical Properties of Variations with Respect to the Mode

The PDF of irradiance variations with respect to the mode $PDF(\Delta S)$ with $\Delta S = S - S^{\mathrm{mode}}$ and the logarithm $\ln PDF(\Delta S)$ are plotted in Figure 2 (at the top

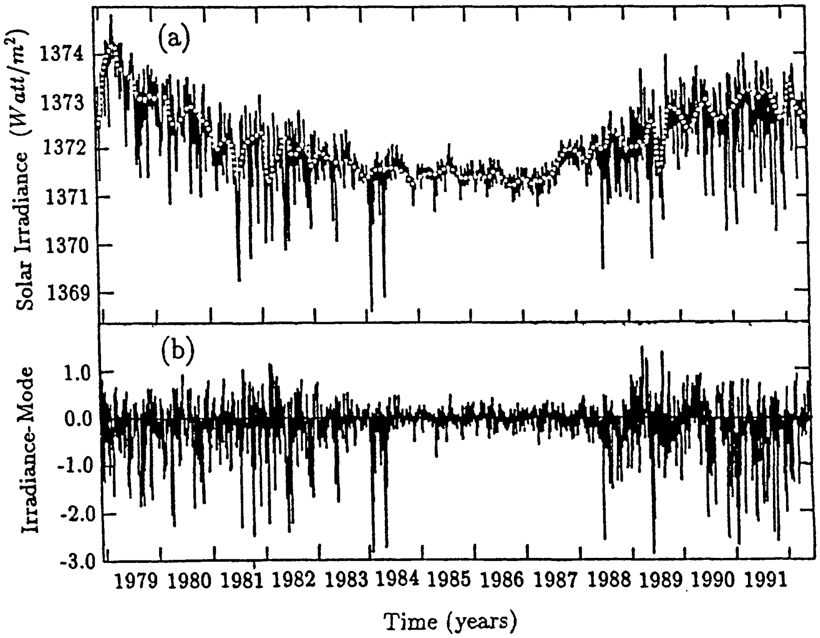

Fig. 1. Time series of ERB total solar irradiance measurements. The dotted line represents the smoothed running mode estimation (a); and the irradiance variations with respect to the mode (b).

and bottom, respectively). The PDF peaks sharply (kurtosis = 4.06), therefore the mode is estimated, as a rule, with high confidence at all times. The error in estimating the mode determined as the difference of quantiles containing 10% of measurements within R around the maximum of PDF rarely exceeds 0.1 W m^{-2} with a typical value of 0.06 W m^{-2}. The PDF which describes irradiance variations is definitively asymmetric (skewness $= -1.53$). The character of the $PDF(\Delta S)$ is quite different for decreasing and increasing changes, and this is clearly seen in $\ln PDF(\Delta S)$ plot. A least-squares fit approximates the curves by Equations (2), which are shown by the dashed lines in Figure 2. Obviously the $PDF(\Delta S)$ significantly departs from a gaussian and is approximated by

$$PDF(\Delta S) = \begin{cases} \exp(\exp(0.68731\Delta S + 1.69283) - 5)\,, & \Delta S < 0\,, \\ \exp(-4.03638\Delta S + 0.51018\,, & \Delta S > 0\,. \end{cases} \tag{2}$$

Therefore, a study of these data using classical statistics is not correct and possibly misleading. The contributions of decreasing and increasing changes to the dispersion are approximately 85% and 15%, respectively. This large difference results from the dips caused by active sunspot groups. These findings do not yet characterize the energetics of the processes. The energy blocked by sunspots and that radiated from faculae can only be estimated as time integrated deviations from the mode. The ratio of integrated positive deviations to the integrated negative ones is about 0.41. Based on this result it may be supposed that the energy radiated from faculae is about 40% of the energy blocked by sunspots. It should be noted that this estimate is only for short-term variations. This estimate does not differ much

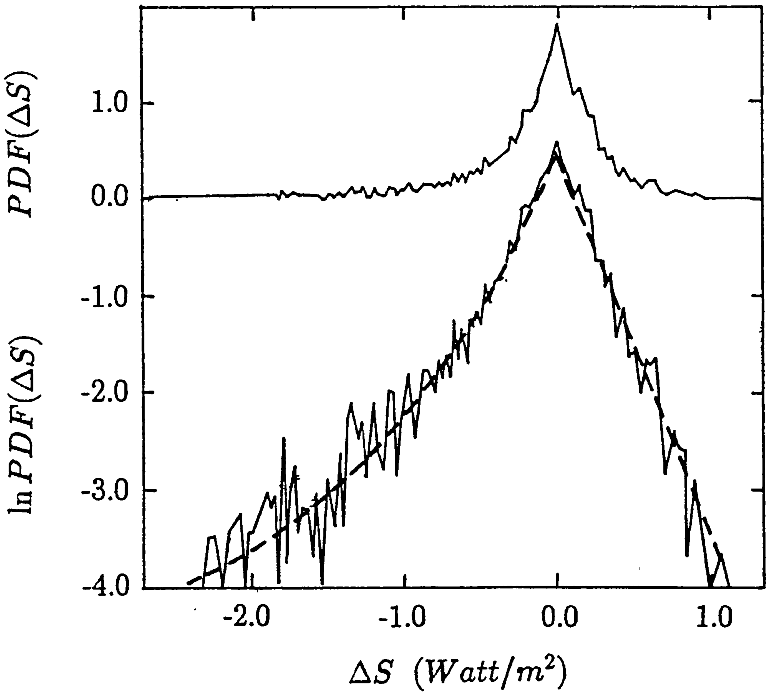

Fig. 2. The probability distribution function of deviations with respect to the mode (*top*) and its logarithm versus ΔS. The dashed lines represent the fits according to Equations (2).

from that made by Chiang and Foukal (1985) which is about 30%. More or less it is consistent with direct photometric measurements of Chapman *et al.* (1984) that show that the facular excess is approximately 57% of the spot deficit, and the finding of Fröhlich (1993) where the share between PSI and $F_{10.7}$ in explaining the solar irradiance variability amounts to 36% for the latter. But this is not consistent with the result of numerical simulations carried out by Fox, Sofia, and Chan (1991), according to which a convective boundary flow that arises around a sunspot flux tube transports to the photosphere most of the energy blocked by a magnetic field shortly after the adjustment.

4. Spectral Relationships and Cross-Covariance of the Deviations

The running mode describes undisturbed irradiance, with respect to which higher-frequency variations occur, and again decreasing deviations from the mode characterize the energy blocked by sunspots, that can be radiated from bright features. In fact, the separation of variations into positive and negative ones is a matter of convention. We deal with only one quantity that describes irradiance changes and is determined by a joint action of sunspots and faculae which could partially compensate each other. Negative deviations occur when the response to sunspots predominates over the facular excess, and positive deviations occur in the opposite case.

Both the negative and positive deviations are related to active regions which are characterized by sunspots and faculae. Let the stored energy blocked by sunspots be input to a physical linear system that after some transformation carries it to the surface of the Sun. So, negative variations are the input $x(t)$, and positive ones $y(t)$ the output of the system. Let us make the crude assumption that when $x(t) < 0$, $y(t) = 0$, and when $y(t) > 0$, let $x(t) = 0$ for the estimation of an inherent transfer function, which may give us a useful model for the energy balance in an active region. With the above assumptions, we can calculate the spectral densities for positive and negative excursions and their cross spectrum. From these the modulus of the transfer function $|H_{xy}(f)|$ (gain) and phase $\varphi_{xy}(f)$ can be calculated (Bendat and Piersol, 1986):

$$|H_{xy}(f)| = \sqrt{G_{xy}(f)/G_{xx}(f)}\,, \tag{3}$$

$$\varphi_{xy} = \mathrm{arctg}[-\mathrm{Im}(G_{xy}(f))/\mathrm{Re}(G_{xy}(f))]\,, \tag{4}$$

where $G_{xx}(f)$, $G_{xy}(f)$ are the spectral densities of the input process and the cross-spectrum between x and y.

The coherence squared, γ^2, modulus, and phase of the transfer function are shown in Figure 3 versus frequency. The coherence squared exceeds 0.5 at frequencies corresponding to periods of 50.7 and 4 days. The latter peak is an artifact related to the operation of Nimbus-7 during the first few years when it was 3 days on 1 day off (Hoyt *et al.*, 1992). There are small maxima of γ^2 and $|H_{xy}|$ at periods $P_i \approx P_c/i$, $i = 1, 2, \ldots, 6$, where P_c is the solar rotation period. Peaks at 9 d and shorter seem to be the result of a rotational modulation of irradiance due to the contribution of solar rotation and brightness inhomogeneities (Mordvinov, 1995). The gain has similar behaviour. The errors of estimating γ^2, $|H_{xy}|$, φ_{xy} when $\gamma^2 = 0.25$ at a 95% confidence level are rather large: $\epsilon[\gamma^2] = 0.35$, $\epsilon[|H|] = 0.20$, $\epsilon[\varphi] = 12°$ (effective degree of freedom = 74). The fact that the coherence takes small values may suggest besides a low signal-to-noise ratio also a nonlinear relation between the variations. If a nonlinear relation exists between the variations, an exchange of energy between the harmonics becomes possible. In particular fast changes can be transformed into slow changes.

Cross-covariance analysis offers a clearer possibility of studying the energy propagation (Bendat and Piersol, 1980). The cross covariance function $R_{xy}(\tau)$ with lag τ for deviations from the mode $|x(t)|$, $y(t)$ is displayed in Figure 4. The absolute value of negative variations, $x(t)$, characterizes the energy stored by sunspots, and positive variations, $y(t)$, represent the release of the energy no matter what physical mechanisms are responsible for the effects. The normalized mean-square error of estimating the cross-covariance is about 14%, when cross-correlation $\rho_{xy} = 0.1$. There are peaks which are difficult to relate to known characteristic time scales of the energy propagation from the convective zone to the photosphere (Foukal and Vernazza, 1979). The fact that the higher maxima of

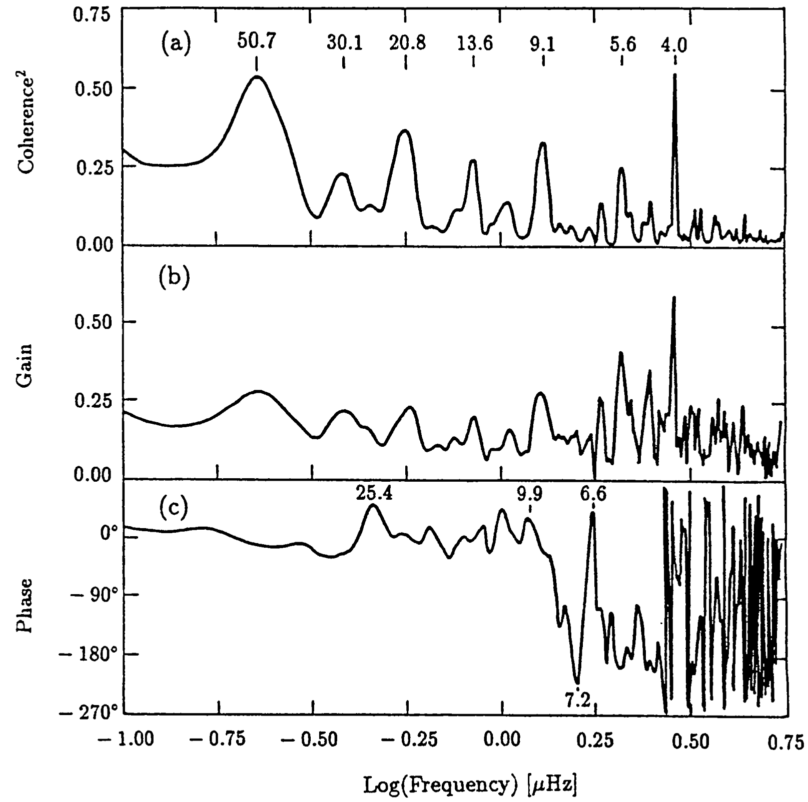

Fig. 3. The coherence squared (a); gain (b); phase (c) versus logarithm frequency to base 10, with frequency measured in μHz. The dominant periods are indicated on the top of each plot.

the cross-covariance function are found at positive lags might be interpreted as a manifestation of a causal relation between the two types of deviations and the delays occurring for the energy blocked by sunspots to get to the surface. It is possible that the peaks of the cross-covariance function at lags of 15, 23, 32, 49, 59, 77 d represent unknown time scales or characterize 'channels' along which the energy propagates outside, after some transformations. There are theoretical reasons to expect long storage times predicted by thermal models of the heat blocked by spots (Chiang and Foukal, 1985).

5. Summary and Discussion

Robust nonlinear filtering has been applied in the study of slow changes of solar irradiance. The background irradiance is defined as that which corresponds to the most probable state of the system and is estimated as the running mode of the time series. Statistical analysis of short-term deviations from the mode show a probability distribution function of fluctuations which is asymmetric and departs significantly from a gaussian.

The spectral relationships between decreasing and increasing changes are studied as the input and output of a linear physical system. The modulus and phase of the

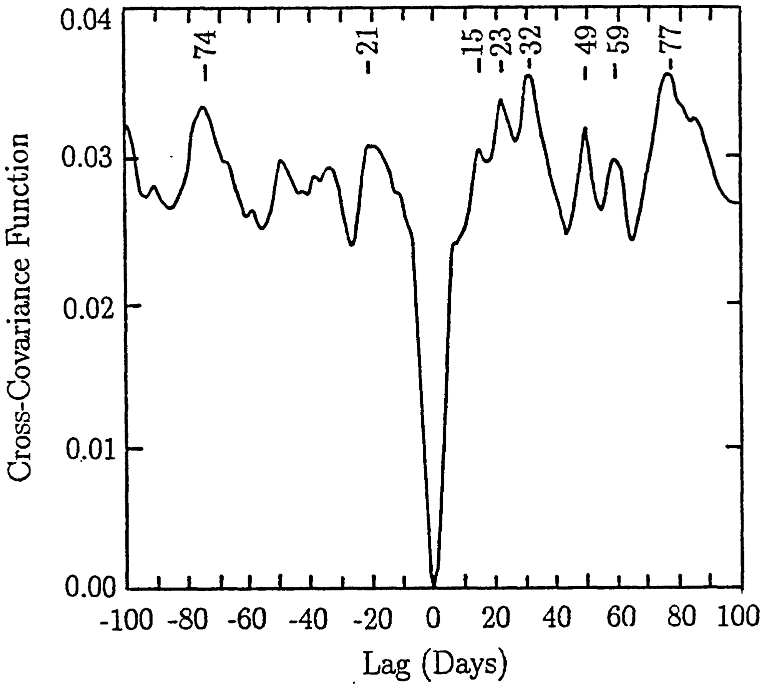

Fig. 4. Cross-covariance function for decreasing and increasing irradiance deviations.

transfer function are estimated, which characterize the mechanism of stored energy transformation to irradiance radiated from faculae. The gain and phase exhibit a complicated behaviour which is hard to explain in detail. Amplitudes of all spectral harmonics are decreased after the conversion. The fact that squared coherence takes small values, seems to suggest a nonlinear relation between variations of the decreasing or increasing type.

At short time scales, the ratio of time integrated energy radiated from faculae to that blocked by sunspots is estimated to be about 0.4 or 40%. The remainder of the energy could be transformed into other kinds of energy and/or contribute to long-term irradiance variations. The fact that not all the stored energy is radiated from long-lived faculae implies that there is energy passing to much larger time scales. The heat dissipated from faculae and the magnetic network contributes to slow irradiance changes forming the energy passing to even longer time scales. If high-frequency irradiance variations (and the stored energy) partially transforms into slow changes, this could explain why solar luminosity is greatest at the epoch of the maximum: the highest magnetic activity increases background irradiance, which manifests itself as an enhanced radiation of the magnetic network (Foukal and Lean, 1988). It is generally believed that the excessive irradiance is radiated from faculae and the magnetic network, but the results do not contradict the concept about cyclic latitudinal temperature variations (Kuhn, Libbrecht, and Dicke, 1988), which may contribute to slow irradiance changes and do not influence short-term variations very much.

The cross-covariance analysis shows time recurrences which might be interpreted as long-term delays in the release of the energy blocked by sunspots, which propagates along different 'channels'. In particular, a time shift of about 49 d approximately characterizes the delay in facular activity with respect to sunspots (Allen, 1973). There is observational evidence that solar luminosity modulation is delayed with respect to magnetic indices by 40–60 days (Yoshimura, 1994).

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References

Allen, C. W.: 1973, *Astrophysical Quantities*, Athlone, London.

Bendat, J. S. and Piersol, A. G.: 1980, *Engineering Applications of Correlation and Spectral Analysis*, Wiley, New-York.

Bendat, J. S. and Piersol, A. G.: 1986, *Random Data. Analysis and Measurement Procedures*, Wiley, New York.

Chapman, G. A., Herzog, A. D., Lawrence, J. K., and Shelton, J. C.: 1984, *Astrophys. J.* **282**, L99.

Chiang, W. E. and Foukal, P. V.: 1985, *Solar Phys.* **97**, 9.

Donnelly, R. F.: 1987, *Solar Phys.* **109**, 37.

Foukal, P. and Vernazza, J.: 1979, *Astrophys. J.* **234**, 707.

Foukal, P. and Lean, J.: 1988, *Astrophys. J.* **328**, 347.

Fox, P. A., Sofia, S., and Chan, K. L.: 1991, *Solar Phys.* **135**, 15.

Fröhlich, C.: 1993, *Interaction between Global Climate Subsystems. The Legacy of Hann*, Geoph. Monographs 75, IUGG 15, p. 123.

Fröhlich, C. and Pap, J.: 1989, *Astron. Astrophys.* **220**, 272.

Härdle, W.: 1989, *Applied Nonparametric Regression*, Cambridge University Press, Cambridge.

Hoyt, D. V., Kyle, H. L., Hickey, J. R., and Maschhoff, R. H.: 1992, *J. Geophys. Res.* **97**, 51.

Huber, P. J.: 1981, *Robust Statistics*, Wiley, New York.

Kuhn, J., Libbrecht, K., and Dicke, R.: 1988, *Science* **242**, 908.

Mordvinov, A. V.: 1995, *Astron. Astrophys.* **293**, 572.

Schatten, K. H., Miller, N., Sofia, S., Endal, A. S., Chapman, G., and Hickey, J.: 1985, *Astrophys. J.* **294**, 689.

Willson, R. C. and Hudson, H. S.: 1991, *Nature* **351**, 42.

Willson, R. C., Gukis, S., Janssen, M., Hudson, H. S., and Chapman, G. A.: 1981, *Science* **211**, 700.

Yoshimura, H.: 1994, *Astron. Nachr.* **315**, 189.