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- A transformation is observed of a poloidal Alfvén wave into a toroidal wave
- A theoretical interpretation is presented of the observed oscillations

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Experimental evidence for the existence of monochromatic transverse small-scale standing Alfvén waves with spatially dependent polarization

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Abstract A transformation of a monochromatic standing poloidal Alfvén wave into a toroidal wave has been observed. A theoretical interpretation is presented of Alfvén-type monochromatic oscillations observed by the Radiation Belt Storm Probes (RBSP)-A satellite on 23 October 2012 when crossing the plasmapause at 21.45–22.30 UT. It is shown that the process involves a poloidal Alfvén wave transforming into a toroidal Alfvén wave. This transformation can serve as proof that the registered oscillations are a single azimuthally small-scale Alfvén mode. As follows from the results of previous theoretical studies, such Alfvén oscillations are generated on the poloidal resonance shell as poloidal standing Alfvén waves. Next, this wave propagates to the toroidal resonance surface across magnetic shells and transforms into a toroidal standing Alfvén wave. The oscillations are absorbed completely near the toroidal resonance surface due to dissipation of their energy in the ionospheric conductive layer. It is shown that the RBSP-A satellite probably crossed two transformation areas of Alfvén waves, located near the plasmapause transition layer.

1. Introduction

Ultralow-frequency (ULF) waves in the terrestrial magnetosphere can be classified according to their azimuthal wave numbers, m. The geomagnetic field is close to a dipole in the inner part of the magnetosphere. If we neglect the small azimuthal inhomogeneity of the magnetospheric plasma, we can use an axially symmetric model of the medium to describe MHD oscillations in the inner magnetosphere. Any disturbance in such a model can be represented as an expansion in azimuthal harmonics of the form $\exp(im\phi)$, where ϕ is the azimuthal angle and $m = 0, 1, 2, \dots$ is the azimuthal wave number. Low-m waves are supposed to be generated by processes on the magnetospheric boundary or to arrive directly from the solar wind [*Archer et al.*, 2013; *Mazur and Chuiko*, 2013; *Potapov*, 2013; *Hartinger et al.*, 2013].

Such an external source is ineffective for high-m waves, because the magnetosphere is opaque for fast magnetosonic (FMS) waves which can excite the Alfvén waves at the resonance magnetic shells. Therefore, high-m Alfvén waves deep in the magnetosphere can only be excited by sources located at the same magnetic shells where these waves can propagate. Currents in the ionosphere are proposed as such a source in *Leonovich and Mazur* [1993] and magnetospheric nonsteady currents produced by high-energy particles drifting in the magnetosphere in *Pilipenko et al.* [2001], *Mager and Klimushkin* [2008], and *James et al.* [2013]. If the conditions exist for a high-m Alfvén wave instability, the wave amplitude grows in their propagation area in the magnetosphere. These conditions may be due to developing MHD instabilities [*Cheng and Lui*, 1998; *Cheremnykh and Parnowski*, 2004; *Leonovich and Kozlov*, 2014] or to nonequilibrium populations of high-energy charged particles [*Southwood*, 1980; *Chen and Hasegawa*, 1991; *Klimushkin and Mager*, 2011; *James et al.*, 2013; *Liu et al.*, 2013].

Theoretical studies of MHD waves in the magnetosphere first started with *Dungey* [1954], where equations were obtained describing the longitudinal structure and spectrum of standing Alfvén waves with azimuthal wave number $m = 0$, which were called “toroidal” Alfvén waves. Magnetic field lines in these waves oscillate in the azimuthal direction. Later, *Dungey* [1967] and *Radoski* [1967] discussed the opposite limiting case $m \to \infty$. Alfvén waves of this type were called “poloidal.” Magnetic field lines in these waves oscillate in the magnetic meridian plane. The frequencies of poloidal and toroidal waves are slightly different (the so-called “polarization splitting” of the spectrum).

As was shown later, toroidal Alfvén waves with $m \sim 1$ can be excited by large-scale fast magnetosonic (FMS) waves propagating in the magnetosphere, on the toroidal resonance shell, where the local frequency of toroidal Alfvén waves coincides with the frequency of the FMS wave source [*Tamao*, 1965; *Radoski*, 1974; *Glassmeier et al.*, 1999]. The transverse structure of Alfvén waves with $m \sim 1$ across magnetic shells has a characteristic resonance peak, localized near the toroidal resonance shell. Initially, this result was obtained in a 1-D inhomogeneous model of the magnetosphere ("box model," considered in *Southwood* [1974] and *Chen and Hasegawa* [1974]) and then it was shown that the same structure also holds for 2-D inhomogeneous dipole models [*Lifshits and Fedorov*, 1986; *Leonovich and Mazur*, 1989; *Chen and Cowley*, 1989]. These oscillations have the form of standing waves in the geomagnetic field line direction between the magneto-conjugated ionospheres. The toroidal polarization of the resonant standing Alfvén waves with $m \sim 1$ remains the same in all their location regions, as confirmed by ground-based magnetometer network observations [*Samson et al.*, 1996].

The transverse structure of azimuthally small-scale Alfvén waves with $m \gg 1$ was first theoretically studied by *Leonovich and Mazur* [1993]. It was shown that a monochromatic source generates these oscillations as a poloidal standing Alfvén wave on the poloidal magnetic resonance shell, where the local frequency of the poloidal Alfvén wave coincides with the source frequency. Unlike toroidal waves, poloidal Alfvén waves do not retain their polarization in a monotonically inhomogeneous plasma. They slowly (with a much smaller velocity than the Alfvén speed) propagate across magnetic shells to the toroidal resonance surface where they are completely absorbed by plasma ions in the ionospheric conductive layer. The poloidal wave is transformed into a toroidal wave in the process of its propagation. In the direction of magnetic field lines, the wave remains a standing wave. In [*Klimushkin et al.*, 1995] these results were implemented in a 3-D inhomogeneous model of the magnetosphere, and in *Klimushkin* [2000, 2007] and *Klimushkin and Mager* [2011] they were adapted for Alfvén waves interacting with high-energy charged particles. In *Mager and Klimushkin* [2002], *Klimushkin et al.* [2004], and *Kozlov et al.* [2006] these waves were studied in a magnetospheric model with hot plasma, and in *Klimushkin and Mager* [2004], in a model with magnetic field line shear.

However, a transformation of Alfvén waves involving a change in their polarization has not been examined in observations until now. The reason for this may be as follows. If the absorption in the ionosphere is strong enough, poloidal Alfvén waves cannot reach the toroidal resonance surface and are absorbed by ionospheric plasma at a stage when they are not yet transformed into toroidal waves [*Leonovich and Mazur*, 1997]. A second possibility for the existence of poloidal Alfvén waves is magnetospheric resonators existing in areas with extrema in poloidal eigenfrequency distribution across magnetic shells [*Leonovich and Mazur*, 1995; *Klimushkin et al.*, 2004]. A resonator does not allow a poloidal wave to travel across magnetic shells, thus keeping its poloidal polarization throughout the wave localization region. Such resonators extract oscillations with frequencies corresponding to their eigenfrequencies from broadband-source spectra. Alfvén resonators may appear near the plasmapause [*Leonovich and Mazur*, 1995; *Mager and Klimushkin*, 2013] and in the region of the ring current intensity maximum [*Vetoulis and Chen*, 1994; *Yeoman et al.*, 2012].

The sources of monochromatic Alfvén waves are difficult to find in the real magnetosphere. Typically, these sources are broadband, so that observable oscillations represent a superposition of waves excited at different resonance shells [*Leonovich and Mazur*, 1998a, 1998b; *Klimushkin et al.*, 2012; *Dmitrienko*, 2013]. However, observations of almost monochromatic oscillations of the Alfvén type were presented recently by *Dai et al.* [2013], where a transformation is clearly seen of a poloidal Alfvén wave into a toroidal wave. That paper only presents part of the full record of such oscillations, but it is evident even from that part that these oscillations are significantly different from poloidal Alfvén waves usually recorded in observations. Their most interesting feature is a gradual — over many periods — transformation of monochromatic poloidal Alfvén waves into toroidal waves.

In particular, it involves a gradual transformation of the phase shift between the various oscillation components of the wave field. Moreover, a decrease is observed in the amplitudes of the field components corresponding to the poloidal Alfvén wave as well as an increase in the amplitudes of the components corresponding to the toroidal Alfvén wave. The registered oscillations did not exhibit any abrupt phase change throughout the entire record, suggesting that a single Alfvén mode was observed.

This paper presents a complete record of this event as well as their detailed theoretical interpretation. It has the following structure. Section 2 presents the properties of azimuthally small-scale Alfvén waves from previous theoretical studies. Section 3 presents the full record of the oscillations registered by the RBSP-A satellite on 23 October 2012 and their theoretical interpretation as monochromatic azimuthally small-scale Alfvén waves

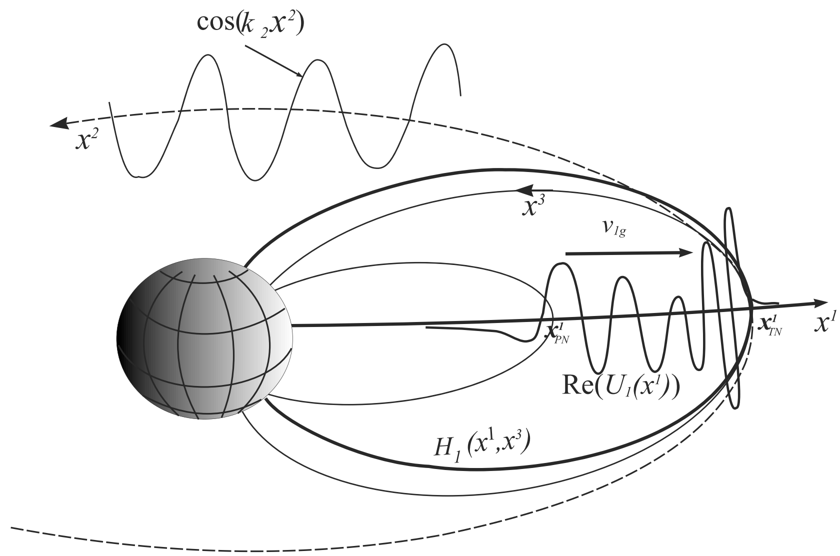

Figure 1. Schematic structure of the fundamental harmonic of standing azimuthally small-scale Alfvén waves and the curvilinear coordinate system (x^1, x^2, x^3) tied to geomagnetic field lines. Here x^1_{PN} and x^1_{TN} are the coordinates of the poloidal and toroidal resonance surfaces.

undergoing a full-phase transformation from poloidal to toroidal Alfvén waves. Oscillation features that are due to satellites crossing the plasmapause are discussed. The main results are summarized in section 4.

2. The Structure of Azimuthally Small-Scale Alfvén Waves

We describe the structure of monochromatic azimuthally small-scale Alfvén waves using a curvilinear orthogonal coordinate system (x^1, x^2, x^3) , in which the x^3 coordinate is directed along the magnetic field lines, x^1 across the magnetic shells and x^2 in the azimuthal direction (see Figure 1). We consider Alfvén waves with $m \gg 1$ as azimuthally small-scale oscillations.

Along magnetic field lines, their structure has the form of standing waves between the magneto-conjugated ionospheres, and their harmonics are numbered by the longitudinal wave number, $N = 1, 2, 3, \ldots$. The complete spatial structure of azimuthally small-scale Alfvén waves excited by a monochromatic source is theoretically investigated in a dipole model of the magnetosphere with a "cold plasma" in *Leonovich and Mazur* [1993] and in models with a "hot plasma" in *Mager and Klimushkin* [2002], *Klimushkin et al.* [2004], and *Kozlov et al.* [2006]. Qualitatively, this structure, based on the theoretical model designed in *Leonovich and Mazur* [1993, 1997], is shown in Figure 1 for the magnetic field components of the fundamental harmonic ($N = 1$).

Across the magnetic shells these modes represent a small-scale wave traveling from the poloidal to the toroidal resonance surface. Such oscillations are generated as a poloidal standing Alfvén wave near the poloidal resonance surface. The source of these oscillations might be, for example, external currents in the ionosphere [*Leonovich and Mazur*, 1996] or magnetosphere [*Mager and Klimushkin*, 2008], whose spectrum contains harmonics with frequencies corresponding to the eigenfrequencies of poloidal standing Alfvén waves on the magnetic shell in question. This monochromatic wave travels to the toroidal resonance surface where it is absorbed completely due to dissipation of its energy in the ionosphere. In the process its polarization changes from poloidal to toroidal. In presence of the kinetic or MHD instabilities, the wave amplitude grows in the course of the transverse propagation. Nevertheless, the wave is attenuated near the toroidal surface [*Klimushkin*, 2000; *Leonovich and Kozlov*, 2014]. Note that in this propagation process the oscillations keep their large-scale structure along the magnetic field lines as a standing wave. In the directions transverse to the field lines (in the azimuthal direction and across the magnetic shells), they are running waves.

The distance between the poloidal and toroidal resonance surfaces strongly depends on the temperature of the background plasma, more precisely on parameter $\beta = 8\pi P_0/B_0^2$, where P_0 is kinetic plasma pressure and B_0 is the magnetic field strength. In the plasmasphere, where $\beta \ll 1$ and the geomagnetic field can be assumed to be almost dipole, Alfvén waves can be described using MHD equations obtained in the "cold plasma" approximation. In the outer magnetosphere, where $\beta \sim 1$, the geomagnetic field is different from a dipole, and equations derived in the "warm plasma" approximation have to be used to describe the oscillations, taking into account the effects associated with finite gas-kinetic plasma pressure.

The distance between the poloidal and toroidal resonance surfaces calculated for the fundamental harmonic of the standing waves ($N = 1$) is much larger than for the higher harmonics of standing waves in "cold plasma" models [see *Leonovich and Mazur*, 1993]. For poloidal Alfvén waves actually observed in the magnetosphere ($m < 300$), the above structure of field oscillations, where numerous wavelengths in the x^1 coordinate fit in between the poloidal and toroidal resonance surfaces, can be expected for the fundamental harmonic only. Accordingly, it is only for this harmonic that we can expect a transformation of a poloidal Alfvén wave into a toroidal wave. In models with a "warm plasma" the calculated distance between the resonance surfaces is large enough for all the main harmonics of standing Alfvén waves which is why such a structure can be observed for all of them [*Mager and Klimushkin*, 2002; *Kozlov et al.*, 2006; *Klimushkin et al.*, 2004].

Let us consider the structure of azimuthally small-scale Alfvén waves based on the findings in *Leonovich and Mazur* [1993]. All field components of such oscillations can be expressed in terms of the electric field scalar potential, φ. In the ideal MHD approximation $E_\parallel = 0$ for all MHD waves. Thus, the wave's electric field is two-dimensional, $\mathbf{E} = (\mathbf{E}_\perp, 0)$, where the subscript $_\perp$ means projection onto the direction perpendicular to the background magnetic field $\mathbf{B_0}$. Like any two-dimensional vector field, the transverse electric field can be decomposed into the sum of the 2-D curl-free and vortical components, where the former can be expressed in terms of the two-dimensional gradient of scalar potential φ, and the latter in terms of the two-dimensional curl of vector potential $\mathbf{\Psi}$:

$$\mathbf{E}_\perp = -\nabla_\perp \varphi + [\nabla_\perp, \mathbf{\Psi}].$$

The first and the second terms in this expression describe the Alfvén and the magnetosonic wavefields, respectively [*Klimushkin*, 1994]. Further we will consider the Alfvén oscillations described by the scalar potential φ only.

The relation of the electric $\mathbf{E}$ and magnetic field $\mathbf{B}$ of the oscillation is given by

$$\mathrm{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}.$$

Hence, for monochromatic waves of the form $\exp(ik_2x^2 - i\omega t)$, where ω is the oscillation frequency and k_2 is the azimuthal wave number (if azimuthal angle ϕ is used as the x^2 coordinate, then $m = 0, 1, 2, 3, \dots$ is the azimuthal wave number), we obtain the following relations between the electromagnetic field components

$$\begin{aligned} E_1 &= -\nabla_1\varphi, \qquad & B_1 &= \frac{ck_2}{\omega}\frac{g_1}{\sqrt{g}}\nabla_3\varphi, \\ E_2 &= -ik_2\varphi, \qquad & B_2 &= i\frac{c}{\omega}\frac{g_2}{\sqrt{g}}\nabla_3\nabla_1\varphi, \\ E_3 &= 0, \qquad & B_3 &= \frac{c}{\omega}\frac{g_3}{\sqrt{g}}\left(\nabla_2E_1 - \nabla_1E_2\right). \end{aligned} \tag{1}$$

where $\nabla_i = \partial/\partial x^i$ ($i = 1, 2, 3$), $g = \sqrt{g_1g_2g_3}$, and g_1, g_2, g_3 are the metric tensor components. In this approximation, the wave's parallel magnetic field is absent, $B_3 = 0$. However, when small corrections in the MHD equations due to the field line curvature and plasma pressure are taken into account, a parallel magnetic field of high-m Alfvén waves emerges. However, it is very sensitive to the pressure gradient of the background plasma and magnetic field geometry [*Leonovich and Mazur*, 1993; *Mager and Klimushkin*, 2002].

Since the wavelength of these oscillations along and across magnetic field lines are very different, we can use the multiple-scale method to describe their structure. As a result, the structure of an individual oscillation harmonic can be presented as

$$\varphi_N(x^1, x^2, x^3) = U_N(x^1)H_N(x^1, x^3)\exp(ik_2x^2 - i\omega t), \tag{2}$$

where $N = 1, 2, 3, \dots$ is the harmonic number of a large-scale standing wave. Function $U_N(x^1)$ in equation (2) describes the oscillation small-scale structure across magnetic shells and $H_N(x^1, x^3)$ the structure of standing Alfvén waves along magnetic field lines.

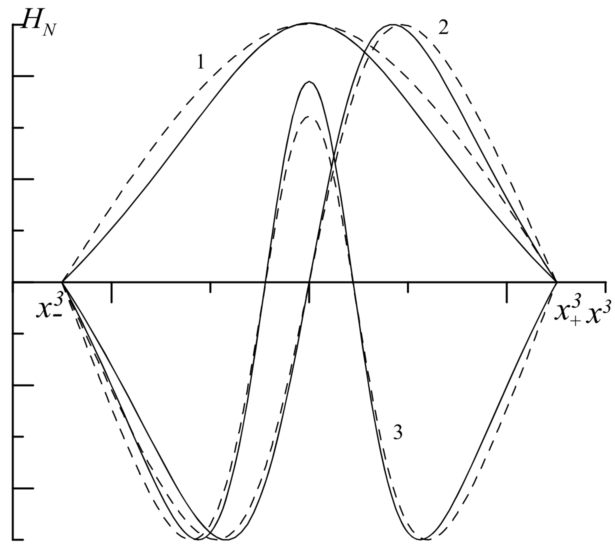

Figure 2. The structure of the first three harmonics of standing poloidal (solid lines) and toroidal (dashed lines) Alfvén waves along geomagnetic field lines.

Let us note an important property of the field structure of these waves following from equation (2). The field structure of a standing wave along magnetic field lines is described by the $H_N(x^1, x^3)$ function, where the dependence on x^1 and x^3 is determined by the background plasma distribution on the magnetic shell under study. Oscillations travelling along the azimuthal x^2 coordinate in the axisymmetric magnetosphere model in question do not change the wave field structure along the magnetic field lines. When waves propagate across magnetic shells, a significant change in their structure can occur only when the plasma distribution along the magnetic field lines changes significantly.

An exact calculation of the $U_N(x^1)$ and $H_N(x^1, x^3)$ functions is rather complicated and can only be carried out numerically. For a qualitative understanding of the structure of azimuthally small-scale Alfvén waves it is sufficient to consider these functions in the WKB approximation in a "cold plasma" model. For the $H_N(x^1, x^3)$ function, describing the structure of the oscillations along magnetic field lines, we have, in the WKB approximation,

$$H_N(x^1, x^3) \sim \sin\left(\int_{x^3_-}^{x^3} k_3(x^1, x^{3\prime})\mathrm{d}x^{3\prime}\right),$$

for the x^3 coordinate. Here $k_3 = \omega\sqrt{g_3(x^1, x^3)}/v_A(x^1, x^3)$ is the wave vector component along the longitudinal coordinate, $v_A(x^1, x^3) = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén speed, and x^3_- is the coordinate of the field line intersection with the ionosphere in the Southern Hemisphere. The structures of the poloidal and toroidal Alfvén waves are identical in this approximation. The exact numerical solutions of MHD equations describing the structure of the poloidal and toroidal waves are shown in Figure 2. It can be seen that the differences are minimal.

For the $U_N(x^1)$ function describing the oscillation structure across magnetic shells, we can use the following representation:

$$U_N(x^1) \sim \begin{cases} \exp\left(i\int_{x^1_{\mathrm{PN}}}^{x^1} k_1(x^{1\prime})\mathrm{d}x^{1\prime}\right), & x^1_{\mathrm{PN}} < x^1 < x^1_{\mathrm{TN}}, \\ \exp\left(-\int_{x^1_{\mathrm{PN}}}^{x^1} |k_1(x^{1\prime})|\mathrm{d}x^{1\prime}\right), & x^1 > x^1_{\mathrm{TN}};\ x^1 < x^1_{\mathrm{PN}}, \end{cases} \quad (3)$$

where one can use the following expression:

$$k_1(x^1) = k_2\frac{x^1 - x^1_{\mathrm{PN}}}{x^1_{\mathrm{TN}} - x^1}$$

for a qualitative description of the $k_1(x^1)$ wave vector [see *Leonovich and Mazur*, 1997; *Klimushkin et al.*, 2004]. The upper expression in equation (3) describes the oscillations in the transparency region $x^1_{\mathrm{PN}} < x^1 < x^1_{\mathrm{TN}}$; the lower expression does the same for the opacity regions. The complete spatial structure of the fundamental harmonic ($N = 1$) for the oscillation magnetic field components is shown qualitatively in Figure 1.

To compare the oscillation structure calculated theoretically to the observed oscillation structure, let us proceed from the expressions obtained in the curvilinear coordinate system to the expressions in the local

Euclidean frame. Let us subscript the components of the vectors over the radial (transverse to the magnetic shells), azimuthal, and parallel (along the magnetic field lines) coordinates with r, a, and p respectively. We have

$$E_r = \frac{E_1}{\sqrt{g_1}}, \qquad E_a = \frac{E_2}{\sqrt{g_2}}, \qquad E_p = \frac{E_3}{\sqrt{g_3}}, \tag{4}$$

$$B_r = \frac{B_1}{\sqrt{g_1}}, \qquad B_a = \frac{B_2}{\sqrt{g_2}}, \qquad B_p = \frac{B_3}{\sqrt{g_3}}, \tag{5}$$

and from equation (2)

$$E_r = -\frac{i}{k_a}\nabla_r E_a, \qquad B_a = \frac{i}{k_a}\nabla_r B_r, \tag{6}$$

where $k_a = k_2/\sqrt{g_2}$ is the azimuthal component of the wave vector and $\nabla_r = g_1^{-1/2}\nabla_1$ is the derivative along the radial coordinate.

Note the polarization features of the oscillations. Since the oscillations are standing waves along magnetic field lines, it follows from equations (2), (4), and (5) that the phase shift between their E_a and B_r components is $\pi/2$. As follows from equation (6), in the opacity regions over the radial x^1 coordinate and near the poloidal resonance surface ($x^1 = x^1_{\mathrm{PN}}$) the phase shift between the B_a and B_r components is also $\pi/2$, which corresponds to an oscillation that is not propagating along the x^1 coordinate. However, in the transparency region ($x^1_{\mathrm{PN}} < x^1 < x^1_{\mathrm{TN}}$), when moving away from the poloidal resonance surface toward the toroidal surface, the phase shift between these components gradually changes to π (assuming that k_r and k_a have the same sign), which is typical of a traveling wave along the radial coordinate.

Dai et al. [2013] discovered evidence that the oscillations are amplified due to resonant interaction with charged high-energy particles. Note that the instability must be considered as a wave amplification mechanism rather than the source. To set this mechanism to work, some initial perturbation is required. This initial perturbation can be provided by ionospheric or magnetospheric currents. If the instability conditions are satisfied, then, as the wave travels across magnetic shells, the amplitudes of all wave electromagnetic field components first rise due to interaction with the particles when moving away from the poloidal surface and then decrease when approaching the toroidal surface. The oscillation polarization changes from poloidal (with dominant E_a and B_r components) to toroidal (with dominant E_r and B_a components) [*Klimushkin*, 2000; *Klimushkin and Mager*, 2011]. Irrespective of whether an instability is present or absent, the waves are fully absorbed near the toroidal resonance surface due to dissipation of their energy in the ionosphere [*Klimushkin*, 2007].

3. A Theoretically Calculated Structure of Azimuthally Small-Scale Alfvén Waves as Compared to RBSP-A Satellite Observations

Almost monochromatic Alfvén type oscillations with frequency ~12 mHz were recorded by the RBSP-A satellite on 23 October 2012 at 21.45–22.30 UT [*Dai et al.*, 2013]. The full record of the oscillation electromagnetic field components are shown in Figure 3. This record is one of the few where such oscillations are observed for numerous periods with no abrupt phase change, which can be regarded as one and the same mode observed while the satellite flew through its localization region. What catches one's attention is that the oscillations are localized in the range of magnetic shells $4.4 < L < 5.2$ (altitudes from $\approx 26,000$ km to $\approx 21,000$ km) and their polarization changes during the recording. Three intervals are identified corresponding to the three regions with different polarizations the satellite crossed, as shown in Figure 5b.

We note especially that the polarization change in the RBSP-A observed oscillations cannot be explained by phase mixing in a wave packet that initially has a poloidal polarization, which later changes to toroidal in a strongly inhomogeneous plasma (as, for example, in *Mann and Wright* [1995], *Leonovich and Mazur* [1998a], and *Klimushkin et al.* [2012]). The frequency of such oscillations should change significantly with L shells. In contrast, the RBSP-A observed oscillations exhibit very high monochromaticity for many oscillation periods. Moreover, during these observations the RBSP-A satellite shifts strongly across the magnetic shells, where the plasma parameters vary greatly. However, the wave frequency remained the same. It is a clear indication that the pulsation was generated by a monochromatic rather than time-dependent source. On the other hand,

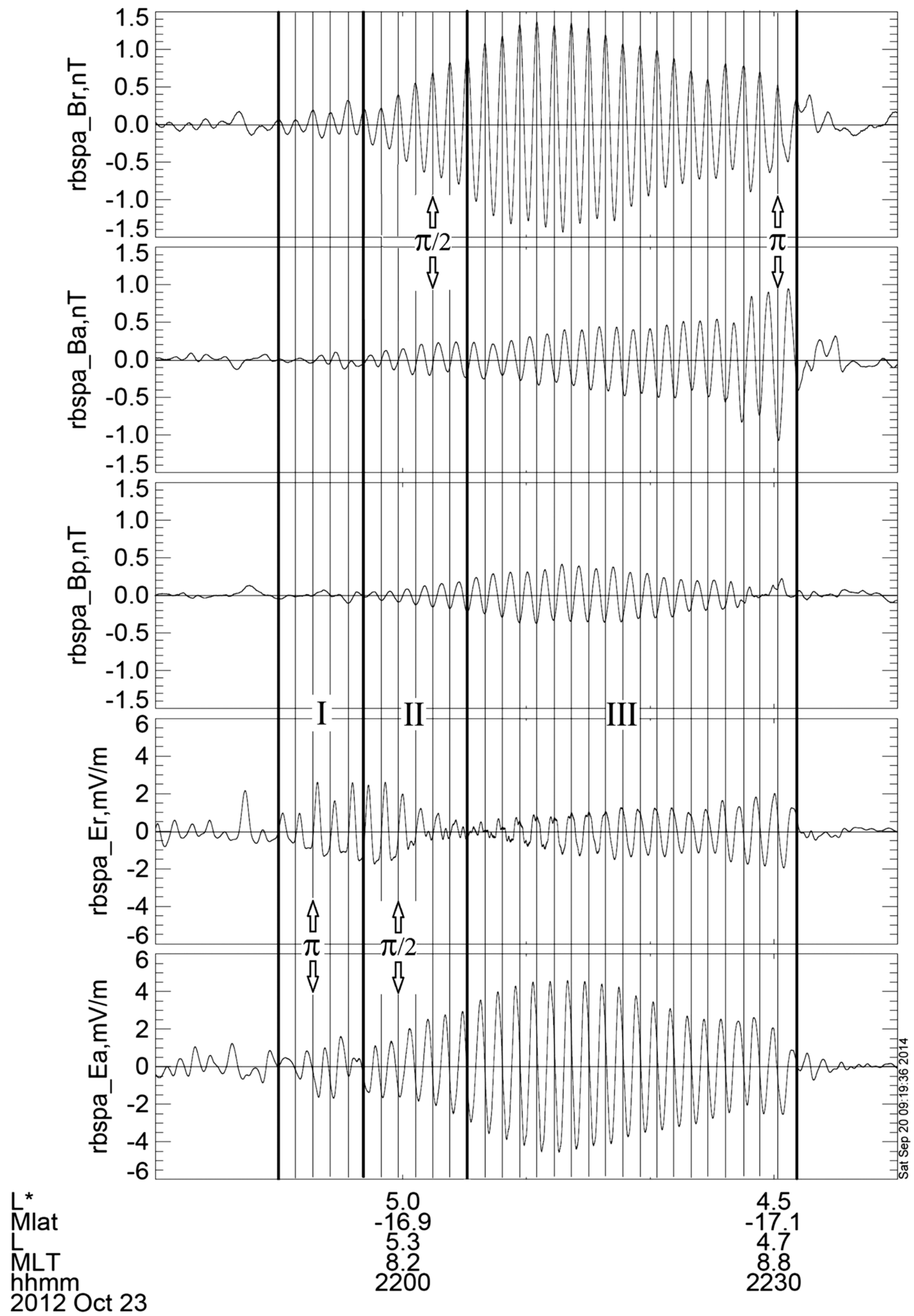

Figure 3. RBSP-A-recorded oscillations of the electromagnetic field components (23 October 2012). Three regions are highlighted with different polarization of oscillations corresponding to the satellite crossing two transparency regions (I and III) separated by an opacity region (II), as shown in Figure 5b.

the polarization change can very well be explained in the framework of the above proposed model of high-m Alfvén waves generated by an ionospheric or magnetospheric current and amplified by a flow of high-energy particles. Thus, the observed polarization change can only be explained by the presence of a single monochromatic oscillation mode having the poloidal polarization at one of the magnetic shells, which changes to the toroidal polarization at the others shells.

Let us now attempt to treat these oscillations in terms of the theory of azimuthally small-scale Alfvén waves in the previous section. We assume that the satellite observed oscillations with frequency $\bar{\omega} = \omega + k_r v_{\mathrm{sat},r}$, where $k_r = k_1/\sqrt{g_1}$ is the radial component of the wave vector and $v_{\mathrm{sat},r}$ is the radial component of the satellite velocity. Inside the transparency region, $k_r \sim m/a$, $\Omega_{P1} \sim v_A/a$, where a is the field line equatorial radius.

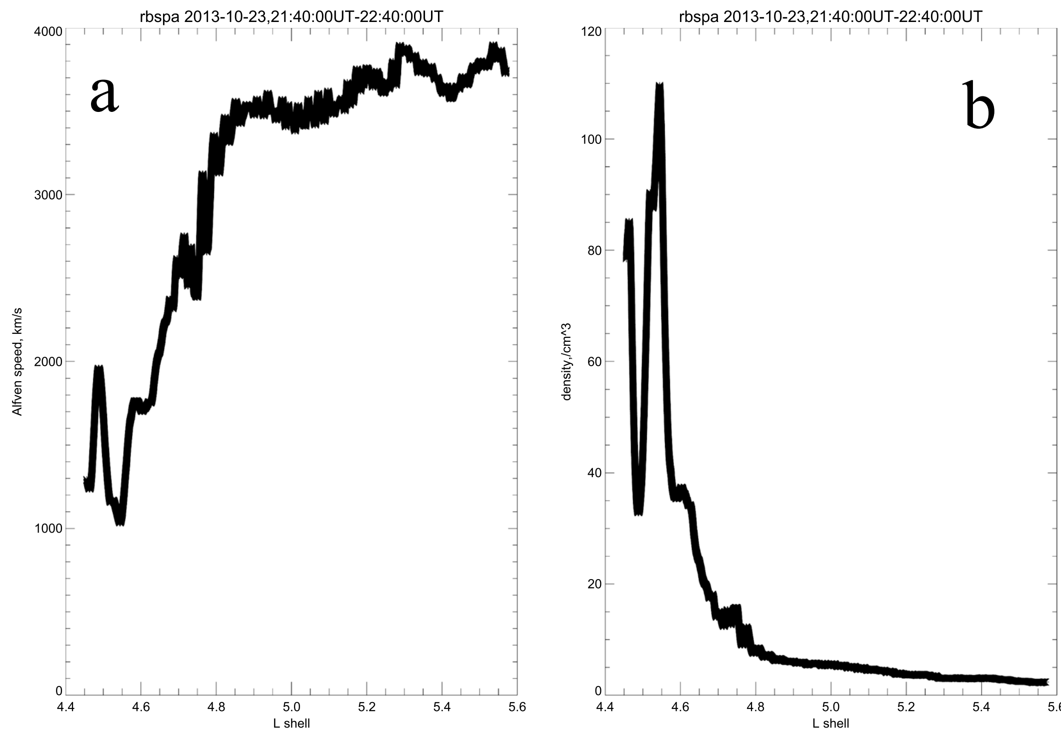

Figure 4. (a) Alfvén speed (b) and plasma density distribution across magnetic shells, as registered by RBSP-A satellite in the 23 October 2012 event.

Given that $v_A \approx 1000$ km/s, $v_{\mathrm{sat},r} \approx 1$ km/s and $m < 100$, we have $\bar{\omega} \approx \omega$; i.e., the oscillation frequency was determined by the source not by the satellite crossing the oscillation small-scale radial structure.

The fact that the oscillations were registered in a limited range of magnetic shells can be regarded as their being localized between the poloidal and toroidal resonance surfaces. The phase shift between the components of E_a and B_r is $\pi/2$. This corresponds to the structure of a standing wave along magnetic field lines. In magnetic field oscillations, the observations begin with the dominant poloidal B_r component, its amplitude first increasing and then decreasing gradually. At the end of the recording the toroidal B_a component is dominant. This can be regarded as the satellite entering the transparency region from the direction of the poloidal resonance surface and moving towards the toroidal surface. The phase shift between the B_a and B_r components is $\pi/2$ at first and then gradually changes to π, which is also fully consistent with the structure of the oscillations in the previous section.

Oscillations of the electric field components exhibit a somewhat unexpected pattern. They start before the oscillations of the magnetic components. At first they are dominated by the toroidal E_r component, which becomes almost zero at magnetic shell $L = 4.9$ but gradually increases and at the end of the recording becomes dominant again. Of course, one can consider this part of the record as unrelated to subsequent oscillations in magnetic components. However, the oscillation frequency is very close to the frequency of subsequent oscillations in the magnetic components. This may indicate a single-wave process. If this is true, then we can offer the following interpretation for it.

Figure 4 shows the Alfvén speed and plasma density distribution as recorded by RBSP-A as it crossed magnetic shells during the 23 October 2012 event. This distribution implies that the satellite was in the plasmapause region [see, e.g., *Sheeley et al.*, 2001]. We assume $\beta \ll 1$ inside and $\beta \sim 0.1-0.3$ outside the plasmasphere. This estimate of β outside the plasmasphere can be regarded only as a rough one based on its typical values near the plasmapause. β varying within this range does not change qualitatively the structure of high-m Alfvén waves across the magnetic shells [*Mager and Klimushkin*, 2002; *Klimushkin et al.*, 2004].

Therefore, the theory describing the Alfvén waves in a "cold plasma" model is applicable here. The results of this theory imply that the above oscillation structure can only be observed for the fundamental harmonic

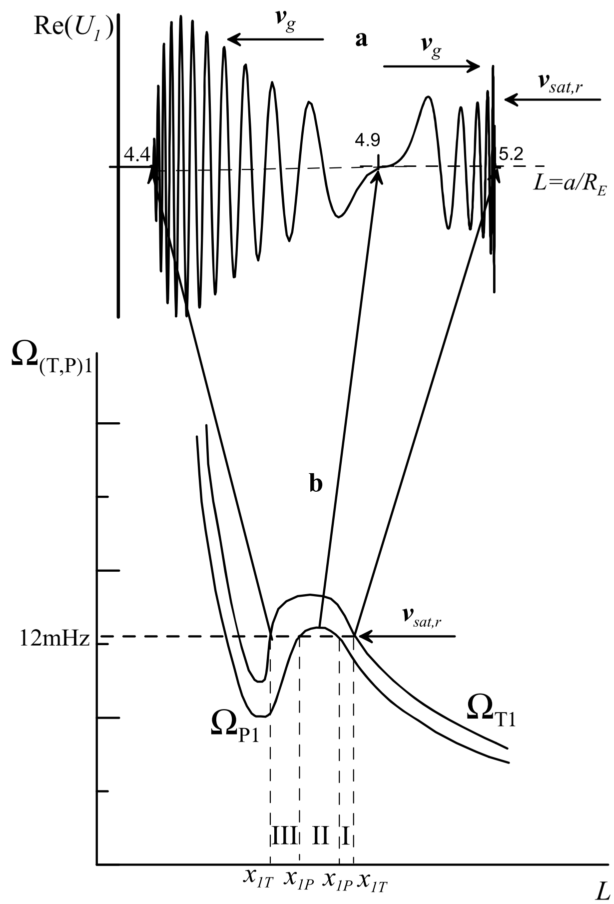

Figure 5. (a) Transverse radial structure for oscillations in the two transparency regions crossed by RBSP-A satellite. (b) A schematic representation of the satellite crossing the localization region of azimuthally small-scale Alfvén waves near the plasmapause, with two transparency regions (I and III) separated by an opacity region (II).

of standing Alfvén waves ($N = 1$). The poloidal and toroidal standing Alfvén wave eigenfrequency distribution for the fundamental harmonic is shown qualitatively in Figure 5b for a wide range of magnetic shells.

The above scheme implies that near the plasmapause the satellite must cross two transparency regions (I and III in Figure 5b) separated by a small opacity region (II). In this scheme, the first transparency region (I) is adjacent to the plasmapause from the outer-magnetosphere side, the opaque region (II) is located at the outer boundary of the plasmapause transition layer, and the second transparency region (III) is inside this transitional layer.

The expected field structure of the azimuthally small-scale oscillations is shown in Figure 5a. The satellite enters into the first transparency region (I) from the side of the toroidal resonance shell. Therefore, the initial polarization of the oscillations is toroidal. The phase shift between the E_a and E_r components is π, which corresponds to a wave traveling toward the toroidal resonance surface. This is in line with our expectations, because in the first and second transparency regions the waves run toward the toroidal surface in opposite directions (the radial $k_r{=}k_1/\sqrt{g_1}$ wave vector component has different signs in these transparency regions). The absence of the oscillations in the magnetic field when crossing the first transparency region can be explained by the fact that at the time, the satellite may have been near the node in the longitudinal distribution of the wave magnetic field components (see Figure 1).

As it travels the satellite shifts toward the poloidal resonance surface and the toroidal component, E_r, becomes much less than the poloidal component, E_a. In this case, the phase shift between E_a and E_r changes to $\pi/2$. At nearly the same time the oscillations begin in the magnetic B_a and B_r components, with a phase shift of $\pi/2$. This corresponds to the satellite entering a small opacity region (II). It then crosses this opacity region and enters the second transparency region (III) from the direction of the poloidal resonance surface. When the satellite enters the second transparency region, it is far enough from the standing wave node, and the oscillations in magnetic components become noticeable. We do not consider the parallel component of the wavefield, B_p, because, as noted in the previous section, it is very sensitive to β parameter variations. In view of the absence of data on the field-aligned β distribution, making predictions about B_p behavior in observations is not possible.

Let us note that RBSP-A was at a nearly constant magnetic latitude ($\sim -17°$) during the observations. Thus, the satellite coming out of the node region of oscillation magnetic field is not due to its motion along the geomagnetic field lines. Most likely it is due to greatly changing Alfvén speed distribution along geomagnetic field lines when the satellite is crossing through the plasmapause transition layer. The fundamental harmonic node is located at different points (relative to the magnetic equator) for different magnetic shells. Therefore, while crossing different magnetic shells, the satellite finds itself at different distances from the node of the observed standing waves.

Oscillations in the second transparency region begin as poloidal (with the oscillation field dominated by the E_a and B_r components) and gradually transform into toroidal oscillations (with dominant E_r and B_a components). The phase shift between the E_a and E_r components and the B_a and B_r components gradually changes from $\pi/2$ to π, which corresponds to a wave traveling from the poloidal to toroidal resonance surface (but with the opposite sign of the wave vector k_r component compared to region (II)). We suppose that the sign of the k_a component of the wave vector remains the same. When crossing the toroidal resonance surface, the oscillations disappear.

4. Summary

Let us summarize the main results of this work.

1. The basic properties of monochromatic azimuthally small-scale Alfvén waves are described in a magnetospheric model with a dipole magnetic field. The oscillations have the form of standing waves along magnetic field lines between the magneto-conjugated ionospheres. They are small-scale harmonic traveling waves in the azimuthal direction. In the radial direction (across magnetic shells) the waves have the following structure. The waves are generated by a monochromatic source at the resonance magnetic shell, where the source frequency coincides with the eigenfrequency of a poloidal standing Alfvén wave. From there the wave escapes toward the toroidal magnetic resonance shell where its frequency coincides with the eigenfrequency of a toroidal standing Alfvén wave. In the process the standing poloidal wave gradually transforms into a toroidal standing Alfvén wave. Near the toroidal resonance shell, the wave is completely absorbed due to the dissipation of its energy in the ionospheric conductive layer.
2. The structure of Alfvén waves recorded on 23 October 2012 by RBSP-A satellite at 21.45 – 22.30 UT is compared with a theoretically calculated structure of azimuthally small-scale Alfvén waves. It is shown that the observed picture of oscillations can be explained by the satellite crossing two transparency regions for azimuthally small-scale Alfvén waves, located near the plasmapause. The satellite must have observed the fundamental harmonic of standing Alfvén waves, because it is only for that harmonic that poloidal Alfvén oscillations can be observed to transform into toroidal oscillations.

When it crossed the first transparency region, the satellite was near the node of the standing Alfvén wave magnetic field. Therefore, oscillations are only observed in electrical components, which have an antinode in this area. When crossing the second transparency region, the satellite was far away from the standing wave node, and the oscillations were observed in both the electric and magnetic field components. In either transparency region, a transformation of poloidal into toroidal Alfvén waves was observed.

Here are a few comments on some complications of the proposed interpretation. The chief is the fact that the proposed pattern of azimuthally small-scale Alfvén waves can only be observed for a narrow enough wave packet, both in the k_2 (or m) wave number space and in the frequency spectrum. It is hard to imagine a magnetospheric process in which oscillations could be generated that have a narrow enough band in azimuthal wave numbers and are almost monochromatic. Their source must exist in a wide enough longitudinal range, within which a nearly sinusoidal wave has to be generated and to travel in the azimuthal direction. It is known from optical observations, however, that such structures are observed in the high-latitude ionosphere as discrete auroral arcs. A similar process might take place here.

We suppose that the observed polarization change cannot be explained by phase mixing of the Alfvén waves generated by a nonstationary process. The wave frequency would then depend on L shell [*Mann and Wright*, 1995] and time [*Leonovich and Mazur*, 1998a], but neither phenomenon had been observed. Thus, an explanation in terms of a monochromatic standing Alfvén wave propagating across magnetic shells looks most plausible.

The high monochromaticity of the recorded oscillations can be explained as follows. It can be seen from the Alfvén speed distribution in Figure 4 that the speed is almost constant in the plasmapause transition layer. The eigenfrequencies of poloidal standing Alfvén waves may exhibit the same distribution in this area. In this case, the transition layer has many resonant surfaces for poloidal standing Alfvén waves with almost identical eigenfrequencies. Therefore, these oscillations stand out against the background of other oscillations in the spectrum of a broadband source (external currents in the ionosphere?), and we see them as monochromatic oscillations. For oscillations at other frequencies there is only one resonance surface and they are not visible on the background of the main oscillation frequency corresponding to the maximum in the poloidal eigenfrequency distribution across magnetic shells (see Figure 5b).

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References

Archer, M. O., T. S. Horbury, J. P. Eastwood, J. M. Weygand, and T. K. Yeoman (2013), Magnetospheric response to magnetosheath pressure pulses: A low-pass filter effect, *J. Geophys. Res. Space Physics*, *118*, 5454–5466, doi:10.1002/jgra.50519.

Chen, L., and S. C. Cowley (1989), On field line resonances of hydromagnetic Alfvén waves in dipole magnetic field, *Geophys. Res. Lett.*, *16*, 895–897, doi:10.1029/GL016i008p00895.

Chen, L., and A. Hasegawa (1974), A theory of long-period magnetic pulsations: 1. Steady state excitation of field line resonance, *J. Geophys. Res.*, *79*, 1024–1032, doi:10.1029/JA079i007p01024.

Chen, L., and A. Hasegawa (1991), Kinetic theory of geomagnetic pulsations: 1. Internal excitations by energetic particles, *J. Geophys. Res.*, *96*, 1503–1512, doi:10.1029/90JA02346.

Cheng, C. Z., and A. T. Y. Lui (1998), Kinetic ballooning instability for substorm onset and current disruption observed by AMPTE/CCE, *Geophys. Res. Lett.*, *25*, 4091–4094, doi:10.1029/1998GL900093.

Cheremnykh, O. K., and A. S. Parnowski (2004), The theory of ballooning perturbations in the inner magnetosphere of the Earth, *Adv. Space Res.*, *33*, 769–773, doi:10.1016/S0273-1177(03)00642-2.

Dai, L., et al. (2013), Excitation of poloidal standing Alfvén waves through drift resonance wave-particle interaction, *Geophys. Res. Lett.*, *40*, 4127–4132, doi:10.1002/grl.50800.

Dmitrienko, I. S. (2013), Evolution of FMS and Alfvén waves produced by the initial disturbance in the FMS waveguide, *J. Plasma Phys.*, *79*, 7–17, doi:10.1017/S0022377812000608.

Dungey, J. W. (1954), *Electrodynamics of the Outer Atmospheres*, vol. 69, Ionos. Sci. Rep., Cambridge, Pa.

Dungey, J. W. (1967), Hydromagnetic waves, in *Physics of Geomagnetic Phenomena*, edited by S. Matsushita and W. H. Campbell, p. 913, Academic Press, New York and London.

Glassmeier, K.-H., et al. (1999), Magnetospheric field line resonances: A comparative planetology approach, *Surv. Geophys.*, *20*, 61–109, doi:10.1023/A:1006659717963.

Hartinger, M. D., D. L. Turner, F. Plaschke, V. Angelopoulos, and H. Singer (2013), The role of transient ion foreshock phenomena in driving Pc5 ULF wave activity, *J. Geophys. Res. Space Physics*, *118*, 299–312, doi:10.1029/2012JA018349.

James, M. K., T. K. Yeoman, P. N. Mager, and D. Yu. Klimushkin (2013), The spatio-temporal characteristics of ULF waves driven by substorm injected particles, *J. Geophys. Res. Space Physics*, *118*, 1737–1749, doi:10.1002/jgra.50131.

Klimushkin, D. Yu. (1994), Method of description of the Alfvén and magnetosonic branches of inhomogeneous plasma oscillations, *Plasma Phys. Rep.*, *20*, 280–286.

Klimushkin, D. Yu. (2000), The propagation of high-m Alfvén waves in the Earth's magnetosphere and their interaction with high-energy particles, *J. Geophys. Res.*, *105*, 23,303–23,310, doi:10.1029/1999JA000396.

Klimushkin, D. Yu. (2007), How energetic particles construct and destroy poloidal high-m Alfvén waves in the magnetosphere, *Planet. Space Sci.*, *55*, 722–730, doi:10.1016/j.pss.2005.11.006.

Klimushkin, D. Yu., and P. N. Mager (2004), The structure of low-frequency standing Alfvén waves in the box model of the magnetosphere with magnetic field shear, *J. Plasma Phys.*, *70*, 379–395, doi:10.1017/S0022377803002563.

Klimushkin, D. Yu., and P. N. Mager (2011), Spatial structure and stability of coupled Alfvén and drift compressional modes in non-uniform magnetosphere: Gyrokinetic treatment, *Planet. Space Sci.*, *59*, 1613–1620, doi:10.1016/j.pss.2011.07.010.

Klimushkin, D. Yu., A. S. Leonovich, and V. A. Mazur (1995), On the propagation of transversally small-scale standing Alfvén waves in a three-dimensionally inhomogeneous magnetosphere, *J. Geophys. Res.*, *100*, 9527–9534, doi:10.1029/94JA03233.

Klimushkin, D. Yu., P. N. Mager, and K.-H. Glassmeier (2004), Toroidal and poloidal Alfvén waves with arbitrary azimuthal wavenumbers in a finite pressure plasma in the Earth's magnetosphere, *Ann. Geophys.*, *22*, 267–287, doi:10.5194/angeo-22-267-2004.

Klimushkin, D. Yu., P. N. Mager, and K.-H. Glassmeier (2012), Spatio-temporal structure of Alfvén waves excited by a sudden impulse localized on an L-shell, *Ann. Geophys.*, *30*, 1099–1106, doi:10.5194/angeo-30-1099-2012.

Kozlov, D. A., A. S. Leonovich, and J. B. Cao (2006), The structure of standing Alfvén waves in a dipole magnetosphere with moving plasma, *Ann. Geophys.*, *24*, 263–274, doi:10.5194/angeo-24-263-2006.

Leonovich, A. S., and D. A. Kozlov (2014), Coupled guided modes in the magnetotails: Spatial structure and ballooning instability, *Astrophys. Space Sci.*, *353*, 9–23, doi:10.1007/s10509-014-1999-3.

Leonovich, A. S., and V. A. Mazur (1989), Resonance excitation of standing Alfvén waves in an axisymmetric magnetosphere (monochromatic oscillations), *Planet. Space Sci.*, *37*, 1095–1116, doi:10.1016/0032-0633(89)90081-0.

Leonovich, A. S., and V. A. Mazur (1993), A theory of transverse small-scale standing Alfvén waves in an axially symmetric magnetosphere, *Planet. Space Sci.*, *41*, 697–717, doi:10.1016/0032-0633(93)90055-7.

Leonovich, A. S., and V. A. Mazur (1995), Magnetospheric resonator for transverse-small-scale standing Alfvén waves, *Planet. Space Sci.*, *43*, 881–883, doi:10.1016/0032-0633(94)00206-7.

Leonovich, A. S., and V. A. Mazur (1996), Penetration to the Earth's surface of standing Alfvén waves excited by external currents in the ionosphere, *Ann. Geophys.*, *14*, 545–556, doi:10.1007/s00585-996-0545-1.

Leonovich, A. S., and V. A. Mazur (1997), A model equation for monochromatic standing Alfvén waves in the axially-symmetric magnetosphere, *J. Geophys. Res.*, *102*, 11,443–11,456, doi:10.1029/96JA02523.

Leonovich, A. S., and V. A. Mazur (1998a), Standing Alfvén waves with $m \gg 1$ in an axisymmetric magnetosphere excited by a non-stationary source, *Ann. Geophys.*, *16*, 914–920, doi:10.1007/s00585-998-0914-z.

Leonovich, A. S., and V. A. Mazur (1998b), Standing Alfvén waves with $m \gg 1$ in an axisymmetric magnetosphere excited by a stochastic source, *Ann. Geophys.*, *16*, 900–913, doi:10.1007/s00585-998-0900-5.

Lifshits, A. E., and E. N. Fedorov (1986), Hydromagnetic oscillations of the magnetospheric-ionospheric resonator, *Akad. Nauk SSSR Dokl.*, *287*, 90–94.

Liu, W., J. B. Cao, X. Li, T. E. Sarris, Q.-G. Zong, M. Hartinger, K. Takahashi, H. Zhang, Q. Q. Shi, and V. Angelopoulos (2013), Poloidal ULF wave observed in the plasmasphere boundary layer, *J. Geophys. Res. Atmos.*, *118*, 4298–4307, doi:10.1002/jgra.50427.

Mager, P. N., and D. Yu. Klimushkin (2002), Theory of azimuthally small-scale Alfvén waves in an axisymmetric magnetosphere with small but finite plasma pressure, *J. Geophys. Res.*, *107*(A11), 1356, doi:10.1029/2001JA009137.

Mager, P. N., and D. Yu. Klimushkin (2008), Alfvén ship waves: High-m ULF pulsations in the magnetosphere generated by a moving plasma inhomogeneity, *Ann. Geophys.*, *26*, 1653–1663, doi:10.5194/angeo-26-1653-2008.

Mager, P. N., and D. Yu. Klimushkin (2013), Giant pulsations as modes of a transverse Alfvénic resonator on the plasmapause, *Earth Planets Space*, *65*, 397–409, doi:10.5047/eps.2012.10.002.

Mann, I. R., and A. N. Wright (1995), Finite lifetimes of ideal poloidal Alfvén waves, *J. Geophys. Res.*, *100*, 23,677–23,686, doi:10.1029/95JA02689.

Mazur, V. A., and D. A. Chuiko (2013), Kelvin-Helmholtz instability on the magnetopause, magnetohydrodynamic waveguide in the outer magnetosphere, and Alfvén resonance deep in the magnetosphere, *Plasma Phys. Rep.*, *39*, 488–503, doi:10.1134/S1063780X13060068.

Pilipenko, V., N. Kleimenova, O. Kozyreva, M. Engebretson, and O. Rasmussen (2001), Long-period magnetic activity during the May 15, 1997 storm, *J. Atmos. Sol. Terr. Phys.*, *63*(5), 489–501, doi:10.1016/S1364-6826(00)00189-9.

Potapov, A. S. (2013), ULF wave activity in high-speed streams of the solar wind: Impact on the magnetosphere, *J. Geophys. Res. Atmos.*, *118*, 6465–6477, doi:10.1002/2013JA019119.

Radoski, H. R. (1967), Highly asymmetric MHD resonances: The guided poloidal mode, *J. Geophys. Res.*, *72*, 4026–4027, doi:10.1029/JZ072i015p04026.

Radoski, H. R. (1974), A theory of latitude dependent geomagnetic micropulsations: The asymptotic fields, *J. Geophys. Res.*, *79*, 595–603, doi:10.1029/JA079i004p00595.

Samson, J. C., L. L. Cogger, and Q. Pao (1996), Observations of field line resonances, auroral arcs, and auroral vortex structures, *J. Geophys. Res.*, *101*, 17,373–17,384, doi:10.1029/96JA01086.

Sheeley, B. W., M. B. Moldwin, H. K. Rassoul, and R. R. Anderson (2001), An empirical plasmasphere and trough density model: CRRES observations, *J. Geophys. Res.*, *106*, 25,631–25,642, doi:10.1029/2000JA000286.

Southwood, D. J. (1974), Some features of field line resonances in the magnetosphere, *Planet. Space Sci.*, *22*, 483–491, doi:10.1016/0032-0633(74)90078-6.

Southwood, D. J. (1980), Low frequency pulsation generation by energetic particles, *J. Geomagn. Geoelec., Suppl. II*, *32*, 75–88.

Tamao, T. (1965), Transmission and coupling resonance of hydromagnetic disturbances in the non-uniform Earth's magnetosphere, *Sci. Rep. Tohoku Univ.*, *17*, 43–54.

Vetoulis, G., and L. Chen (1994), Global structures of Alfvén-ballooning modes in magnetospheric plasmas, *Geophys. Res. Lett.*, *21*, 2091–2094, doi:10.1029/94GL01703.

Yeoman, T. K., M. James, P. N. Mager, and D. Yu. Klimushkin (2012), SuperDARN observations of high-m ULF waves with curved phase fronts and their interpretation in terms of transverse resonator theory, *J. Geophys. Res.*, *117*, A06231, doi:10.1029/2012JA017668.