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Turbulent transport of magnetic fields in a highly conducting rotating fluid and the solar cycle

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Abstract. This paper discusses effects of turbulent transport of mean axisymmetric magnetic fields in inhomogeneous rotating fluids. Rotationally-induced anisotropy was shown to enable turbulence transporting toroidal and poloidal components of the field in different directions. Qualitative interpretations of the findings are suggested. For the Rossby numbers typical of deep layers of the solar convection zone, the poloidal field is transported towards the poles while toroidal field moves equatorwards. The vertical component of toroidal field transport changes its sign at a latitude $\lambda^* \simeq 30°$, being directed downward at higher and upward at lower latitudes. Typical velocities and times of the field transports in the solar convection zone are estimated, and the possible role of the transport effects in the mean field dynamics over a solar cycle is discussed.

Key words: hydromagnetics – turbulence – Sun: magnetic field, rotation

1. Introduction

It is widely accepted that magnetic fields of the Sun are generated by dynamo mechanisms operating in convection zone. The dynamo-models are known to require an increase in angular velocity of global rotation with depth to reproduce the observed equatorward drift of solar activity. However, recent helioseismology data (Harvey 1988; Libbrecht 1988; Narrow 1988) revealed a near-constancy of angular velocity with depth at middle and low latitudes. Hence, the drift of dynamo-waves along isorotational surfaces (Yoshimura 1975) cannot account for the equatorward migration of sunspot activity. This suggests that traditional dynamo-models of the solar cycle miss some physics which is important for the mean magnetic field transport in the convection zone of the Sun.

The solar dynamo literature concentrates mainly upon two effects of convection: turbulent diffusion and the so-called alpha-effect (Steenbeck et al. 1966) of cyclonic turbulence. This is, probably, because these two effects, together with inhomogeneity of global rotation, were found sufficient to get oscillatory solutions of the induction equation and to model solar cyclicity. It has long been known, however, that there is, at least, one more effect of turbulence, namely turbulent transport of magnetic field, which may be important for the solar magnetic field dynamics.

Zeldovich (1957) and Spitzer (1957) discovered diamagnetism of inhomogeneously-turbulent conducting fluids: the mean magnetic field is transported with the velocity $\boldsymbol{v}_d = -\nabla\eta_t/2$, where η_t is turbulent diffusivity of the field. However, the inhomogeneity of turbulent diffusivity in convection zone of the Sun is small (Spruit 1974). Density inhomogeneity is more pronounced by far. This inhomogeneity was also found capable of causing turbulent transport of mean magnetic field. Drobyshevski (1977) considered the two-dimensional random flow of a density-stratified rotating conducting fluid and has found that the mean magnetic field is transported with the effective velocity $\boldsymbol{v}_d \simeq (\nabla\rho)\eta_t/\rho$, though the fluid as a whole is at rest. Later on, Vainshtein (1978) demonstrated this effect to disappear for three-dimensional locally-isotropic turbulence. However, the transport reappears again when turbulence has an anisotropy with preferred direction different from that of density gradient (Vainshtein 1978; Plieva 1987). The required anisotropy can be induced by rotational influence on turbulence.

This paper considers the mean magnetic field transport by turbulence in density-stratified rotating fluids. We shall find that rotational influence on convection (turbulence) causes the transport effect to acquire a remarkable property: the direction of magnetic field transport depends on the orientation of the field. The poloidal axisymmetric field is transported polewards. On the contrary, the toroidal axisymmetric field is transported equatorward in latitude, whereas radial transport velocity of this field has no definite sign and is positive (upward) at low latitudes and negative (downward) at high latitudes for values of the Rossby numbers typical of the large-scale solar convection. These findings are consistent with the observed poleward migration of solar poloidal fields (Howard 1974; Makarov et al. 1983) and with the equatorward transport of the toroidal fields implied by the observed drift of sunspot activity. The estimated transport velocities are close to the required values, and the latitude of a change of sign of the radial velocity of toroidal field transport roughly corresponds to the maximal latitude of the sunspot activity zone. The same features of the mean field transport by random convective motions are expected for any star whose rotation is sufficiently rapid. When the angular velocity of global rotation tends to zero, velocities of turbulent transport of toroidal and poloidal fields tend to coincide and to be purely radial, as should be the case.

Our treatment starts with a consideration in Sect. 2 of two simple examples of the turbulent transport of the mean magnetic field which, though being not directly applicable to the solar convection zone, serve well to illustrate the physical nature of the transport mechanisms discussed in the next sections. Velocities of axisymmetric mean field transport by turbulence in density-stratified rotating fluids will be derived in Sect. 3. Section 4 considers implications for mean magnetic field dynamics in the Sun. Section 5 contains brief final remarks.

2. Two simple examples

In this section we consider two relatively simple cases of turbulence. The consideration reproduces the main features of the mean field transport effects found in the subsequent treatment of rotating turbulence and simplifies greatly the qualitative interpretation of the findings.

A description of dynamics of mean magnetic field $\boldsymbol{B}=\langle \boldsymbol{H}\rangle$ (magnetic field $\boldsymbol{H}$ is a superposition of mean, $\boldsymbol{B}$, and fluctuating, $\boldsymbol{h}$, components: $\boldsymbol{H}=\boldsymbol{B}+\boldsymbol{h}$, $\langle \boldsymbol{h}\rangle=0$) requires derivation of the mean electromotive force (EMF), $\varepsilon=\langle \boldsymbol{u}\times\boldsymbol{h}\rangle$, which contributes to the averaged induction equation.

$$\partial\boldsymbol{B}/\partial t=\nabla\times\langle \boldsymbol{u}\times\boldsymbol{h}\rangle+\eta\Delta\boldsymbol{B}, \tag{2.1}$$

where velocity $\boldsymbol{u}$ is assumed to have a zero mean value, and η is magnetic diffusivity.

2.1. First example: two-dimensional turbulence of a density-stratified fluid

Let us consider a turbulent conducting fluid with an inhomogeneous density distribution. We assume the density gradient, $\nabla\rho$, to have the same direction (downward) everywhere and the density profile to be steady,

$$\operatorname{div}\rho\boldsymbol{u}=0. \tag{2.2}$$

Note that Eq. (2.2) naturally results from the anelastic approximation for subsonic convection (see, e.g., Gilman & Glatzmaier 1981) which is widely applied to the Sun. Equation (2.2) makes it more convenient to use solenoidal momentum density $\boldsymbol{p}=\rho\boldsymbol{u}$, instead of velocity $\boldsymbol{u}$. Let turbulence be two-dimensional with the flow velocity, $\boldsymbol{u}$, not varying along a horizontal (normal to density gradient) direction defined by a unit vector $\boldsymbol{e}$. (Note that parallel velocities $u_{\parallel}=(\boldsymbol{u}\cdot\boldsymbol{e})$ need not be zero but merely independent of the coordinate $x=(\boldsymbol{r}\cdot\boldsymbol{e})$.) Assume further that turbulence is statistically steady and quasi-isotropic in planes normal to vector $\boldsymbol{e}$. Inhomogeneous turbulence cannot be strictly isotropic. The term "quasi-isotropy" means inhomogeneous but as close to isotropy as possible (Vainshtein 1978; Kichatinov 1987). The spectral tensor for such turbulence is

$$\langle \hat{p}_i(z,\omega)\hat{p}_j(z',\omega')\rangle=\frac{\hat{E}(k,\omega,\boldsymbol{\kappa})}{4\pi k}[\delta_{ij}-k_ik_j/k^2 + (\kappa_ik_j-\kappa_jk_i)/2k^2]\;\delta(\boldsymbol{k}\cdot\boldsymbol{e})\;\delta(\omega+\omega'), \tag{2.3}$$

where

$$\boldsymbol{k}=(z-z')/2 \quad \text{and} \quad \boldsymbol{\kappa}=z+z'.$$

The circumflex above letters in (2.3) and below means Fourier-amplitudes, e.g.,

$$\boldsymbol{p}(\boldsymbol{r},t)=\int\exp(iz\cdot\boldsymbol{r}-i\omega t)\,\hat{\boldsymbol{p}}(z,\omega)\,dz\,d\omega.$$

The spectral tensor (2.3) differs from that derived by Kichatinov (1987) for three-dimensional quasi-isotropic turbulence by the presence of the delta-function $\delta(\boldsymbol{k}\cdot\boldsymbol{e})$, which means two-dimensionality, and by the normalizing coefficient. We keep in (2.3) only terms of up to first order in the ratio κ/k which is equivalent to l/L, where l is a typical scale of turbulent motions, and L is the spatial scale of variations of mean quantities.

The function $\hat{E}(k,\omega,\boldsymbol{\kappa})$ in (2.3) is a Fourier-transform of a local spectrum, i.e.,

$$E(k,\omega,\boldsymbol{r})=\int\hat{E}(k,\omega,\boldsymbol{\kappa})\exp(i\boldsymbol{\kappa}\cdot\boldsymbol{r})\,d\boldsymbol{\kappa}$$

is a local spectrum:

$$\langle\rho_\perp^2\rangle=\rho^2\langle u_\perp^2\rangle=\int_0^\infty\int_0^\infty E(k,\omega,\boldsymbol{r})\,dk\,d\omega,$$

where the "$\perp$" sign means a component of a vector normal to $\boldsymbol{e}$. Mean amplitudes of the velocities normal and parallel to direction $\boldsymbol{e}$ are equal for the turbulence defined by (2.3):

$$\langle u_{\parallel}^2\rangle=\langle(\boldsymbol{e}\cdot\boldsymbol{u})^2\rangle=\langle u_\perp^2\rangle.$$

We assume that $\langle u^2\rangle$ is much less inhomogeneous than density. It may be shown that under this assumption the local spectrum can be written as

$$E(k,\omega,\boldsymbol{r})=\rho^2(\boldsymbol{r})\,q(k,\omega),$$

where q is the spectrum of fluctuating velocities:

$$\langle u_\perp^2\rangle=\int_0^\infty\int_0^\infty q(k,\omega)\,dk\,d\omega.$$

The derivation of mean EMF will be made using the first order smoothing approximation (FOSA). Though this approximation may be (and has been) the subject of some criticism, it remains the basic tool of the mean-field electrodynamics. We sidestep further discussions of FOSA because this has been done in detail elsewhere (see, e.g., Moffatt 1978). We note only that it is convenient for our purposes to represent the products u_iB_j in the induction equation as $u_iB_j=p_i(B_j/\rho)$ to deal with solenoidal field $\boldsymbol{p}=\rho\boldsymbol{u}$ instead of the velocity field $\boldsymbol{u}$.

Fluctuating magnetic field $\boldsymbol{h}'$ resulting from wiggling of mean field lines of force by random motions can be found from the equation for Fourier-amplitudes:

$$(\eta k^2-i\omega)\hat{h}_i'(k,\omega)=ik_j\int[\hat{p}_i(\boldsymbol{k}-\boldsymbol{q},\omega)\,(B_j/\rho)(\boldsymbol{q}) - \hat{p}_j(\boldsymbol{k}-\boldsymbol{q},\omega)(B_i/\rho)(\boldsymbol{q})]\,d\boldsymbol{q}. \tag{2.4}$$

We neglect spatial inhomogeneity of the field $\boldsymbol{B}$ when deriving mean EMF. This means the neglect of eddy diffusivity. By successively forming a cross product of (2.4) with $\hat{\boldsymbol{p}}$, averaging, using (2.3) and transforming to real variables with taking first order terms in l/L into account, one gets

$$\langle\boldsymbol{u}\times\boldsymbol{h}'\rangle=\boldsymbol{v}_{\mathrm{D}}\times\boldsymbol{B}-2\boldsymbol{v}_{\mathrm{D}}\times\boldsymbol{e}(\boldsymbol{e}\cdot\boldsymbol{B}), \tag{2.5}$$

where velocity $\boldsymbol{v}_{\mathrm{D}}$ is

$$\boldsymbol{v}_{\mathrm{D}}=-\boldsymbol{\lambda}\frac{1}{2}\int_0^\infty\int_0^\infty\frac{k^2\eta\,q(k,\omega)}{k^4\eta^2+\omega^2}dk\,d\omega,\qquad \boldsymbol{\lambda}=\nabla\rho/\rho.$$

Substitution of (2.5) into (2.1) yields

$$\partial\boldsymbol{B}/\partial t=\nabla\times(\boldsymbol{v}_{\mathrm{D}}\times\boldsymbol{B}-2\boldsymbol{v}_{\mathrm{D}}\times\boldsymbol{e}(\boldsymbol{e}\cdot\boldsymbol{B})), \tag{2.6}$$

where the diffusive term is omitted. The first term on the right-hand side of (2.6) corresponds, obviously, to mean field transport with effective velocity $\boldsymbol{v}_D$. To understand the meaning of the second term, let us consider a two-dimensional magnetic field which does not vary along direction $\boldsymbol{e}$. In a Cartesian coordinate system with the x-axis parallel to $\boldsymbol{e}$ and the z-axis directed along the density gradient (downward) such a two-dimensional field can be represented as

$$\boldsymbol{B}=\boldsymbol{e}\,B(y,z)+\nabla\times(\boldsymbol{e}A(y,z)), \qquad (2.7)$$

where scalar B is a field component parallel to the x-axis, henceforth referred to as the p-field, and $\boldsymbol{e}A$ is a vector potential for the field normal to $\boldsymbol{e}$, which will be named t-field.

Substitution of (2.7) into (2.6) yields equations for components of the mean field,

$$\partial B/\partial t=-\partial(v_D B)/\partial z, \qquad (2.8a)$$

$$\partial A/\partial t=v_D\partial A/\partial z, \qquad (2.8b)$$

where v_D is an absolute value of vector $\boldsymbol{v}_D$. If the same algebra were made with the equation

$$\partial \boldsymbol{B}/\partial t=\nabla\times(\boldsymbol{v}_D\times\boldsymbol{B}), \qquad (2.9)$$

one finds, instead of (2.8), the following equations

$$\partial B/\partial t=\partial(v_D B)/\partial z, \qquad (2.10a)$$

$$\partial A/\partial t=v_D\partial A/\partial z. \qquad (2.10b)$$

Equations (2.10) differ from (2.8) by the sign of the right-hand side of the first equation only. Equations (2.10) are merely another representation of (2.9) and describe certainly the transport of both p- and t-fields upward with the same velocity v_D. The difference of signs between the right-hand sides of (2.8a) and (2.10a) shows that (2.8) describes the transport of p-field and t-field in *opposite* directions. Equation (2.6) describes certainly the same process. p-field is transported downward, whereas t-field still moves upward. This means that the term $-2\boldsymbol{v}_D\times\boldsymbol{e}(\boldsymbol{e}\cdot\boldsymbol{B})$ in the expression (2.5) for mean EMF represents the effect of an additional transport of p-field with velocity $-2v_D$.

It may be shown that nothing changes in the above considerations when A in (2.7) is x-dependent, i.e., when t-field varies along direction $\boldsymbol{e}$. Therefore, it may be concluded that if the mean magnetic field is a superposition of magnetic p-field parallel to direction $\boldsymbol{e}$ and magnetic t-field normal to this direction, the two-dimensional turbulence considered separates these two components and transports the p-field down and the t-field up with the same velocity v_D.

We must be careful, however, when applying these conclusions to arbitrary three-dimensional fields. Any magnetic field can be certainly decomposed into t-type and p-type vectors. However, these vectors cannot always be treated as magnetic (divergence-free) fields. If the p- and t-components of the mean magnetic field are not solenoidal, the effects produced by Eq. (2.6) cannot be reduced to transport of the field components. Nevertheless, in the remainder of the paper we shall be dealing with mean fields which possess a symmetry sufficient to ensure that EMF with the structure like (2.5) produce transport effects only.

It is known that rotational influence on large-scale solar convection makes the convective elements elongate along the axis of rotation (cf. Gilman & Miller 1986). In other words, rotation changes convection towards two-dimensionality. The above findings make it tempting to anticipate that the influence of rotation should result in convective transport of the poloidal component of the global magnetic field toward the axis of rotation and the toroidal component outward of the axis. We shall discuss this issue in more detail in Sects. 3 and 4.

Let us now proceed with the interpretation of the turbulent transport effects. Consider first the transport of p-fields. Figure 1A shows cross-sections of p-field flux tubes which experience vertical displacements. The density of the lines of force is shown by dots. The plane of Fig. 1 is supposed normal to vector $\boldsymbol{e}$. When a p-field flux tube moves up, it swells up, and the flux density decreases. If a tube moves down, it is compressed and the field strength in it increases. Therefore, mixing of an initially homogeneous p-field by two-dimensional turbulence results in an increase of the field strength with depth, i.e., the field is transported downward. This is just Drobyshevski's (1977) mechanism.

The situation with t-field is quite different. Let a line of force of this field be horizontal (for simplicity) at an initial instant. This line is shown dashed in Fig. 1B. At later instants the line is perturbed by fluid motion and assumes the form shown as the solid line in Fig. 1B. If the field is frozen-in to the fluid, the line of force is a (flexible) boundary which cannot be crossed by the fluid. Therefore, masses of matter on each side of the line of force do not change with time. In other words, the masses of the fluid (for unit length of the x-axis) of dashed areas S_1 and S_2 are equal for the steady density distribution considered. Hence, a downward increase of density requires the inequality $S_1>S_2$. Consequently, an arbitrary perturbation should displace a greater portion of t-field line of force upward. For this reason, the two-dimensional mixing of a density-stratified fluid transports mean t-field in the direction opposite to density gradient.

2.2. *Second example: three-dimensional MHD turbulence of a density-stratified fluid*

If the derivations of the preceding section were made for three-dimensional quasi-isotropic hydrodynamic turbulence, the mean field transport effect would disappear (Vainshtein 1978). The situation changes, however, with allowance for fluctuating magnetic fields of turbulent origin. These fields are usually ignored in mean field dynamo literature and some introductory discussion of their origin and nature seems to be appropriate.

Traditional kinematic approaches assume that the only source of fluctuating magnetic fields is turbulent wiggling of mean field lines of force. However, this is not the case when magnetic Reynolds numbers are sufficiently large. It was firstly suggested

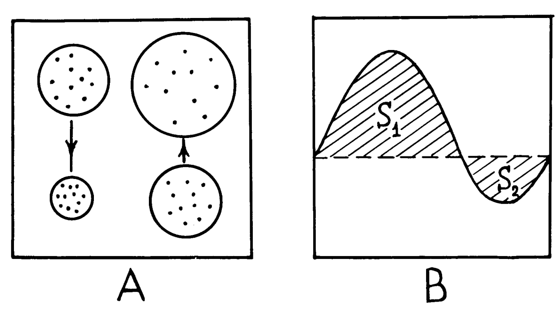

Fig. 1. **A** The downward displacements of a p-field flux tube increases the field strength because of the tube compression (left). The line-of-force density is shown by dots. An up-going tube swells and the field strength in the tube decreases (right). Hence, the two-dimensional mixing of p-field results in downward field transport. **B** Any disturbance of an initially horizontal t-field line lifts the larger portion of the line up

by Batchelor (1950) that highly conducting turbulent fluids may be unstable to weak seed random magnetic fields with a typical spatial scale of the order of that of turbulent motions, i.e., the small-scale fields may be amplified by turbulence. Later on Kraichnan & Nagarajan (1967) demonstrated the extreme complexity of the small-scale dynamo problem which is fundamentally impossible to solve by approximate methods; this is probably why the fluctuating fields produced by small-scale dynamos are usually ignored. At the same time, Kraichnan & Nagarajan (1967) have shown that using Lagrangian history direct interaction approximation results in asymptotic exponential growth of fluctuating magnetic fields with time. Further, Kazantsev (1968) demonstrated that the kinematic small-scale dynamo problem can be solved exactly for the case of infinitely small Strouhal numbers and also found turbulent amplification of the fields at sufficiently high magnetic Reynolds numbers. The dynamic problem with Lorentz forces included was treated by Pouquet et al. (1976) using the eddy-damped quasi-normal Markovian approximation. They also found initial growth of magnetic fluctuations, with the growth stabilized at subsequent moments in the inertial interval of wave numbers at near equipartition of kinetic and magnetic energies. That paper was probably the first to note an important role played by fluctuating fields produced by small-scale dynamo in the dynamics of large-scale magnetic fields. Finally, direct three-dimensional numerical simulations done by Meneguzzi et al. (1981) also yield a dynamo-amplification of small-scale magnetic fields, though magnetic Reynolds numbers ($\simeq 100$) were not far above the instability threshold. Though these results do not resolve completely the small-scale dynamo problem, they leave little doubt that any turbulent motion is unstable to fluctuating magnetic fields at sufficiently large magnetic Reynolds numbers. Amplification of the fields is probably stabilized at the near-equipartition of kinetic and magnetic energies. Observations of fine-structured magnetic fields of Kilogauss strength on the Sun (Stenflo 1973) show that small-scale dynamo is probably operating in the solar convection zone. Note again that the small-scale dynamo generates fluctuating magnetic fields from these same fields and the mean magnetic field is not needed for this process.

Let us now consider an MHD turbulence with fluctuating magnetic field $\boldsymbol{h}$ supported by small-scale dynamo and with no mean field $\boldsymbol{B}$ present. Magnetohydrodynamic equations are invariant with a change of sign of magnetic field. Therefore, a random flow $\boldsymbol{u}$ generates a fluctuating field $\boldsymbol{h}$ with the same probability as $-\boldsymbol{h}$, and it may be assumed that the correlation $\langle u_i h_j \rangle$ equals zero for the MHD turbulence considered. For this reason, $\langle \boldsymbol{u} \times \boldsymbol{h} \rangle = 0$.

Let a weak mean magnetic field $\boldsymbol{B}$ be imposed on the turbulent fluid. The field $\boldsymbol{B}$ perturbs the MHD turbulence and fluctuating fields are now equal to $\boldsymbol{u}+\boldsymbol{u}'$ and $\boldsymbol{h}+\boldsymbol{h}'$, where small perturbations $\boldsymbol{u}'$ and $\boldsymbol{h}'$ are induced by the mean magnetic field influence. The mean EMF $\langle (\boldsymbol{u}+\boldsymbol{u}') \times (\boldsymbol{h}+\boldsymbol{h}') \rangle = \langle \boldsymbol{u} \times \boldsymbol{h}' \rangle + \langle \boldsymbol{u}' \times \boldsymbol{h} \rangle$ may no longer be zero (the term $\langle \boldsymbol{u}' \times \boldsymbol{h}' \rangle$ is neglected).

If FOSA is used, the perturbations $\boldsymbol{h}'$ satisfy Eq. (2.4) and are produced by wiggling of mean field lines of force. The contribution of $\langle \boldsymbol{u} \times \boldsymbol{h}' \rangle$ to mean EMF is just what is derived by traditional kinematic theories. We assume in this section that the MHD turbulence is quasi-isotropic (locally isotropic but inhomogeneous) and momentum density $\boldsymbol{p} = \rho \boldsymbol{u}$ to satisfy Eq. (2.2). The contribution of $\langle \boldsymbol{u} \times \boldsymbol{h}' \rangle$ to mean EMF is zero for this case (Vainshtein 1978).

Let us consider the contribution of $\langle \boldsymbol{u}' \times \boldsymbol{h} \rangle$. The perturbation $\boldsymbol{u}'$ is produced by Lorentz force $(\nabla \times \boldsymbol{h}) \times \boldsymbol{B}/\mu$. We apply FOSA and shall find momentum density perturbations $\boldsymbol{p}' = \rho \boldsymbol{u}'$ from the Fourier-transformed equation of motion

$$[-i\omega + \nu k^2 + i\nu(\boldsymbol{k}\cdot\boldsymbol{\lambda})]\,\hat{p}_i'(\boldsymbol{k}, \omega) = i(\boldsymbol{k}\cdot\boldsymbol{B})\,\hat{h}_i(\boldsymbol{k}, \omega)/\mu, \tag{2.11}$$

where $\boldsymbol{\lambda} = \nabla\rho/\rho$, μ is vacuum permeability, and ν is molecular viscosity. We neglected inhomogeneity of the mean field $\boldsymbol{B}$ and used (2.2) when deriving (2.11). Vector $\boldsymbol{\lambda}$ may be considered constant because our derivations are linear in the parameter l/L.

Fluctuating magnetic field will be assumed quasi-isotropic:

$$\langle \hat{h}_i(z, \omega)\,\hat{h}_j(z', \omega) \rangle = \frac{\hat{H}(k, \omega, \boldsymbol{\kappa})}{16\,\pi k^2}[\delta_{ij} - k_i k_j/k^2 + (\kappa_i k_j - \kappa_j k_i)/2k^2]\,\delta(\omega + \omega'), \tag{2.12}$$

where $\boldsymbol{k} = (z - z')/2$, $\boldsymbol{\kappa} = z + z'$, and $\hat{H}$ is the Fourier-transform of the local spectrum:

$$H(k, \omega, \boldsymbol{r}) = \int \exp(i\boldsymbol{\kappa}\cdot\boldsymbol{r})\,\hat{H}(k, \omega, \boldsymbol{\kappa})\,d\boldsymbol{\kappa},$$

$$\langle h^2 \rangle = \int_0^\infty \int_0^\infty H(k, \omega, r)\,dk\,d\omega.$$

For the sake of simplicity, we assume energy equipartition for the local spectra:

$$H(k, \omega, \boldsymbol{r}) = \rho(\boldsymbol{r})\,\mu\, q(k, \omega), \tag{2.13}$$

where q is the velocity spectrum; remember that inhomogeneity of $\langle u^2 \rangle$ is neglected in comparison with a more pronounced density inhomogeneity.

By forming cross product of (2.11) with $\hat{\boldsymbol{h}}$, using (2.12) and transforming to real variables, we find

$$\varepsilon = \langle \boldsymbol{u}' \times \boldsymbol{h} \rangle = \boldsymbol{v}_m \times \boldsymbol{B},$$

where

$$\boldsymbol{v}_m = \lambda \frac{1}{6} \int_0^\infty \int_0^\infty \frac{\nu k^2 q(k, \omega)}{\nu^2 k^4 + \omega^2}\,d\omega\,dk. \tag{2.14}$$

Hence, the presence of fluctuating magnetic fields produced by small-scale dynamo results in the downward transport of the mean magnetic field.

Consider now a possible interpretation of the transport effect. Velocity $\boldsymbol{u}'$ may be estimated as $\boldsymbol{u}' \simeq \tau(\nabla \times \boldsymbol{h}) \times \boldsymbol{B}/\mu\rho$, where τ is a typical lifetime of the fluctuations. These are certainly not the velocities $\boldsymbol{u}'$ which transport the field $\boldsymbol{B}$. In the opposite case, velocity $\boldsymbol{v}_m$ would depend on $\boldsymbol{B}$. The velocities $\boldsymbol{u}$ can also not directly contribute to the transport effect considered because they were not explicitly involved in derivations of (2.14).

The following interpretation of the obtained transport effect may be suggested. The intensity of magnetic fluctuations $\langle h^2 \rangle = \mu\rho \langle u^2 \rangle$ increases downward because the intensity of fluctuating velocities, $\langle u^2 \rangle$, was assumed to vary with position much less than the density. Amplitudes of fluctuating currents $\boldsymbol{j} = (\nabla \times \boldsymbol{h})/\mu$ should also increase along the z-axis (see Fig. 2). Let the mean field $\boldsymbol{B}$ be parallel to the y-axis as shown in Fig. 2. The random Lorentz force $\boldsymbol{j} \times \boldsymbol{B}$ produces small-scale motions with velocities $\boldsymbol{u}'$ isotropic in the xz plane. Let us consider a level shown by the dashed line in Fig. 2. Vertical motions u_z' will carry to this level the fluid elements from other levels together with fluctuating currents flowing in them. The velocities u_z' results from x-components, j_x,

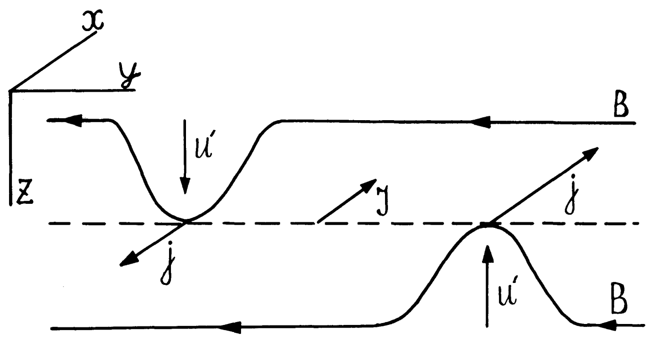

Fig. 2. An illustration of mean field transport due to fluctuating magnetic fields produced by small-scale dynamo (see text)

of the currents. The components u'_z and j_x always have different signs (see Fig. 2), i.e., the signs of j_x for upward and downward motions are opposite. However, the up-going fluid elements carry relatively large currents. This results in mean current $\boldsymbol{J}$ along the x-axis. The current $\boldsymbol{J}$ produces a new mean field which decreases the original mean magnetic field at upper levels and decreases it at lower levels. The resulting change of the mean field $\boldsymbol{B}$ distribution is equivalent to the field transport with the effective velocity $\boldsymbol{v}_m$(2.14), though there are no real hydrodynamic motions which move the field $\boldsymbol{B}$ with such a velocity.

3. Mean field transport by rotating turbulence

Let the turbulent fluid rotate as a whole with an angular velocity $\boldsymbol{\Omega}$. The equation for fluctuating momentum density $\boldsymbol{p}=\rho\boldsymbol{u}$ in a co-rotating coordinate system is

$$[\nu k^2+i\nu(\boldsymbol{k}\cdot\boldsymbol{\lambda})-i\omega]\hat{\boldsymbol{p}}(\boldsymbol{k},\omega)+2k^{-2}(\boldsymbol{k}\cdot\boldsymbol{\Omega})(\boldsymbol{k}\times\hat{\boldsymbol{p}}(\boldsymbol{k},\omega)) -\hat{f}^s(\boldsymbol{k},\omega)=i\hat{\boldsymbol{h}}(\boldsymbol{k},\omega)(\boldsymbol{k}\cdot\boldsymbol{B})/\mu, \tag{3.1}$$

where pressure was eliminated with the help of the anelasticity condition (2.2), and $\hat{\boldsymbol{f}}^s$ is the solenoidal part of a random force $\hat{\boldsymbol{f}}$: $\hat{\boldsymbol{f}}^s=\hat{\boldsymbol{f}}-\boldsymbol{k}(\boldsymbol{k}\cdot\hat{\boldsymbol{f}})/k^2$. We assume the Strouhal numbers for the turbulence to be small, which allows the neglect of nonlinear terms when deriving (3.1) (FOSA is used).

Let us consider first the turbulence not perturbed by mean magnetic field. The neglect of the right-hand side of (3.1) leads to the usual (Rüdiger 1989) linear relation:

$$\hat{p}_i(\boldsymbol{k},\omega)=D_{ij}(\boldsymbol{k},\omega,\boldsymbol{\Omega})\,\hat{p}^\circ_j(\boldsymbol{k},\omega), \tag{3.2}$$

where

$$\hat{p}^\circ_j(\boldsymbol{k},\omega)=\hat{f}^s(\boldsymbol{k},\omega)/(\nu k^2+i\nu(\lambda\cdot\boldsymbol{k})-i\omega), \tag{3.3}$$

and tensor $\hat{D}$ is defined as

$$D_{ij}=(\delta_{ij}+\alpha\sigma\varepsilon_{ijp}k_p/k)/(1+\alpha^2\sigma^2), \tag{3.4}$$

$\sigma=\boldsymbol{k}\cdot\boldsymbol{\Omega}/(k\Omega)$ is a cosine between vectors $\boldsymbol{k}$ and $\boldsymbol{\Omega}$, $\alpha=2\Omega/(\nu k^2+i\nu(\lambda\cdot\boldsymbol{k})-i\omega)$. The random field $\hat{\boldsymbol{p}}^\circ$ in Eqs. (3.2) and (3.3) corresponds to the so-called "original turbulence" which would take place under real sources of turbulence but in the absence of rotation (Rüdiger 1989). The original turbulence will be assumed quasi-isotropic (Kichatinov 1987):

$$\langle\hat{p}^\circ_i(z,\omega)\hat{p}^\circ_j(z',\omega')\rangle=\frac{\hat{E}(k,\omega,\boldsymbol{\kappa})}{16\pi k^2}[\delta_{ij}-k_ik_j/k^2 +(\kappa_ik_j-\kappa_jk_i)/2k^2]\,\delta(\omega+\omega'),$$

$$\boldsymbol{k}=(z-z')/2,\ \boldsymbol{\kappa}=z+z', \tag{3.5}$$

where again only terms of up to first order in $\kappa/k\simeq l/L$ are kept. The spectral tensor for rotating turbulence can be found by using (3.2):

$$\langle\hat{p}_i(z,\omega)\hat{p}_j(z',\omega')\rangle=D_{im}(z,\omega,\boldsymbol{\Omega})\,D_{jn}(z',\omega',\boldsymbol{\Omega}) +\langle\hat{p}^\circ_m(z,\omega)\hat{p}^\circ_n(z',\omega')\rangle. \tag{3.6}$$

Magnetic Reynolds numbers will be assumed large; this means that the small-scale dynamo discussed in the preceding section is operating and produces fluctuating magnetic fields $\boldsymbol{h}$. The small-scale dynamo is an essentially nonlinear process (Kraichnan & Nagarajan 1967; Kazantsev 1968; Pouquet et al. 1967; Meneguzzi et al. 1981) which is principally impossible to describe using FOSA. Nevertheless, the perturbations $\boldsymbol{u}'$ and $\boldsymbol{h}'$ which the mean field $\boldsymbol{B}$ produces in the fluctuations $\boldsymbol{u}$ and $\boldsymbol{h}$ can be derived within the framework of FOSA, and this will be done below.

The spectral tensor of fluctuating magnetic fields will be assumed to have the same structure as the spectral tensor (3.6) of random motions generating these fields,

$$\langle\hat{h}_i(z,\omega)\hat{h}_j(z',\omega')\rangle=D_{im}(z,\omega,\boldsymbol{\Omega})\,D_{jn}(z',\omega',\boldsymbol{\Omega}) +\langle\hat{h}^\circ_m(z,\omega)\hat{h}^\circ_n(z',\omega')\rangle, \tag{3.7}$$

where $\langle\hat{h}^\circ_m\hat{h}^\circ_n\rangle$ is the quasi-isotropic spectral tensor (2.12).

One more essential point should be discussed before proceeding with derivation of the mean field transport velocities. In contrast to the examples of the preceding section, the transport effects are not all nondiffusive (proportional to $\boldsymbol{B}$ but not to spatial derivatives of $\boldsymbol{B}$) contributions to mean EMF for rotating turbulence. There is also the so-called alpha-effect by Steenbeck et al. (1966). This paper is restricted to discussion of mean field transport and does not consider the alpha-effect. The question arises, however, as to how one from another can be distinguished. It is appropriate at this point to note that the turbulent transport of the field must be independent of the direction of rotation of convection shell. Therefore, the corresponding contributions to the mean EMF must be even functions of angular velocity. We shall see below that these even contributions coincide in their structure with Eq. (2.5) and do, indeed, represent the sought-for transport effects. On the other hand, it may be shown that the remainder of mean EMF which is an odd function of Ω has a structure typical (Rüdiger 1978) of the alpha-effect: $\alpha_{ij}B_j$, where

$$\alpha_{ij}=\alpha_1\delta_{ij}+\alpha_2(g_ie_j+g_je_i)+\alpha_3e_ie_j$$

is a symmetric pseudo-tensor (α_n are pseudo-scalars, and $\boldsymbol{g}$ and $\boldsymbol{e}$ are unit vectors in radial direction and along the rotation axis, respectively). The contribution of $\alpha_{ij}B_j$ to the mean EMF generates a toroidal field from a poloidal one and vice versa, i.e., this actually is an alpha-effect.

We derive below the part of mean EMF which is symmetric with respect to a change of sign of the angular velocity. Similar to the derivations made above, there are contributions of two kinds:

$$\boldsymbol{\varepsilon}=\langle\boldsymbol{u}\times\boldsymbol{h}'\rangle+\langle\boldsymbol{u}'\times\boldsymbol{h}\rangle, \tag{3.8}$$

where $\boldsymbol{h}'$ and $\boldsymbol{u}'$ are perturbations of fluctuating magnetic field and velocity resulting from mean magnetic field influence on turbulence. The first term in the right-hand side of (3.8) is the usual kinematic contribution. The second term is brought about by fluctuating magnetic fields generated by small-scale dynamo. We shall derive these two contributions separately.

3.1. Contribution of fluctuating velocities

The perturbation $\boldsymbol{h}'$ is found from (2.4) to be

$$\hat{h}'_i(\boldsymbol{k}, \omega) = \frac{ik_j}{\eta k^2 - i\omega} \int [\hat{p}_i(\boldsymbol{k}-\boldsymbol{q}, \omega)\,(B_j/\rho)(\boldsymbol{q}) - \hat{p}_j(\boldsymbol{k}-\boldsymbol{q}, \omega)\,(B_i/\rho)/(\boldsymbol{q})]\,d\boldsymbol{q}. \tag{3.9}$$

The sought-for contribution $\langle \boldsymbol{u} \times \boldsymbol{h}' \rangle = \langle \boldsymbol{p} \times \boldsymbol{h}' \rangle / \rho$ can be derived using (3.9) and the spectral tensor of momentum density $\boldsymbol{p}$ defined by (3.5) and (3.6). After rather involved algebra we find

$$\langle \boldsymbol{u} \times \boldsymbol{h}' \rangle = \boldsymbol{V} \times \boldsymbol{B} - 2(\boldsymbol{V} \times \boldsymbol{e})(\boldsymbol{e} \cdot \boldsymbol{B}), \tag{3.10}$$

here and below $\boldsymbol{e} = \boldsymbol{\Omega}/\Omega$ is a unit vector in the direction of angular velocity (note the difference with the horizontal vector $\boldsymbol{e}$ of the preceeding section), and the velocity $\boldsymbol{V}$ is

$$\boldsymbol{V} = -\boldsymbol{\lambda}_\perp \frac{1}{8} \int_0^\infty \int_0^\infty \frac{\eta k^2 q(k, \omega)}{\eta^2 k^4 + \omega^2} I_1(k, \omega, \Omega)\,dk\,d\omega. \tag{3.11}$$

In this equation, $\boldsymbol{\lambda}_\perp = \boldsymbol{\lambda} - \boldsymbol{e}(\boldsymbol{e} \cdot \boldsymbol{\lambda})$ is a component of vector $\boldsymbol{\lambda} = (\nabla \rho)/\rho$ normal to the rotation axis, and I_1 is defined as

$$I_1(k, \omega, \Omega) = \frac{1}{\hat{\Omega}^2} \left\{ \frac{3\cos\varphi + \hat{\Omega}^2}{\hat{\Omega}\cos(\varphi/2)} \left[\tan^{-1}\left(\frac{\hat{\Omega} - \sin(\varphi/2)}{\cos(\varphi/2)} \right) + \tan^{-1}\left(\frac{\hat{\Omega} + \sin(\varphi/2)}{\cos(\varphi/2)} \right) \right] - \frac{3(1-\cos\varphi)}{2\hat{\Omega}\sin(\varphi/2)} \ln\left(\frac{\hat{\Omega}^2 - 2\hat{\Omega}\sin(\varphi/2) + 1}{\hat{\Omega}^2 + 2\hat{\Omega}\sin(\varphi/2) + 1} \right) - 6 \right\}, \tag{3.12}$$

where

$$\hat{\Omega} = 2\Omega/\sqrt{\nu^2 k^4 + \omega^2}, \qquad \cos\varphi = (\nu^2 k^4 - \omega^2)/(\nu^2 k^4 + \omega^2).$$

It may be shown that the contribution of (3.10) to mean EMF represents transport of toroidal (t-field) and poloidal (p-field) components of an axisymmetric field $\boldsymbol{B}$ in opposite directions with velocities $\boldsymbol{V}$ and $-\boldsymbol{V}$, respectively. To demonstrate this, consider an axisymmetric field

$$\boldsymbol{B} = \boldsymbol{e}_\varphi B + \nabla \times (\boldsymbol{e}_\varphi A), \tag{3.13}$$

where $\boldsymbol{e}_\varphi$ is an azimuthal unit vector, B is t-field, and $\boldsymbol{e}_\varphi A$ is a vector potential for p-field. Let the field (3.13) satisfy the equation,

$$\partial \boldsymbol{B}/\partial t = \nabla \times \varepsilon, \tag{3.14a}$$

$$\varepsilon = \boldsymbol{v} \times \boldsymbol{B} + (\boldsymbol{v}' \times \boldsymbol{e})(\boldsymbol{e} \cdot \boldsymbol{B}), \tag{3.14b}$$

where $\boldsymbol{v}$ and $\boldsymbol{v}'$ also possess axial symmetry. If Eq. (3.14) is written in cylindrical coordinates z, r, and φ, we find

$$\partial B/\partial t = -\partial(v_z B)/\partial z - \partial(v_r B)/\partial r,$$

$$\partial A/\partial t = -v_z \partial A/\partial z - (v_r + v'_r) r^{-1} \partial(rA)/\partial r. \tag{3.15}$$

Equations (3.13)–(3.15) clearly demonstrate that $\boldsymbol{v}'_\perp = \boldsymbol{v}' - \boldsymbol{e}(\boldsymbol{e} \cdot \boldsymbol{v}')$ is an additional velocity of p-field transport. Comparison of (3.10) with (3.14b) shows that in (3.10) $\boldsymbol{v}' = -2\boldsymbol{V}$ and $\boldsymbol{v} = \boldsymbol{V}$. Therefore, (3.10) represents the p-field transport with the velocity $\boldsymbol{v} + \boldsymbol{v}' = -\boldsymbol{V}$.

This result is easy to explain within the framework of the interpretation of mean field transport by two-dimensional turbulence suggested in the preceding section. Rotational influence is known to elongate convective cells along the axis of rotation. This effect is accounted for by Eq. (3.2). It may be understood as the appearance in rotating turbulence of a two-dimensional component, whose properties vary slowly along direction $\boldsymbol{e}$. Such a two-dimensional turbulence transports p- and t-fields in opposite directions. The t-field is transported to regions of relatively low fluid density with the velocity $\boldsymbol{V}$, in accordance with the interpretation of the preceding section (Fig. 1B). The p-field is transported in opposite direction (Fig. 1A). The presence in p-field of a component normal to $\boldsymbol{e}$ plays no role because this component is parallel to the velocity $\boldsymbol{V}$ (3.11).

Let us return to Eqs. (3.11) and (3.12). If rotation is slow ($\hat{\Omega} \ll 1$), the function I_1 of (3.12) is small:

$$I_1 = 8\hat{\Omega}^2(2\cos\varphi - 1)/15,$$

where only the lowest-order term in $\hat{\Omega}$ is kept. The velocity (3.11) for this case,

$$\boldsymbol{V} = -\boldsymbol{\lambda}_\perp \Omega^2 \frac{4}{15} \int_0^\infty \int_0^\infty \frac{\eta k^2 (\nu^2 k^4 - 3\omega^2) q(k, \omega)}{(\nu^2 k^4 + \omega^2)^2 (\eta^2 k^4 + \omega^2)} dk\,d\omega, \tag{3.16}$$

is small as well.

For the opposite limit of rapid rotation ($\hat{\Omega} \gg 1$), from (3.12) we find

$$I_1 = \pi/(|\hat{\Omega}|\cos(\varphi/2)),$$

where we keep only the lowest order term in $\hat{\Omega}^{-1}$. Therefore, the velocity (3.11) for this case,

$$\boldsymbol{V} = -\boldsymbol{\lambda}_\perp \frac{\pi\eta}{16|\Omega|\nu} \int_0^\infty \int_0^\infty \frac{\nu^2 k^4 + \omega^2}{\eta^2 k^4 + \omega^2} q(k, \omega)\,dk\,d\omega, \tag{3.17}$$

is also small.

The kinematic transport effect is, probably, most pronounced in the moderate rotation case when the parameter $\hat{\Omega} = 2\Omega/\sqrt{\nu^2 k^4 + \omega^2}$ is of order unity for those wave numbers and frequencies, which make a dominant contribution to the integral (3.11). If rotation is slow, rotationally induced anisotropy, which is necessary for the transport effect to arise, is small. This is probably why the velocity (3.16) decreases with decreasing Ω. If rotation is rapid, rotational suppression of turbulence causes the velocity (3.17) to decrease with increasing Ω.

3.2. Contribution of fluctuating magnetic fields

Let us now consider the contribution of $\langle \boldsymbol{u}' \times \boldsymbol{h} \rangle$ to the mean EMF (3.8). This contribution may be written as $\langle \boldsymbol{p}' \times \boldsymbol{h} \rangle / \rho$ in terms of momentum density $\boldsymbol{p}$. The perturbation $\boldsymbol{p}'$ can be found from (3.1),

$$\hat{p}'_i(\boldsymbol{k}, \omega) = \frac{iD_{ij}(\boldsymbol{k}, \omega, \boldsymbol{\Omega})\hat{h}_j(\boldsymbol{k}, \omega)\,(\boldsymbol{k} \cdot \boldsymbol{B})}{\mu[\nu k^2 + i\nu(\boldsymbol{k} \cdot \boldsymbol{\lambda}) - i\omega]}. \tag{3.18}$$

A rather involved algebra with the use of (3.7), (2.12), and (2.13) yields

$$\langle \boldsymbol{u}' \times \boldsymbol{h} \rangle = \boldsymbol{V}_m \times \boldsymbol{B} + (\boldsymbol{V}'_m \times \boldsymbol{e})(\boldsymbol{e} \cdot \boldsymbol{B}), \tag{3.19}$$

where

$$\boldsymbol{V}_m = \boldsymbol{\lambda} \frac{1}{8} \int_0^\infty \int_0^\infty \frac{\nu k^2 q(k, \omega)}{\nu^2 k^4 + \omega^2} I_2(k, \omega, \Omega)\,dk\,d\omega, \tag{3.20a}$$

$$\boldsymbol{V}'_m = -\boldsymbol{\lambda}_\perp \frac{1}{8} \int_0^\infty \int_0^\infty \frac{\nu k^2 q(k, \omega)}{\nu^2 k^2 + \omega^2} I_3(k, \omega, \Omega)\,dk\,d\omega, \tag{3.20b}$$

and the functions I_2 and I_3 are

$$I_2(\omega, k, \Omega)=\frac{1}{\hat{\Omega}^2\cos^2(\varphi/2)}\left\{1+\frac{\hat{\Omega}^2-1}{2\hat{\Omega}\cos(\varphi/2)}\times\left[\tan^{-1}\left(\frac{\hat{\Omega}-\sin(\varphi/2)}{\cos(\varphi/2)}\right)+\tan^{-1}\left(\frac{\hat{\Omega}+\sin(\varphi/2)}{\cos(\varphi/2)}\right)\right]\right\}. \quad (3.21)$$

$$I_3(\omega, k, \Omega)=\frac{1}{\hat{\Omega}^2\cos^2(\varphi/2)}\left\{1+2\frac{\hat{\Omega}^2\cos\varphi+1}{\hat{\Omega}^4+2\hat{\Omega}^2\cos\varphi+1}+\frac{\hat{\Omega}^2-3}{2\hat{\Omega}\cos(\varphi/2)}\left[\tan^{-1}\left(\frac{\hat{\Omega}-\sin(\varphi/2)}{\cos(\varphi/2)}\right)+\tan^{-1}\left(\frac{\hat{\Omega}+\sin(\varphi/2)}{\cos(\varphi/2)}\right)\right]\right\},$$

$$\hat{\Omega}=2\Omega/\sqrt{v^2k^4+\omega^2}, \qquad \cos\varphi=(v^2k^4-\omega^2)/(v^2k^4+\omega^2).$$

It has been shown in Sect. 3.1 that mean EMF with the structure (3.19) represents the t-field transport with the velocity $\boldsymbol{V}_m$ and the p-field with velocity $\boldsymbol{V}_m+\boldsymbol{V}'_m$, i.e., $\boldsymbol{V}'_m$ is an additional velcoity of p-field transport.

If $\hat{\Omega}$ tends to zero (no rotation), it may be shown using (3.21) that $I_2=4/3$ and $I_3=0$. Equation (3.20a) transforms in this case into expression (2.14) for a nonrotating fluid, and $\boldsymbol{V}'_m=0$. In other words, the additional velocity $\boldsymbol{V}'_m$ of p-field transport is rotationally induced. In the case of rapid rotation ($\hat{\Omega}\gg 1$), from (3.21) we find

$$I_2=I_3=\pi/(2|\hat{\Omega}|\cos^3(\varphi/2)).$$

In this case, the velocities (3.20) decrease with increasing angular velocity which probably results from rotational suppression of turbulence.

3.3. Mixing-length approximation

The results found above are difficult to apply to the Sun or to other objects because of lack of knowledge of the parameters involved. The aim of this section is to adapt our findings to applicational purposes. The widely used mixing-length approximation seems to meet this aim quite well. This approximation will be understood as one replacing nonlinear terms together with time derivatives in the equations for fluctuating fields by τ-relaxation terms, i.e., instead of (3.1) and (3.9) we now have

$$\hat{\boldsymbol{p}}/\tau+2k^{-2}(\boldsymbol{k}\cdot\boldsymbol{\Omega})(\boldsymbol{k}\times\hat{\boldsymbol{p}})-\hat{\boldsymbol{f}}^{\mathrm{s}}=i\hat{\boldsymbol{h}}(\boldsymbol{k}\cdot\boldsymbol{B})/\mu,$$

$$\hat{h}'_i/\tau=ik_j\int[\hat{p}_i(\boldsymbol{k}-\boldsymbol{q})(B_j/\rho)(\boldsymbol{q})-\hat{p}_j(\boldsymbol{k}-\boldsymbol{q})(\boldsymbol{B}_i/\rho)(\boldsymbol{q})]\,d\boldsymbol{q}, \quad (3.22)$$

where

$$\tau\simeq l/\sqrt{\langle u^2\rangle}$$

is a typical lifetime of a convective eddy, and l is the mixing length. Reynolds numbers are assumed small, $\tau\ll l^2/v$, $\tau\ll l^2/\eta$, and viscous terms are neglected in (3.22). We assume next in the spirit of the mixing-length approximation that the spectrum of turbulent motions is single-scaled:

$$q(k)\sim\delta(k-l^{-1}).$$

There is no need to repeat all the preceding derivations starting from (3.22) to find the mixing-length representation of the above findings. It may be shown that the mixing-length analogs of Eqs. (3.11) and (3.20) are obtainable formally by substitution into these equations of the expressions to follow

$$q(k,\omega)=2\langle u^2\rangle^\circ\,\delta(k-l^{-1})\,\delta(\omega),$$

$$vk^2=\eta k^2=\tau^{-1}. \quad (3.23)$$

Equations (3.23) link the above FOSA-findings to the mixing-length approximation.

Substitution of (3.23) into (3.11) and (3.20) yields

$$\boldsymbol{V}=-\boldsymbol{\lambda}_\perp\tau\langle u^2\rangle^\circ\,\varphi_1(\hat{\Omega}),$$

$$\boldsymbol{V}_m=\boldsymbol{\lambda}\tau\langle u^2\rangle^\circ\,\varphi_2(\hat{\Omega}),$$

$$\boldsymbol{V}'_m=-\boldsymbol{\lambda}_\perp\tau\langle u^2\rangle^\circ\,\varphi_3(\hat{\Omega}), \quad (3.24)$$

where $\hat{\Omega}=2\tau\Omega$ is the Coriolis number (reciprocal of the Rossby number), $\langle u^2\rangle^\circ$ is mean intensity of fluctuating velocities for original turbulence, and the functions $\varphi_n(\hat{\Omega})=\frac{1}{8}I_n(\Omega,k,\omega)\Big|_{\omega=0,\ vk^2=1/\tau}$ are

$$\varphi_1(\hat{\Omega})=\frac{1}{4\hat{\Omega}^2}\left[\frac{\hat{\Omega}^2+3}{\hat{\Omega}}\tan^{-1}(\hat{\Omega})-3\right],$$

$$\varphi_2(\hat{\Omega})=\frac{1}{8\hat{\Omega}^2}\left[1+\frac{\hat{\Omega}^2-1}{\hat{\Omega}}\tan^{-1}(\hat{\Omega})\right],$$

$$\varphi_3(\hat{\Omega})=\frac{1}{8\hat{\Omega}^2}\left[1+\frac{2}{1+\hat{\Omega}^2}+\frac{\hat{\Omega}^2-3}{\hat{\Omega}}\tan^{-1}(\hat{\Omega})\right]. \quad (3.25)$$

It has been shown above that $-2\boldsymbol{V}$ and $\boldsymbol{V}'_m$ in the second terms of the right-hand sides of (3.10) and (3.19), respectively, are additional velocities of transport of p-field. Using (3.10) and (3.19) we express the velocities of turbulent transport of the toroidal ($\boldsymbol{V}^{\mathrm{t}}$) and poloidal ($\boldsymbol{V}^{\mathrm{p}}$) components of an axisymmetric mean field embedded into a rotating density-stratified highly conducting fluid in terms of the velocities (3.24):

$$\boldsymbol{V}^{\mathrm{t}}=\boldsymbol{V}+\boldsymbol{V}_{\mathrm{m}}, \quad (3.26a)$$

$$\boldsymbol{V}^{\mathrm{p}}=-\boldsymbol{V}+\boldsymbol{V}_{\mathrm{m}}+\boldsymbol{V}'_m. \quad (3.26b)$$

Note again that $\boldsymbol{V}^{\mathrm{t}}$ and $\boldsymbol{V}^{\mathrm{p}}$ differ substantially.

4. In convection zone of the Sun

Solar dynamo is believed to operate in deep regions of convection zone far below the supergranulation layer. The deviation of stratification from adiabaticity at these depths is very small (cf. Gough & Weiss 1976) and the density distribution is almost spherically symmetric. Therefore

$$\boldsymbol{\lambda}=\boldsymbol{e}_{\mathrm{r}}\frac{1}{\rho}\frac{\partial\rho}{\partial r}=-\boldsymbol{e}_{\mathrm{r}}\frac{1}{\gamma-1}\frac{g}{c_{\mathrm{p}}T}, \quad (4.1)$$

where $\boldsymbol{e}_{\mathrm{r}}$ is radial unit vector, T is temperature, g is gravity, c_{p} is specific heat at constant pressure, and γ is the ratio of specific heats.

By definition, the quantity $\langle u^2\rangle^\circ$ is the intensity of convective velocities which would take place with actual sources of convection (actual superadiabaticity) if there were no rotation. A satisfactory estimation for this quantity seems to be the known mixing-length relation $\langle u^2\rangle^\circ=-\nabla\Delta T\,l^2g/(4T)$ where $\nabla\Delta T$ is superadiabatic temperature gradient. However, deriving the superadiabaticity of stratification leads to a rather complicated problem of convective heat transport in rotating fluids which is beyond the scope of this paper. We shall use the estimate $\tau\langle u^2\rangle^\circ = 3\kappa$, where $\kappa \simeq 10^{13}$ cm^2 s^{-1} is turbulent diffusivity supplied by models of nonrotating convection zone (Spruit 1974; Gough & Weiss 1976). Note, however, that such an estimation lowers the value of $\langle u^2\rangle^\circ$ because it neglects rotational influence which is known to increase superadiabaticity.

By using (3.24)–(3.26) and (4.1), we find

$$V_\theta^{\rm p} = -\frac{3\kappa g}{(\gamma-1)c_{\rm p}T}\cos\lambda\sin\lambda[\varphi_1(\hat\Omega)-\varphi_3(\hat\Omega)], \tag{4.2a}$$

$$V_r^{\rm p} = -\frac{3\kappa g}{(\gamma-1)c_{\rm p}T}[\varphi_2(\hat\Omega)+\cos^2\lambda(\varphi_1(\hat\Omega)-\varphi_3(\hat\Omega))], \tag{4.2b}$$

$$V_\theta^{\rm t} = \frac{3\kappa g}{(\gamma-1)c_{\rm p}T}\cos\lambda\sin\lambda\,\varphi_1(\hat\Omega), \tag{4.2c}$$

$$V_r^{\rm t} = -\frac{3\kappa g}{(\gamma-1)c_{\rm p}T}[\varphi_2(\hat\Omega)-\cos^2\lambda\,\varphi_1(\hat\Omega)]. \tag{4.2d}$$

In these equations and below λ is latitude (scalar) but not the density gradient (vector) used above.

Durney & Latour (1978) estimated the Coriolis number for giant solar convection to be $\hat\Omega \simeq 6$. The functions φ_n for this magnitude of $\hat\Omega$ assume the following values

$$\varphi_1 = 0.0426, \quad \varphi_2 = 0.0319, \quad \varphi_3 = 0.0305. \tag{4.3}$$

Velocities (4.2a) and (4.2c) have different signs when (4.3) is used. $V_\theta^{\rm p}$ is negative while $V_\theta^{\rm t}$ is positive, i.e., the poloidal and toroidal fields are transported towards the poles and the equator, respectively.

The radial velocity (4.2d) of toroidal field transport changes sign at latitude $\lambda^* = \cos^{-1}(\sqrt{\varphi_2/\varphi_1})$, being negative (downward) for $\lambda > \lambda^*$ and positive (upward) for $\lambda < \lambda^*$. Using (4.3) we find

$\lambda^* \simeq 30°$.

It is widely believed that sunspot activity is associated with strong subphotospheric toroidal magnetic fields. Absence of spots at high latitudes is usually interpreted as indicating that the strong t-fields are lacking there. However, our findings suggest that the t-fields need not be weak at high latitudes but are locked near the base of convection zone by downward turbulent transport. At $\lambda < \lambda^*$ the vertical transport velocity changes sign and may lift the t-fields to the upper layers of convection zone, thus stimulating sunspot activity. Vertical transport of the field may be affected by magnetic buoyancy. However, Kleeorin et al. (1989) have recently found that pressure of a mean field embedded in a highly conducting turbulent fluid is *negative* which greatly questions the applicability of the buoyancy concept to *mean* fields.

LaBonte & Howard (1982) found a close relation between magnetic activity and the 2/hemisphere mode of solar torsional oscillations (Howard & LaBonte 1980). The torsional waves are believed to be of magnetic origin (Schüssler 1981; Yoshimura 1981; Rüdiger et al. 1986). At low latitudes, the shear region of torsional wave zonal velocity coincides with the region of maximal rate of emergence of new magnetic flux which, probably, is a manifestation of concentrated subphotospheric toroidal magnetic fields. The torsional wave amplitude changes little when the wave travels from pole to equator. It is tempting, in the light of the above findings, to speculate that toroidal fields of the strength sufficient to produce torsional waves are present at high latitudes as well but are hidden there deep in the convection zone by downward turbulent transport.

The $\sin\lambda\cos\lambda$ dependence of the velocity $V_\theta^{\rm t}$ (4.2c) on latitude agrees qualitatively with the observed variations (initial increase and subsequent decrease) of velocity of torsional wave latitudinal propagation (LaBonte & Howard 1982).

The time interval Δt of t-field transport from a latitude λ_1 to a latitude $\lambda_2 < \lambda_1$ with the velocity (4.2c) is

$$\Delta t = r\int_{\lambda_2}^{\lambda_1}\frac{d\lambda}{V_\theta^{\rm t}(\lambda)} = r\frac{(\gamma-1)c_{\rm p}T\ln(tg\lambda_1/tg\lambda_2)}{3\kappa g(r)\varphi_1(\hat\Omega)} \tag{4.4}$$

where distance r is assumed independent of λ. Suppose that the transport occurs near the base of convection zone, i.e., $r = 0.7\,R$. The plausible estimates for this location are $c_{\rm p} = 3.4\,10^8$ cm^2 s^{-2} K^{-1} (fully ionized hydrogen), $T = 2\,10^6$ K, and $\gamma = 5/3$. Neglecting self-gravitation, we assume next $g = g_0(R/r)^2$, where $g_0 = 2.74\,10^4$ cm s^{-2} is the surface value of solar gravity. Substitution of these values into (4.4) and using (4.3) yields

$$\Delta t = 9.8\ln(tg\lambda_1/tg\lambda_2)\ \text{yr}. \tag{4.5}$$

If we adopt $\lambda_1 = 70°$ and $\lambda_2 = 10°$ as starting and final positions, Eq. (4.5) gives $\Delta t = 27$ yr. This value does not differ greatly from 22 yr which a torsional wave takes to travel from polar to equatorial regions. Probably, the slight overestimation of Δt results from underestimation of the velocities (4.2) under the assumption $\tau\langle u^2\rangle^\circ = 3\kappa$ discussed above.

The base of convection zone is known to be a favourable site for the solar dynamo operation (Galloway & Weiss 1981; Golub et al. 1981; Spruit & van Ballegooijen 1982). The mechanisms placing dynamo at this site are uncertain, however. Turbulent transport with negative (downward) radial velocities $V_r^{\rm p}$ (4.2b) and $V_r^{\rm t}$ (for $\lambda < \lambda^*$) represents one of the possibilities.

Expressions (3.25) show that the inequalities $\varphi_1 > \varphi_3$ and $\varphi_1 > \varphi_2$ hold for any star whose rotation is sufficiently rapid ($\hat\Omega > 3.4$). We may expect from Eqs. (4.2) that general features of mean field migration on such a star must be the same as on the Sun, i.e., p-fields should migrate poleward and t-field should move equatorward producing lowlatitudinal activity belt.

Finally, we note that the equatorward turbulent transport of t-fields may serve to reconcile the near-constancy of angular velocity with depth inferred from helioseismology data (Harvey 1988; Libbrecht 1988; Narrow 1988) with the solar dynamo models. The direct tree-dimensional simulations of solar dynamo (Glatzmaier 1985) may be losing the turbulent transport effects because the Reynolds numbers in the simulations are small. The small-scale dynamo does not work in this case, and the velocities V_m and V'_m fall to nought. The velocity V (3.11) is small for the case of low Reynolds numbers.

5. Final remarks

We have seen that the rotational influence on MHD turbulence imparts a new property to the effects of turbulent transport of mean magnetic fields: poloidal and toroidal fields are transported with different velocities. The main danger for the applicability of the expressions found for these velocities to the Sun seems to come from the lack of reliable information on small-scale dynamo operation in density stratified rotating fluids. For this reason, we cannot judge with confidence on how far from reality are our assumptions about the properties of fluctuating magnetic fields used when deriving the turbulent transport effects. Nevertheless, these assumptions seem plausible on physical grounds and the turbulent transport effects reproduce quite well (qualitatively, at least) the observed redistribution of magnetic fields over a solar cycle, which probably mean that the relevant physics was adequately covered by our treatment.

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