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Standing Alfven waves in the magnetosphere from a localized monochromatic source

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Abstract

We have constructed a theory for standing Alfven waves driven in the magnetosphere by a monochromatic source localized in the ionosphere. It is shown that a dominant role in such oscillations is played by harmonics with large azimuthal wave numbers with $m \gg 1$. We explore the possibility of such oscillations being excited during active experiments on periodic (with the period of Alfven eigenmodes of the magnetosphere) ionospheric modifications. Our study has made it apparent that the measured distance between local maxima of the amplitude of oscillations excited in such an experiment can be used to infer the value of the polarization splitting of the spectrum between poloidal and toroidal eigenmodes of the magnetosphere.

1. Introduction

It is generally agreed that credit for the pioneering theoretical investigation into standing Alfven waves in the magnetosphere is due to *Dungey* [1954]. He obtained equations describing the field of monochromatic Alfven oscillations in an axisymmetric magnetosphere. For two limiting values of the azimuthal wave number $m = 0$ and $m = \infty$, equations describing the longitudinal (along geomagnetic field lines) structure of the field of Alfven waves split off from a complicated system of MHD equations. Oscillations of this type were given the title toroidal ($m = \infty$) and poloidal ($m = 0$) eigenmodes. Subsequently, these equations were repeatedly solved both analytically [*Radoski,* 1967; *Radoski and Carovillano,* 1969; *Krylov and Lifshitz,* 1984] and numerically [*Cummings et al.,* 1969]. Those calculations resulted in a description of the longitudinal structure of standing Alfven waves of the poloidal and toroidal type and of the sets of their eigenfrequencies.

For oscillations with $m \neq 0$ and $m \neq \infty$, the system of MHD equations is not split into separate, simpler equations for the individual modes of MHD oscillations. In the cold magnetosphere model, equations for Alfven waves and equations for fast magnetosound are associated in this system of equations. For monochromatic oscillations with a fixed frequency ω this system has singularities at some points along the transverse coordinate, namely, on resonant magnetic shells. The transverse structure of the oscillation field near these resonance shells was treated in the theory of field line resonance [*Southwood,* 1974; *Chen and Hasegawa,* 1974]. Originally, this theory was constructed for a model of the magnetosphere that is inhomogeneous only in one, transverse, coordinate. Subsequently, by considering a magnetosphere that is inhomogeneous both in one of the transverse coordinates and in the longitudinal coordinate, solutions of MHD equations were obtained describing both the transverse and longitudinal structure of the oscillation fields near these resonance shells [*Kivelson and Southwood,* 1986; *Leonovich and Mazur,* 1989; *Chen and Cowley,* 1989].

It should be noted that the cited papers considered the following excitation mechanism for oscillations on resonance magnetic shells. The magnetosound wave driven on the magnetopause penetrates deep into the magnetosphere and excites, via resonance, standing Alfven waves on those magnetic shells where its frequency coincides with the eigenfrequency of the Alfven oscillations. However, such an excitation mechanism for standing Alfven waves is effective only for harmonics with the azimuthal wave number $m \sim 1$. Since the greater part of the magnetosphere is an opacity region for the magnetosound waves under consideration, their amplitude drops exponentially inside the magnetosphere with increasing m, and the excitation of standing Alfven waves becomes ineffective.

For standing Alfven waves with $m \gg 1$ the oscillation source must lie on the same field lines where these waves are excited. *Southwood and Saunders* [1985], *Walker* [1987], and *Taylor and Walker* [1987] take, as such a source, slow magnetosound waves which for harmonics with $m \gg 1$ propagate, as do Alfven waves, along geomagnetic field lines. This provides a means for the resonance interaction of these modes of the MHD oscillations. *Walker and Pekrides* [1996] considered this process by taking into account the influence of the ring current and the electric field of the geomagnetic tail. *Leonovich and Mazur* [1993] investigated in detail the complete spatial structure of a monochromatic standing wave with $m \gg 1$ in a cold magnetosphere, the sources of which are external currents in the ionosphere. Similar studies for a plasma with $\beta \sim 1$ were carried out by *Klimushkin* [1998].

The cited references all addressed the spatial structure of one Fourier harmonic with a fixed frequency ω and the azimuthal wave number m. It is amply evident that real sources of Alfven waves generate oscillations in the magnetosphere with a broad spectrum of frequencies as well as of wave numbers. In this context, to investigate the field of oscillations generated by real sources, it is necessary to pass from a description of a single Fourier harmonic to the field of oscillations generated by broadband sources.

In this paper we proceed to a study of the field of oscillations with a fixed frequency ω generated in the magnetosphere by a source that is spatially localized in the ionosphere. This means that the oscillation source has an arbitrary structure across field lines and hence a broad spectrum in azimuthal wave numbers m. Particular attention will be given to the study of the oscillations from a strongly localized source. An HF radar that heats the ionosphere at regular intervals with a frequency near the frequency of Alfven eigenoscillations of the magnetosphere on the magnetic shell under consideration can presumably be used to generate such oscillations in the magnetosphere.

The possibility that magnetospheric MHD oscillations can be excited by artificial means has been discussed in the literature for a long time. *Greifinger* [1972] considered the possible excitation of the oscillations using large (~ 100 km in size) antennas with high-current generators. *Cornwall* [1972] discussed an alternative possibility of generating ULF waves that involves injecting a charged particle cloud into the auroral magnetosphere or into the ring current region. However the most realistic possibility appears to be the artificial excitation of magnetospheric MHD eigenoscillations using powerful HF radio transmitters. Experiments made with such transmitters [*Fraser-Smith and Cole*, 1975; *Willis and Davis*, 1976; *Getmantsev et al.*, 1977] showed that the ionospheric modification with electromagnetic pulses modulated with ~ 1 Hz frequencies is sometimes followed by MHD oscillations with frequencies close to the modulation frequency. This suggested that the observed oscillations are caused by the preceding ionospheric modification with modulated emission of HF transmitters. Unfortunately, we are unaware of like experiments on artificial excitation of lower frequency magnetospheric MHD oscillations at frequencies close to fundamental-harmonic frequencies of standing Alfven waves at geomagnetic midlatitudes.

This paper can be regarded as a theoretical justification for the possibility of carrying out such an experiment. The distinctive characteristics of the spatial distribution of the amplitude of such oscillations can be used to measure the polarization splitting of the spectrum of Alfven oscillations into toroidal and poloidal eigenmodes.

2. Basic Equations

In this paper we shall use three coordinate systems connected with geomagnetic field lines; they are presented in Figure 1. The curvilinear orthogonal coordinate system is (x^1, x^2, x^3) in which the coordinate x^2 defines a field line on a given magnetic shell x^1 and x^3 defines a point on a given field line. The local Cartesian coordinate system is (n, y, l). Physical components of an arbitrary vector $\boldsymbol{a}$ in this system are related to covariant components in curvilinear coordinates by the following relations:

1

$$a_n = \sqrt{g_1}a^1, \qquad a_y = \sqrt{g_2}a^2, \qquad a_l = \sqrt{g_3}a^3,$$

where g_i $(i = 1, 2, 3)$ are the metric tensor components of the curvilinear coordinate system. The local Cartesian coordinate system is (x, y, z) near the ionosphere in which the axis z is normal to Earth's surface, the axis x is directed from south to north, and the axis y is directed from west to east. Note that near the ionosphere, the axis y in the coordinate systems (x, y, z) and (n, y, l) coincides, the axis x is the axis n projected onto the ionosphere along geomagnetic field lines, and the axis z is in opposition to the axis l projected onto the direction of the axis z.

A disturbance in the monochromatic wave of the form $\mathrm{e}^{-i\omega t}$, where ω is the oscillation frequency, t is a time, will be described by the electric potential Φ related to the electromagnetic components of the wave field by the relations

$$\begin{aligned} E_1 &= -\nabla_1\Phi, \qquad B_1 = -i\frac{c}{\omega}\sqrt{\frac{g_1}{g_2}}\nabla_2\frac{1}{\sqrt{g_3}}\nabla_3\Phi, \\ E_2 &= -\nabla_2\Phi, \qquad B_2 = i\frac{c}{\omega}\sqrt{\frac{g_2}{g_1}}\nabla_1\frac{1}{\sqrt{g_3}}\nabla_3\Phi, \\ E_3 &= 0. \end{aligned} \tag{1}$$

The expression for the component B_3 is more unwieldy and is not reproduced here (it may be found in a paper by *Leonovich and Mazur* [1993]). They also showed that the field structure of the standing Alfven wave with a fixed frequency ω and the azimuthal wave number m may be represented as

$$\Phi_N = \bar{U}_N(x^1, k_2, \omega)H_N(x^1, k_2, x^3, \omega)\mathrm{e}^{-i\omega t + ik_2x^2},$$

where $N = 1, 2, 3, ...$ is the longitudinal eigenmode number of standing waves. If the azimuthal angle φ is used as the azimuthal coordinate x^2, then $k_2 = m$ is the azimuthal wave number. In this paper we employ a more general notation, i.e. x^2 and k_2, because we shall consider a large number of harmonics from $m \sim 1$ to $m = \infty$, and the variation of k_2 will be considered continuous.

The function H_N describes the field structure of the standing Alfven wave along a geomagnetic field line. This function was investigated at length by *Leonovich and Mazur* [1989, 1993]. At this point, we give only the expression obtained in the WKB approximation in longitudinal coordinate x^3 (or, the equivalent, in coordinate l, $\mathrm{d}l = \sqrt{g_3}\mathrm{d}x^3$) applicable to longitudinal harmonics of standing waves with $N \gg 1$:

$$H_N = C_N \sin\left(\Omega_N \int_{l_-}^{l} \frac{\mathrm{d}l'}{A}\right). \tag{2}$$

Here C_N is the normalizing factor independent of the longitudinal coordinate, A is the Alfven velocity, and

$\Omega_N = 2\pi N/t_A$ is the eigenoscillation frequency of the Nth harmonic of the standing waves, where

$$t_A = \oint \frac{\mathrm{d}l'}{A} \equiv 2\int_{l_-}^{l_+} \frac{\mathrm{d}l}{A}$$

is the transit time with the Alfven velocity in the magnetosphere along a field line from the ionosphere of the southern hemisphere ($l = l_-$) to the ionosphere of the northern hemisphere ($l = l_+$) and back. Note that (2) does not take into account the frequency differences Ω_N for eigenoscillations of the toroidal and poloidal types. However, the exact values of the frequencies of eigenoscillations of these types (Ω_{TN} and Ω_{PN}) are somewhat different:

$$\Omega_{TN} - \Omega_{PN} = \Delta\Omega_N \ll \Omega_N. \qquad (3)$$

The function $\bar{U}_N$ describes the field structure of the standing Alfven wave across the magnetic shells. The scale of its variation in coordinate x^1 is much smaller than the scale of variation of H_N in this coordinate. This lets us obtain the equation for $\bar{U}_N$ using the method of different scales. An exact equation for azimuthal harmonics with $m \gg \Omega_N/\Delta\Omega_N \gg 1$ was obtained by *Leonovich and Mazur* [1993]. *Leonovich and Mazur* [1997] obtained a model equation enabling $\bar{U}_N$ to be investigated almost throughout the entire range of wave numbers m (or k_2)

$$\alpha_T \nabla_1 \left[(\omega + i\gamma_N)^2 - \Omega_{TN}^2\right] \nabla_1 \bar{U}_N - \alpha_P k_2^2 \left[(\omega + i\gamma_N)^2 - \Omega_{PN}^2\right] \bar{U}_N = \bar{I}_N. \qquad (4)$$

Here we have introduced the following designations:

$$\alpha_T = \oint \sqrt{\frac{g_2}{g_1}} \frac{H_N^2}{A^2} \mathrm{d}l,$$

$$\alpha_P = \oint \sqrt{\frac{g_1}{g_2}} \frac{H_N^2}{A^2} \mathrm{d}l,$$

$$\gamma_N = \left(G_N^+ + G_N^-\right) / \alpha_T \omega^2; \quad G_N = \sqrt{\frac{g_2}{g_1}} v \left(\frac{\partial H_N}{\partial l}\right)^2,$$

$$I_N = j_N^+ - j_N^-; \quad j_N = 2\sqrt{g_1 g_2} \frac{\partial H_N}{\partial l} \frac{j_\parallel}{V},$$

$$v = \frac{c^2 \cos\chi}{4\pi \Sigma_p}, \quad V = \frac{\Sigma_p}{\cos\chi},$$

where Σ_p is the integral Pedersen conductivity of the ionosphere, χ is the angle of magnetic declination (see Figure 1), and $j_\parallel$ is the external longitudinal (along geomagnetic field lines) current density on the upper boundary of the ionosphere. The plus and minus signs imply that the quantities being considered are taken at points where a field line traverses the ionosphere of the northern and southern hemisphere, respectively. The quantity γ_N is the damping decrement of standing waves associated with their ohmic dissipation in the ionosphere (the dissipation is assumed to be weak, $\gamma_N \ll \Omega_N$). The function I_N, related to external currents on the upper boundary of the ionosphere, represents in this equation the source of the standing waves in the magnetosphere of interest (for more details, see *Leonovich and Mazur* [1996]). In the limiting case $m \gg \Omega_N/\Delta\Omega_N \gg 1$, (4) transforms to the equations that follows from a rigorous theory (eq.(84) and eq.(96) of *Leonovich and Mazur*, [1993]).

The frequencies of toroidal Ω_{TN} and poloidal Ω_{PN} eigenoscillations are functions of the magnetic shell (i.e., they depend on the coordinate x^1). As shown in our earlier work, the scale of localization of the waves under consideration in coordinate x^1, Δ_N, is much smaller than the typical scale of variation of the functions $\Omega_{TN}(x^1)$ and $\Omega_{PN}(x^1)$ in this coordinate, l_N. The region of localization of the waves lies between two resonant magnetic shells $x^1 = x^1_{TN}$ where the source frequency coincides with the toroidal eigenfrequency $\omega = \Omega_{TN}(x^1_{TN})$ and $x^1 = x^1_{PN}$, where $\omega = \Omega_{PN}(x^1_{PN})$:

$$\Delta_N = x^1_{TN} - x^1_{PN} \ll l_N.$$

In the region of localization of the wave the functions $\Omega_{TN}(x^1)$ and $\Omega_{PN}(x^1)$ can be linearized:

$$\begin{aligned} \Omega_{TN}(x^1) &\approx \omega\left(1 - \frac{x^1 - x^1_{TN}}{2l_N}\right), \\ \Omega_{PN}(x^1) &\approx \omega\left(1 - \frac{x^1 - x^1_{PN}}{2l_N}\right). \end{aligned} \qquad (5)$$

The value of the splitting of the resonance surfaces Δ_N is related to the polarization splitting of the spectrum $\Delta\Omega_N$ by the relation $\Delta_N = l_N \Delta\Omega_N/\omega$. Substituting (5) into (4) gives

$$\frac{\partial}{\partial\xi}(\xi + i\varepsilon)\frac{\partial \bar{U}_N}{\partial\xi} - k_y^2(\xi + 1 + i\varepsilon)\bar{U}_N = \bar{b}_N, \qquad (6)$$

where the following designations are used: $\xi = (x^1 - x^1_{TN})/\Delta_N$, $k_y = k_2\Delta_N\sqrt{\alpha_T/\alpha_P}$, $\varepsilon = 2\gamma_N l_N/\omega\Delta_N$, and $\bar{b}_N = \bar{I}_N l_N \Delta_N/\alpha_T\omega^2$. *Leonovich and Mazur* [1997] solved (6) for the case where $\bar{I}_N$ is independent of transverse coordinates (x^1, x^2).

In this paper we shall examine the case where the source $\bar{I}_N$ has an arbitrary structure in transverse coordinates. Since in (6) the coefficients are linear in ξ, in solving it we use the Fourier method by substituting $\bar{U}_N$ in the form

$$\bar{U}_N(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}_N(k)\mathrm{e}^{ik\xi}\mathrm{d}k. \qquad (7)$$

On substituting this expression into (6), we obtain for $\tilde{U}_N$ a first-order differential equation (with respect to k) which is readily solved. Substituting the resulting solution into (7) gives

$$\bar{U}_N(\xi) = \frac{i}{2\pi}\int_{-\infty}^{\infty}\bar{b}_N(\xi', k_y)\mathrm{d}\xi' \times$$
$$\int_{-\infty}^{\infty}\frac{\mathrm{e}^{i\Psi_N(\xi,k,k_y)}}{\sqrt{k^2+k_y^2}}\mathrm{d}k\int_{-\infty}^{k}\frac{\mathrm{e}^{-i\Psi_N(\xi',k',k_y)}}{\sqrt{k'^2+k_y^2}}\mathrm{d}k', \qquad (8)$$

where

$$\Psi_N(\xi,k,k_y) = k(\xi+i\varepsilon) + |k_y|\arctan\frac{k}{|k_y|}.$$

If $\bar{b}_N$ is independent of ξ, then the integral taken over ξ' leads to the $\delta(k')$ function, and the solution (8) becomes

$$\bar{U}_N(\xi) = i\frac{\bar{b}_N}{|k_y|}\int_0^{\infty}\frac{\mathrm{e}^{i\Psi_N(\xi,k,k_y)}}{\sqrt{k^2+k_y^2}}\mathrm{d}k,$$

as obtained by *Leonovich and Mazur* [1997].

The solution (8) describes the Fourier harmonic of the oscillations with a fixed value of the azimuthal wave number m (or k_2). If the source has an arbitrary structure in azimuthal coordinate, that is,

$$b_N(\xi,\eta) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{b}_N(\xi,k_y)\mathrm{e}^{ik_y\eta}\mathrm{d}k_y,$$

where $\eta = \sqrt{\alpha_T/\alpha_P}x^2/\Delta_N$, then a complete solution will have the form

$$U_N(\xi,\eta) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\mathrm{d}\eta'\int_{-\infty}^{\infty}b_N(\xi',\eta')V_N(\xi,\xi',\eta,\eta')\mathrm{d}\xi', \qquad (9)$$

where

$$V_N = i\int_{-\infty}^{\infty}\mathrm{d}k_y\int_{-\infty}^{\infty}\frac{\mathrm{e}^{i(\Psi_N(\xi,k,k_y)+k_y\eta)}}{\sqrt{k^2+k_y^2}}\mathrm{d}k \times$$
$$\int_{-\infty}^{k}\frac{\mathrm{e}^{-i(\Psi_N(\xi',k',k_y)+k_y\eta')}}{\sqrt{k'^2+k_y^2}}\mathrm{d}k'$$

represents the transverse structure of the Nth eigenharmonic of the standing waves excited by a source of the form $\delta(\xi-\xi_0)\delta(\eta-\eta_0)$, where (ξ_0,η_0) is the point of localization of the source.

3. Transverse Structure of Standing Waves From a Source Strongly Localized in One of the Transverse Coordinates

In order to appreciate qualitatively the structure of the solution obtained in (9), we consider two opposite limiting cases. Let b_N in (9) have the form $b_N = \bar{b}_N\delta(\xi-\xi_0)\mathrm{e}^{i\bar{k}_y\eta}$, where $\bar{b}_N$ is independent of ξ and η. Then

$$U_N = \bar{B}_N\int_{-\infty}^{\infty}\frac{\mathrm{e}^{i\Psi_N(\xi,k,\bar{k}_y)}}{\sqrt{k^2+\bar{k}_y^2}}\mathrm{d}k\int_{-\infty}^{k}\frac{\mathrm{e}^{-i\Psi_N(\xi_0,k',\bar{k}_y)}}{\sqrt{k'^2+\bar{k}_y^2}}\mathrm{d}k', \qquad (10)$$

where $\bar{B}_N = i\bar{b}_N\mathrm{e}^{i\bar{k}_y\eta}/2\pi$. Let $\bar{k}_y\Delta_N \gg 1$, then both in the inner integral over k' and in the outer integral over k the stationary phase method can be used in an approximate calculation [*Budden,* 1961]. The saddle points $\bar{k}'$ and $\bar{k}$ in these integrals are determined from the conditions $\partial\Psi_N(\xi_0,k',\bar{k}_y)/\partial k'|_{k'=\bar{k}'} = 0$ and $\partial\Psi_N(\xi,k,\bar{k}_y)/\partial k|_{k=\bar{k}} = 0$ and have the form $\bar{k}' = \pm\bar{k}_y\kappa_N(\xi_0)$ and $\bar{k} = \pm\bar{k}_y\kappa_N(\xi)$, where

$$\kappa_N = \sqrt{\frac{\xi+1}{-\xi}}. \qquad (11)$$

By successively taking these integrals, we get

$$U_N = 2\pi\frac{\omega}{l_N}\bar{B}_N\left[v_N v_N^0(\bar{k}'^2+\bar{k}_y^2)(\bar{k}^2+\bar{k}_y^2)\right]^{-1/2}\times$$
$$\left\{\theta(\xi-\xi_0)\mathrm{e}^{i\Psi_N^0} + i\mathrm{e}^{-i(\Psi_N+\Psi_N^0)} + \theta(\xi_0-\xi)\mathrm{e}^{i(\bar{\Psi}_N-\Psi_N^0)}\right\},$$

where $\theta(\xi)$ is the unit step Heaviside function;

$$v_N(\xi,\omega) = \frac{\omega}{\bar{k}_y}\frac{\Delta_N}{l_N}(\xi+1)^{1/2}\xi^{3/2}$$

is the group velocity of the Nth harmonic of the standing waves in coordinate ξ, $v_N^0 = v_N(\xi_0,\omega)$, $\Psi_N = \Psi_N(\xi,\bar{k},\bar{k}_y)$, and $\Psi_N^0 = \Psi_N(\xi_0,\bar{k}',\bar{k}_y)$. Inside the region of localization of the wave ($-1 \le \xi; \xi_0 \le 0$) the first term in braces of expression U_N describes the wave traveling from the point of localization of the source ($\xi = \xi_0$) to the poloidal resonance surface ($\xi = -1$), the second term represents the wave reflected from the poloidal surface and traveling to the toroidal resonance surface ($\xi = 0$), and the third term characterizes the wave traveling from the point of localization of the source to the toroidal surface.

Another limiting case is the source localized azimuthally at the point $\eta = \eta_0$: $\quad b_N = \bar{b}_N \delta(\eta - \eta_0)$. For such a source, we have

$$\nabla_2 U_N = -i\bar{B}_{N1} \int_0^\infty \left[\frac{1}{\Psi_N(\xi,\kappa) + \eta - \eta_0} + \frac{1}{\Psi_N(\xi,\kappa) - \eta + \eta_0} \right] \frac{d\kappa}{\sqrt{1+\kappa^2}} \qquad (12)$$

being a function defining the radial structure of the B_1 and E_2 components of the wave field (see (1)). Here $\bar{B}_{N1} = i\bar{b}_N \sqrt{\alpha_T/\alpha_P}/2\pi\Delta_N$, and it is designated

$$\Psi_N(\xi,\kappa) = \kappa(\xi + i\varepsilon) + \arctan\kappa.$$

The integral (12) is obtained from (8) by successive integration over η', ξ', k' and k_y and by changing the integration variable $\kappa = k/|k_y|$. The function $\nabla_1 U_N$ that defines the radial structure of the B_2 and E_1 components of the field differs from (12) by the presence of an additional term κ in the integrand and by the amplitude $\bar{B}_{N2} = \sqrt{\alpha_P/\alpha_T}\bar{B}_{N1}$.

On the basis of the structure of the integrand of (12) it can be said that the main contribution to the integral is made by the origin of the integration path $0 \le \kappa \le \bar{\kappa} \ll 1$ and of the points $\kappa_\pm$ where there are singularities defined by the zeroes of the denominators $\Psi_N(\xi,\kappa_\pm) \pm (\eta - \eta_0) = 0$. When $\kappa < \bar{\kappa}$, an approximate expression $\Psi_N \approx (\xi+1+i\varepsilon)\kappa$ holds good for Ψ_N, and the integral for $\kappa \le \bar{\kappa}$ is readily evaluated

$$\nabla_2 U_N|_{\kappa\le\bar{\kappa}} \approx \frac{-i\bar{B}_{N1}}{\xi+1+i\varepsilon} \ln \frac{(\eta-\eta_0)^2 - \bar{\kappa}^2(\xi+1+i\varepsilon)^2}{(\eta-\eta_0)^2}.$$

This expression involves a singularity when $\eta = \eta_0$ which is not regularized by the presence of the dissipation ε. For this singularity to be regularized, it is sufficient to "spread" somewhat the source structure in coordinate η using, instead of the $\delta(\eta-\eta_0)$ function in the expression for b_N, the function

$$b_N(\eta) = \bar{b}_N \frac{\Delta}{(\eta-\eta_0)^2 + \Delta^2},$$

which yields $\bar{b}_N \delta(\eta - \eta_0)$ when $\Delta \to 0$. Using such a source model leads in all preceding calculations to the substitution $\eta - \eta_0 \to \eta - \eta_0 + i\Delta$.

As far as the two singularities of the integrand are concerned, it can be shown that they represent the poles at the points $\kappa = \kappa_N(\xi)$, where $\kappa_N(\xi)$ is defined by (11) and the coordinate ξ lies on characteristics defined by the equation

$$\frac{d\eta}{d\xi} = \pm\kappa_N(\xi). \qquad (13)$$

Thus (12) will be determined by the contribution of the region of localization of the source $\eta = \eta_0$ and by the waves propagating from such a source on characteristics of (13).

On the basis of these two limiting cases one would expect the following distribution pattern of the field of oscillations excited by the source localized in both transverse coordinates. A local maximum in the distribution of the oscillation amplitude must be observed at the point of localization of the source (ξ_0, η_0). If the source lies inside the region of localization of the standing wave ($-1 \le \xi_0 \le 0$), then the resonance surface must show amplitude maxima at points of their intersection by characteristics passing through the point of localization of the source. Also, since two characteristics corresponding to different signs in (13) pass through a given point (ξ_0, η_0), local maxima on resonance surfaces will be symmetric about $\eta = \eta_0$.

4. Transverse Structure of Standing Alfven Waves From a Localized Source

In section 3 we have analyzed qualitatively the propagation of standing Alfven waves from the source in the ionosphere localized in one of the transverse coordinates x or y. In this section we shall integrate numerically the solution (9) for the case where the source is strongly localized in both transverse coordinates. For the sake of definiteness, we shall consider the transverse propagation of the waves projected onto the ionosphere of the northern hemisphere. Since the source is strongly localized, the integration over ξ' and η' in (9) is transferred entirely to the source function $b_N(\xi', \eta')$, and in the expression for V_N the substitution $\xi' \to \xi_0$ and $\eta' \to \eta_0$ is made. As a result, the expressions for the components of the field of the generated waves projected onto the ionosphere have the form

$$E_{XN} = E_N \frac{I_1 \cos\chi}{\sqrt{g_1^{(i)}}}, \qquad E_{YN} = \sqrt{\frac{\alpha_T}{\alpha_P}} E_N \frac{I_2}{\sqrt{g_2^{(i)}}}, \qquad (14)$$

$$B_{XN} = \sqrt{\frac{\alpha_T}{\alpha_P}} B_N \frac{I_2 \cos\chi}{\sqrt{g_2^{(i)}}}, \qquad B_{YN} = -B_N \frac{I_1}{\sqrt{g_1^{(i)}}},$$

where $E_N = A_N H_N^{(i)}$ and $B_N = icA_N(\partial H_N/\partial l)^{(i)}/\omega$ are the characteristic amplitudes of the electric and magnetic fields of the oscillations,

$$A_N = 2\frac{l_N}{\omega^2}\left(\frac{\partial H_N}{\partial l}\right)^{(i)} \frac{\cos\chi}{\sqrt{\alpha_P\alpha_T}\Delta_N^2} \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \frac{j_\parallel(x,y)}{\Sigma_P} \mathrm{d}x\mathrm{d}y,$$

and the index (i) implies that the respective quantities are taken on the upper boundary of the ionosphere.

The coordinates x and y have on the ground the directions from south to north and from west to east, respectively (see Figure 1). They are related to the dimensionless coordinates ξ and η by the relations $x = \xi\Delta_N^{(i)}$ and $y = \eta\sqrt{\alpha_P/\alpha_T}\Delta_N^{(i)}\cos\chi$, where $\Delta_N^{(i)} = \Delta_N\sqrt{g_1^{(i)}}/\cos\chi$ is the distance between the resonance shells projected onto the ionosphere. The dimensionless functions $I_1(\xi,\eta)$ and $I_2(\xi,\eta)$ describe the distribution of the oscillation field in dimensionless coordinates ξ and η:

$$I_1 = \int_{-\infty}^{\infty} \frac{\kappa\mathrm{d}\kappa}{\sqrt{1+\kappa^2}} \times \int_{-\infty}^{\kappa} \frac{\mathrm{d}\kappa'}{\sqrt{1+\kappa'^2}} \left[\frac{1}{(\tilde\eta+\eta+i\Delta)^2} + \frac{1}{(\tilde\eta-\eta+i\Delta)^2}\right],$$

$$I_2 = \int_{-\infty}^{\infty} \frac{\mathrm{d}\kappa}{\sqrt{1+\kappa^2}} \times \int_{-\infty}^{\kappa} \frac{\mathrm{d}\kappa'}{\sqrt{1+\kappa'^2}} \left[\frac{1}{(\tilde\eta+\eta+i\Delta)^2} - \frac{1}{(\tilde\eta-\eta+i\Delta)^2}\right],$$

where

$$\tilde\eta = \kappa(\xi+i\varepsilon) - \kappa'(\xi_0+i\varepsilon) + \arctan\kappa - \arctan\kappa',$$

ξ_0 is the shell, on which the source is localized, Δ is the characteristic scale of its localization in coordinate η, and it is assumed that $\eta_0 = 0$.

In numerical calculations we used a dipole model of the geomagnetic field, in which a length element along a field line has the form

$$\mathrm{d}l = a\cos\theta\sqrt{1+3\sin^2\theta}\mathrm{d}\theta,$$

where a is the equatorial radius of the field line and θ is the angle between the radius of the point on the field line and the equatorial plane. The geomagnetic latitude Φ and the equatorial radius a are related by the relation $\cos^2\Phi = a/r_i$, where r_i is the radius of the ionosphere. The angle of magnetic declination is defined as

$$\chi = \arccos\frac{\sqrt{1-r_i/a}}{\sqrt{4-3r_i/a}}.$$

The metric tensor components in dipole coordinates have the form

$$g_1 = \cos^6\theta(1+3\sin^2\theta)^{-1}, \quad g_2 = a^2\cos^6\theta.$$

In addition, we used the following model for the Alfven velocity distribution in the meridional plane $A = A_0(a_0/a)^\mu[\beta(\theta)]^\nu$, where $\beta = (1+3\sin^2\theta)^{1/2}/\cos^6\theta$. This model neglects an abrupt velocity jump on the plasmapause, but it reproduces reasonably well the behavior of A in the greater part of the dayside magnetosphere with the following choice of the parameters: $A_0 = 10^3$ km/c, $a_0 = 4R_E = 2.5\times10^4$ km, $\mu = 3/2$, and $\nu = 1/4$. The typical scale of variation of the functions Ω_{TN} and Ω_{PN} is defined in this case as $l_N = (\partial\ln t_A/\partial a)^{-1} = a/\mu$. The small parameters that regularize the solutions were taken to be $\varepsilon = 10^{-1}$ and $\Delta = 10^{-1}$, which implies a small dissipation of the waves in the ionosphere and the smallness of the scale of localization of the source compared with the scale of localization of the wave Δ_N.

Results of our calculations are presented in Figures 2 and 3. Figure 2 shows the amplitude distributions of the electric field of the oscillations $E_t = \sqrt{|E_{XN}|^2+|E_{YN}|^2}$ in coordinates ξ and η, with the typical unit amplitude of the oscillations $|E_N| = 1$. It should be noted that in the zero order of the WKB approximation (2) the equality $H_N^{(i)} \equiv H_N(x, l_\pm, \omega) = 0$ holds for the ionosphere, so that $|E_N| = 0$ must hold. However, this expression has only an approximate value accurate to small parameters determined by a small dissipation of the waves in the ionosphere. An exact value of $|E_N| \neq 0$. Four different cases of the location of the oscillation source are considered: in the opacity region behind the toroidal resonance surface ($\xi_0 = 1$, Figure 2a), on the toroidal surface ($\xi_0 = 0$, Figure 2b), on the poloidal resonance surface ($\xi_0 = -1$, Figure 2c), and in the opacity region behind the poloidal surface ($\xi_0 = -2$, Figure 2d).

Figur

In the four cases the toroidal surface ($\xi = 0$) and the point of localization of the source ($\xi = \xi_0$ and $\eta = 0$) manifest themselves quite clearly. This can be understood on the basis of our earlier papers, *Leonovich and Mazur* [1993, 1997]. In the case of a small dissipation of the excited waves their amplitude on the toroidal resonance surface is much larger than their amplitude on the poloidal surface. Besides, for harmonics with $m < l_N/\Delta_N$ the poloidal surface is not distinguished by anything at all, whereas on the toroidal surface the oscillation amplitude has a singularity of the type of field line resonance regularized by the small parameter ε. At the point of localization of the source the amplitude also has a singularity regularized by the small parameter Δ . By increasing these parameters, it is possible to decrease the oscillation amplitude in these regions making it comparable with the amplitude on the poloidal surface. In this case, however, the overall amplitude of the oscillations will become so small that it will be next to impossible to observe them.

A further distinctive feature of Figure 2 is that when the source is in the transparency region of the wave ($-1 \leq \xi_0 \leq 0$), the oscillation amplitude is much larger compared with the source located in opacity regions. This means that harmonics with $m \gg l_N/\Delta_N \gg 1$ have the decisive role in the oscillations excited by a strongly localized source. This is also indicative of the presence of two local maxima on the toroidal surface when the source is localized on the poloidal surface. As follows from results reported in section 3, these maxima are associated with the waves propagating from the source along the characteristics described by (13). Just the oscillations with $m \gg l_N/\Delta_N$ are running waves and can be traveled along the characteristics (13). Therefore these oscillations, being excited inside the transparency region, can reach resonance surfaces without decrease of the amplitude. When the source finds itself on the toroidal surface, the length of the characteristics becomes zero, and the three maxima merge to one. When the source of oscillations is located in the opacity region, the amplitudes of harmonics with $m \gg l_N/\Delta_N$ exponentially decrease in the direction to transparency region and they practically do not contribute to common oscillation amplitude. In this case there is only contribution of harmonics with $m \leq l_N/\Delta_N$, which have structure of a type field line resonance and are not traveled across magnetic shells.

Figure 3 presents the hodographs of the oscillations for three cases of the source location inside the transparency region: on the toroidal surface ($\xi_0 = 0$, Figure 3a), between the resonance surfaces ($\xi_0 = -0.5$, Figure 3b), and on the poloidal surface ($\xi_0 = -1$, Figure 3c). Polarization ellipses are presented on the background of a map of lines of equal amplitude level, similar to the one in Figure 2. By representing the components of the electric field of the oscillations obtained by a numerical integration as $E_X = R_X e^{i(\varphi_x - \omega t)}$ and $E_Y = R_Y e^{i(\varphi_y - \omega t)}$, oscillation hodographs can be constructed from the real parts of these expressions

$$\begin{aligned} \mathrm{Re} E_X &= R_X \cos\alpha, \\ \mathrm{Re} E_Y &= R_Y \cos(\alpha + \Delta\varphi), \end{aligned}$$

where $\alpha = \varphi_x - \omega t$ is the changing phase, $\Delta\varphi = \varphi_y - \varphi_x$. Noteworthy in Figure 3 is primarily a clear manifestation of the toroidal polarization of the oscillations on the toroidal surface ($|E_X| \gg |E_Y|$). At the same time the polarization of the oscillations in the region of localization of the source has a tendency to a poloidal polarization ($|E_X| \lesssim |E_Y|$). When the source is located inside the transparency region, the polarization tends to the linear polarization toward the toroidal surface and to the circular polarization toward the poloidal surface.

5. The Possibility of Measuring the Polarization Splitting of the Spectrum of Alfven Eigenoscillations of the Magnetosphere

The artificial stimulation of geomagnetic pulsations through a periodic modification of the ionosphere (with the frequency of Alfven eigenoscillations of the magnetosphere) could be used to measure the splitting of the resonant shells of the magnetosphere $\Delta_N^{(i)}$ and the related polarization splitting of the spectrum $\Delta\Omega_N = \Omega_{TN} - \Omega_{PN}$. Of course, this requires setting up a sufficiently dense observation network (spaced by $\Delta_N^{(i)}/2$, $\Delta_N^{(i)}/4$) in the neighborhood of the oscillation source. Yet this is a distinct possibility if an HF radar is used in observations, as done in a paper by *Walker et al.* [1982]. If the region of ionospheric modification is much smaller than $\Delta_N^{(i)}$, the theory suggested in this paper can be used to describe the field of the excited oscillations.

The most likely occurrence in such an experiment is the penetration of the source into the transparency region between the resonance surfaces. When the source is located in the opacity region, the amplitude of the excited oscillations will be insufficiently large for observations, and the location exactly on the resonance surfaces is unlikely. When the source is in the transparency region, three local maxima of the oscillation amplitude must be observed: one which is associated with the oscillation source and the other two which are symmetric about $y = 0$, on the toroidal surface (see Figures 3b,3c). The distance between the maxima is readily calculated by the equation of characteristics (13). Upon integrating it, we obtain

$$\Delta\eta = 2\int_0^{\Delta\eta/2} \mathrm{d}\eta = 2\int_{\xi_0}^{0} \kappa_N(\xi)\mathrm{d}\xi = 2\left(\sqrt{-\xi_0(1+\xi_0)} + \arcsin\sqrt{-\xi_0}\right). \quad (15)$$

Equation (15) is useful for determining the value of the splitting between the poloidal and toroidal surfaces $\Delta_N^{(i)}$, based on experimentally measured distances between local peaks on the toroidal surface $\Delta_{YN} = \sqrt{\alpha_T/\alpha_P}\Delta\eta\Delta_N^{(i)}$ and from the point of localization of the source to the toroidal surface $\Delta_0 = -\xi_0\Delta_N^{(i)}$. We have

$$\Delta_{YN} = 2\sqrt{\frac{\alpha_T}{\alpha_P}}\left(\sqrt{\Delta_0(\Delta_N^{(i)} - \Delta_0)} + \Delta_N^{(i)} \arcsin\sqrt{\Delta_0/\Delta_N^{(i)}}\right), \quad (16)$$

where $0 < \Delta_0 < \Delta_N^{(i)}$. Equation (16) implicitly defines the value of the splitting of the resonance surfaces $\Delta_N^{(i)}$. When the source is located on the toroidal surface ($\Delta_0 = 0$) $\Delta_{YN} = 0$; that is, all local maxima merge into one. When it is located on the poloidal surface ($\Delta_0 = \Delta_N^{(i)}$), we have $\Delta_{YN} = \pi\Delta_N^{(i)}$; the distance between the maxima on the toroidal surface is largest (the ratio $\alpha_T/\alpha_P \approx 1$ at all geomagnetic latitudes).

Upon determining $\Delta_N^{(i)}$ by this means, it is also possible to infer the polarization splitting of the spectrum

$$\Delta\Omega_N = \Omega_{TN} - \Omega_{PN} = \frac{\Delta_N^{(i)}}{l_N^{(i)}}\omega, \quad (17)$$

where $l_N^{(i)}$ is the typical scale of the magnetospheric plasma inhomogeneity across the magnetic shells projected onto the ionosphere. In our case it is the typical scale of variation of the functions $\Omega_{TN}(x)$ and $\Omega_{PN}(x)$.

4 and 5

To give an idea of the value of the parameters under investigation, Figures 4 and 5 plot the calculated values of $\Delta_N^{(i)}$ and Ω_{TN} versus geomagnetic latitude Φ for the first five harmonics of the eigenoscillations of the magnetosphere. These calculations were done in terms of a model of the medium reported in this paper on the basis of equations obtained by *Leonovich and Mazur* [1993]. It must be emphasized once again that the computational model under consideration neglects an abrupt change of magnetospheric parameters at the plasmapause. Therefore the results presented in Figures 4 and 5 cannot be used in estimating the parameters of our concern directly in the neighborhood of the plasmapause. However, at the distance $\pm(3°\text{-}5°)$ in latitude from the plasmapause location, the above results give reasonably correct estimates of the mean values of $\Delta_N^{(i)}$ and Ω_{TN} both in the outer magnetosphere and in the plasmasphere.

Noteworthy in Figure 4 is an anomalously large value of $\Delta_1^{(i)} \approx 800$ km compared with other $\Delta_N^{(i)}$ $(N = 2,3,4,5) \sim 10 - 50$ km of the oscillations. This furnishes an opportunity to use a less dense observation network and possibly even ground-based measuring facilities. However, the amplitude of this harmonic must be smaller than that of the other harmonics, with the equal power of the source, because $E_N \sim \Delta_N^{-2}$. It follows from Figure 5 that the excitation of eigenoscillations is the easiest to accomplish at low latitudes in view of an increase in their eigenfrequency because the monochromatic source must operate during a reasonably large number of periods.

Conclusion

The main results of this work may be summarized as follows.

1. We have obtained the solution of MHD equations describing the spatial structure of monochromatic standing Alfven waves in the magnetosphere excited by a source localized in the ionosphere.

2. The behavior of the waves excited by the source localized in one of the transverse coordinates has been studied qualitatively. It has been shown that the main part of the wave packet from a localized source propagates along characteristics described by (13).

3. A numerical study was made of the transverse structure of standing Alfven waves excited by a transversely localized monochromatic source in the ionosphere. It has been shown that local maxima must be observed in the oscillation amplitude distribution across field lines. If the oscillation source lies between the poloidal and toroidal resonance surfaces, where the oscillation amplitude is maximal, three local maxima must be observed: one in the source localization region and the other two on the toroidal resonance surface which are related to the wave source by the characteristics of (13).

4. It has been shown that it is possible to measure experimentally the polarization splitting of the spectrum of standing Alfven waves into toroidal and poloidal eigenmodes. When the three local maxima in the oscillation amplitude distribution on the ionospheric surface are observed, we can measure the distance between the maxima on the toroidal surface, Δ_{YN}, and the distance from the source-associated maximum to this toroidal surface, Δ_0. Then it is possible to determine (equation (16)) the splitting between the poloidal and toroidal resonance surfaces projected onto the ionosphere, $\Delta_N^{(i)}$. After that, using (17) one can determine the polarization splitting of the spectrum of magnetospheric Alfven eigenoscillations between the poloidal and toroidal modes $\Delta\Omega_N$.

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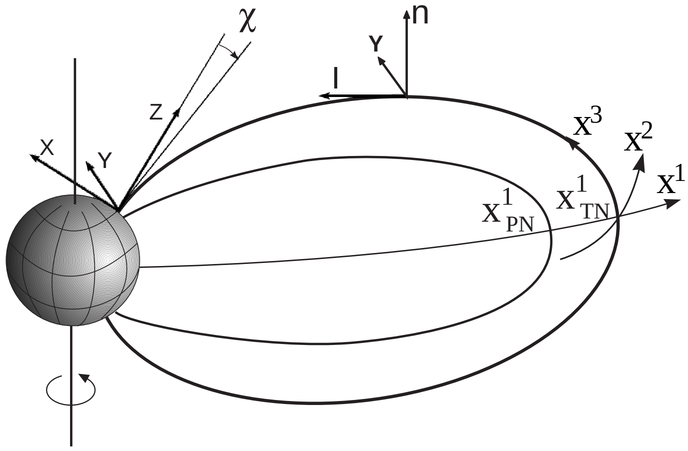

Figure 1. Three coordinate systems connected with geomagnetic field lines: (x^1, x^2, x^3), curvilinear orthogonal coordinate system; (n, y, l), local Cartesian coordinate system in the magnetosphere; and (x, y, z), local Cartesian coordinate system near the ionosphere.

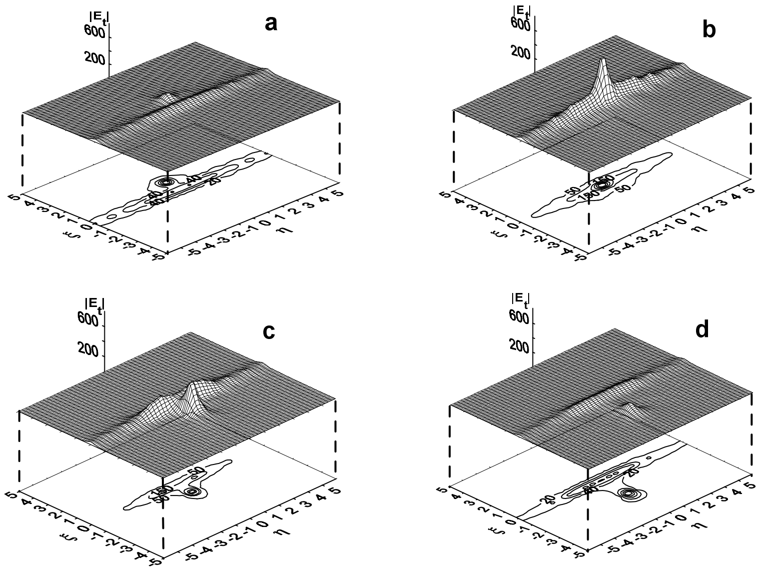

Figure 2. Amplitude distribution of standing Alfven waves excited by a strongly localized monochromatic source in dimensionless radial (ξ) and azimuthal (η) coordinates. Four possible cases of the source location are considered: (a) in the opacity region behind the toroidal surface, (b) on the toroidal resonance surface, (c) on the poloidal resonance surface, and (d) in the opacity region behind the poloidal surface.

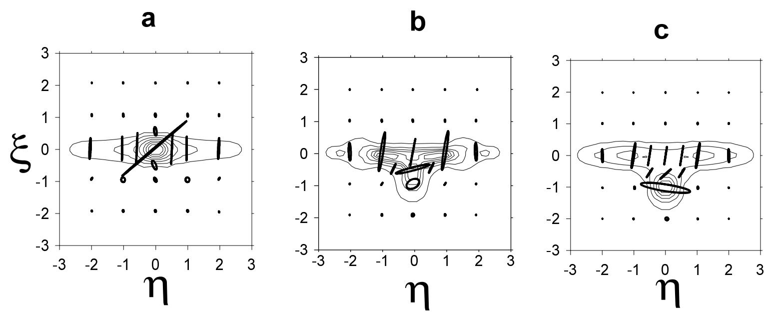

Figure 3. Distribution of hodographs of standing Alfven waves (in electric field oscillations) in the plane of the dimensionless radial (ξ) and azimuthal (η) coordinates in the ionosphere: (a) source on the toroidal resonance surface ($\xi_0 = 0$), (b) source between the poloidal and toroidal resonance surfaces ($\xi_0 = -0.5$), and (c) source on the poloidal resonance surface ($\xi_0 = -1$).

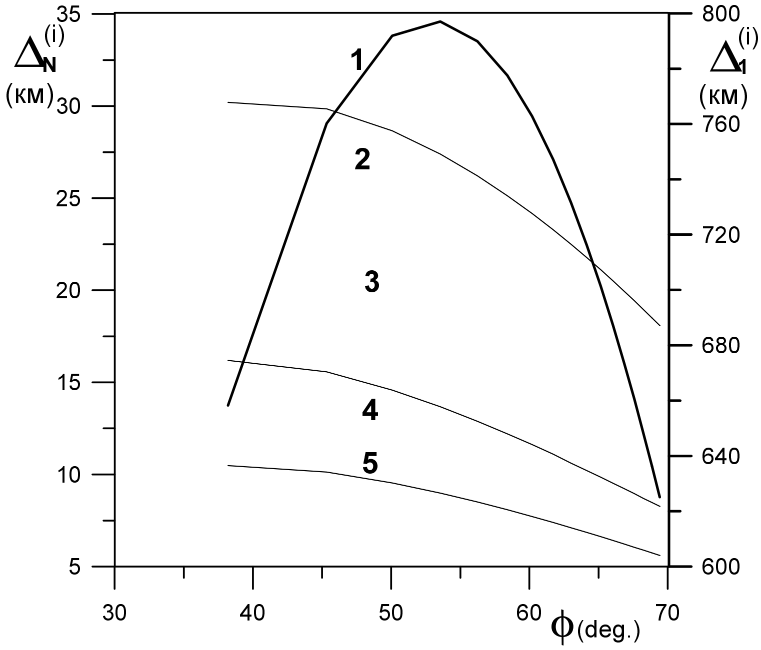

Figure 4. Dependence of the calculated splitting between the toroidal and poloidal resonance magnetic shells on the ionosphere as a function of geomagnetic latitude for the fundamental $\Delta_1^{(i)}$ (axis at the right) and the next four $\Delta_N^{(i)}$ $(N = 2, 3, 4, 5)$ (axis at the left) harmonics of standing Alfven waves.

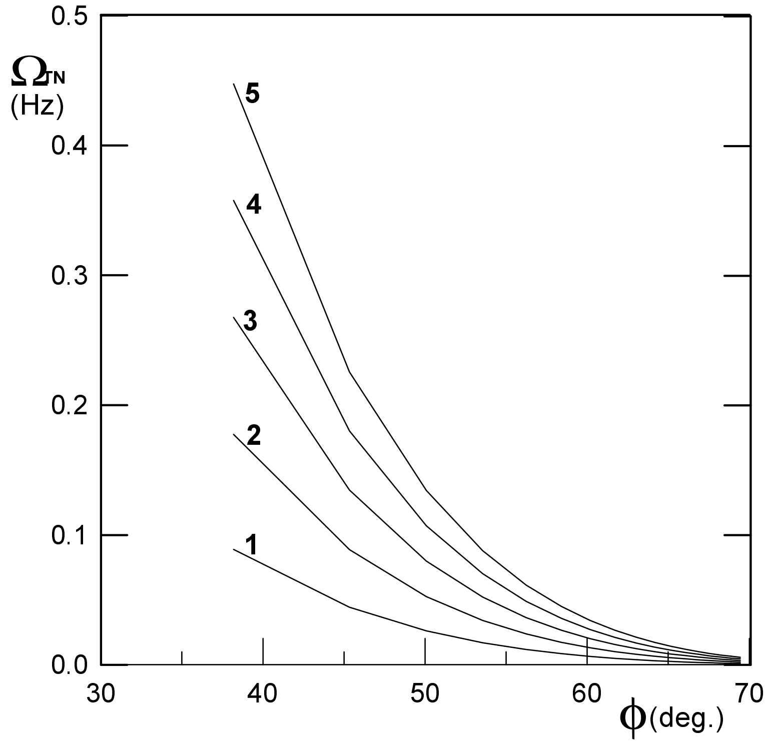

Figure 5. Dependence of calculated eigenfrequencies of toroidal oscillations Ω_{TN} on the geomagnetic latitude for the first five eigenharmonics of standing Alfven waves (N=1,2,3,4,5).