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THE SPATIAL STRUCTURE OF POLOIDAL ALFVEN OSCILLATIONS OF AN AXISYMMETRIC MAGNETOSPHERE

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Abstract—Based on equations of ideal magnetohydrodynamics a theoretical study is made of the spatial structure of poloidal Alfvén oscillations of the axisymmetric magnetosphere with large but finite values of azimuthal wave number m. The study has revealed a transverse dispersion of such oscillations which is attributable to the curvature of geomagnetic field lines that determines the fine transverse structure of the oscillation field. Solutions for different cases of the behaviour of magnetospheric parameters are obtained. By taking the summation over harmonics with different m, we have constructed a solution localized in both transverse coordinates which may be treated as the oscillation of a thin flux tube.

1. INTRODUCTION

Since the first work by Dungey (1954), when investigating MHD oscillations of the axisymmetric magnetosphere, it has been customary to distinguish two particularly simple kinds of natural Alfvén oscillations: toroidal and poloidal modes. To the former ones there corresponds the value of azimuthal wave number $m = 0$. A disturbed electric field in the toroidal mode oscillates in the direction normal (radial) to the field line and the magnetic field and the plasma oscillate in the azimuthal (binormal) direction. Poloidal modes have extremely large values, $m = \infty$. The electric field in them oscillates azimuthally, and the magnetic field and the plasma oscillate normally. In the range $m = 0$ and $m = \infty$ the equations of ideal magnetohydrodynamics split, thus allowing simple solutions to be obtained for Alfvén eigenmodes. These solutions are standing Alfvén waves enclosed between magnetoconjugate ionospheres and concentrated on certain, so-called resonance magnetic surfaces. This means that their eigenfunctions are δ-functions in the direction perpendicular to magnetic shells. This permits us to treat toroidal modes as torsional oscillations of separate magnetic shells and poloidal modes as radial oscillations of individual field lines. The longitudinal (along a field line) structure of standing waves and frequency spectra are determined by corresponding one-dimensional problems for eigenvalues (Radoski, 1967; Radoski and Carovillano, 1969; Cummings *et al.*, 1969; Krylov *et al.*, 1981; Krylov and Lifshitz, 1984). In papers by Walker (1987) and Taylor and Walker (1987) an analysis was made of the influence of finite plasma pressure upon the longitudinal structure of the poloidal Alfvén wave ($m = \infty$). When $\beta \sim 1$, the strong connection between the Alfvén wave and slow magnetosound makes this influence fundamentally important. At the same time the transverse structure of the wave field remains unchanged; the wave field is also concentrated on the resonance surface.

The singularity of eigenfunctions of toroidal and poloidal modes on resonance shells is caused by an excessive idealization and simplification of the problem statement. Taking account of effects beyond the scope of ideal magnetohydrodynamics and considering finite values of m (i.e. $m \neq 0$ and $m \neq \infty$) removes the singularity, thus permitting the fine structure of the oscillation field to be determined. Such an investigation was carried out for Alfvén oscillations with relatively small values of m which have a nearly toroidal character. A first attempt in this direction was made in fundamental papers by Southwood (1974) and Chen and Hasegawa (1974). The subsequent development of the theory was outlined in a review by Southwood and Hughes (1983; and references therein). At that stage the investigation was confined to a very simple magnetosphere model in the form of a flat plasma layer located in a homogeneous magnetic field. In a number of later papers (Lifshitz and Fedorov, 1986; Southwood and Kivelson, 1986) attempts were made to consider oscillations with finite values of m in a curvilinear magnetic field and in a plasma which is inhomogeneous in both the radial and longitudinal directions. This problem was solved in a most thorough and consistent way in papers by Leonovich and Mazur (1989a,b).

As far as poloidal Alfvén oscillations are concerned, their fine transverse spatial structure has not yet been investigated. Meanwhile, without solving this issue, the conclusion itself about the existence of poloidal modes cannot be regarded as justified. The point here is this. The polarization of a poloidal mode assumes that the transverse component of its wave vector is directed azimuthally. It would seem that the limit $m = \infty$ which implies that the azimuthal component of wave vector κ_φ is equal to infinity, ensures this statement; but since the mode is concentrated on a certain magnetic surface, the normal component of wave vector κ_n is also equal to infinity so that the question of the direction of the full transverse component remains open. In this lies the substantial difference of the poloidal mode from the toroidal one. For the latter one, in the zeroth-order approximation $\kappa_\varphi = 0$ and $\kappa_n = \infty$, which agrees quite well with its polarization. Taking account of the various effects determining its fine structure in the direction across field lines leads to finite but small values of κ_φ and to finite but large values of κ_n. In this case the inequality $\kappa_\varphi \ll \kappa_n$ is satisfied, which ensures the toroidal character of the mode.

Thus, there is a pressing need to investigate the fine transverse structure of the poloidal mode. It appears that in this case it suffices to consider large, but finite values of m in the approximation of ideal magnetohydrodynamics. In this also lies the difference of the poloidal mode from the toroidal one, and investigating its transverse structure requires taking account of effects beyond the scope of ideal magnetohydrodynamics.

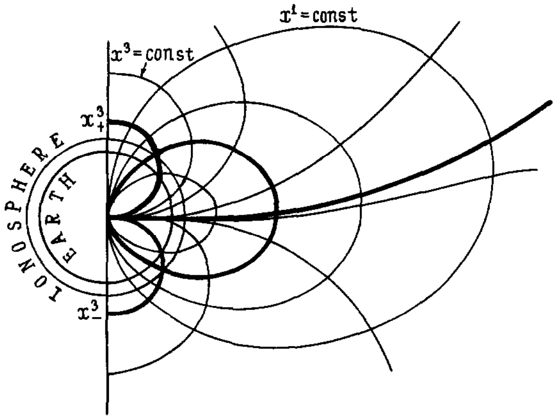

FIG. 1. THE CURVILINEAR ORTHOGONAL COORDINATE SYSTEM (x^1, x^3) IN THE MERIDIONAL PLANE (x^2 = CONST.).
The possible North–South asymmetry of the magnetosphere is especially stressed. The figure also shows: the equatorial line which is a separatrix for curves x^3 = const., one of the magnetic shells and coordinate lines x^3 = const., corresponding to the intersection of this shell with the ionosphere in the Northern (x^3_+) and Southern (x^3_-) Hemispheres.

2. THE DIFFERENTIAL EQUATION FOR POLOIDAL ALFVEN OSCILLATIONS

For describing the axisymmetric magnetosphere, we shall be using an orthogonal curvilinear coordinate system x^1, x^2, x^3 where x^1 = const. coincides with magnetic shells, the coordinate x^2 specifies a field line on a given shell, and x^3 specifies a point on a given field line (see Fig. 1). By x^3_+ and x^3_- we denote the coordinates of intersection of the field line with the ionosphere of conjugate hemispheres. These quantities are functions of the magnetic shell: $x^3_\pm = x^3_\pm(x^1)$. Disturbances in the wave will be described by covariant components of the vector of a disturbed magnetic field

$$B_i = \tilde{B}_i(x^1, x^3)\exp(\mathrm{i}\kappa_2 x^2 - \mathrm{i}\omega t),$$
$$i = 1, 2, 3,$$

where ω is the frequency of the wave, and κ_2 is the covariant azimuthal component of the wave vector. If the azimuthal angle φ is chosen as x^2, then $\kappa_2 = m$ is an azimuthal (integer) wave number. For making order-of-magnitude estimates, we shall also use typical values of the other two components of wave vectors κ_1 and κ_3 which are equal to inverse values of typical spatial scales of the oscillation in the coordinates x^1 and x^3. Note that the respective components of the wave vector in a local Euclidean basis (i.e. physical components) are given by the relationships $\hat{\kappa}_i = \kappa_i/\sqrt{g_i}$, where g_i represents diagonal components of the metric tensor. Note further that the quantities employed in the Introduction are $\kappa_n = \hat{\kappa}_1$ and $\kappa_\varphi = \hat{\kappa}_2$.

The system of equations describing disturbed fields of a monochromatic wave have the form

$$\operatorname{curl}\mathbf{E} = \mathrm{i}\frac{\omega}{c}\mathbf{B}, \quad \operatorname{curl}\mathbf{B} = -\mathrm{i}\frac{\omega}{c}\hat{\varepsilon}\mathbf{E}, \qquad (1)$$

where $\hat{\varepsilon}$ is the tensor of dielectric permittivity of plasma. In the approximation of ideal magnetic hydrodynamics this tensor is a diagonal one, and in a local Euclidean basis

$$\varepsilon_{11} = \varepsilon_{22} = \frac{c^2}{A^2}, \quad \varepsilon_{33} = \infty, \qquad (2)$$

where $A = B_0/\sqrt{4\pi n m_i}$ is the Alfvén velocity. In an axisymmetric magnetospheric model we have $A = A(x^1, x^3)$. From (1) and (2) follows a system of equations for a disturbed magnetic field

$$\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}A^2\frac{g_2}{\sqrt{g}}\frac{\partial \tilde{B}_1}{\partial x^3}+\omega^2\tilde{B}_1=\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}A^2\frac{g_2}{\sqrt{g}}\frac{\partial \tilde{B}_3}{\partial x^1}, \tag{3a}$$

$$\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3}A^2\frac{g_1}{\sqrt{g}}\frac{\partial \tilde{B}_2}{\partial x^3}+\omega^2\tilde{B}_2=\mathrm{i}\kappa_2\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3}A^2\frac{g_1}{\sqrt{g}}\tilde{B}_3, \tag{3b}$$

$$\frac{g_3}{\sqrt{g}}\frac{\partial}{\partial x^1}A^2\frac{g_2}{\sqrt{g}}\frac{\partial \tilde{B}_3}{\partial x^1}-\frac{\kappa_2^2}{g_2}A^2\tilde{B}_3+\omega^2\tilde{B}_3 =\frac{g_3}{\sqrt{g}}\frac{\partial}{\partial x^1}A^2\frac{g_2}{\sqrt{g}}\frac{\partial \tilde{B}_1}{\partial x^3}+\mathrm{i}\frac{\kappa_2}{g_2}A^2\frac{\partial \tilde{B}_2}{\partial x^3}, \tag{3c}$$

where $g = g_1g_2g_3$ is the determinant of the metric tensor. Instead of one of the equations of the system (3), one can use the identity div $\mathbf{B} = 0$ (which follows from this system). In curvilinear coordinates it is of the form

$$\frac{\partial}{\partial x^1}\frac{\sqrt{g}}{g_1}\tilde{B}_1+\mathrm{i}\kappa_2\frac{\sqrt{g}}{g_2}\tilde{B}_2+\frac{\partial}{\partial x^3}\frac{\sqrt{g}}{g_3}\tilde{B}_3=0. \tag{4}$$

If $\tilde{B}_2$ is expressed from it and is substituted into (3c), then a simple manipulation yields

$$\frac{g_3}{\sqrt{g}}\frac{\partial}{\partial x^1}A^2\frac{g_2}{\sqrt{g}}\frac{\partial \tilde{B}_3}{\partial x^1}-\frac{\kappa_2^2}{g_2}A^2\tilde{B}_3+\frac{A^2}{g_3}\frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3}\tilde{B}_3 +\omega^2\tilde{B}_3=-\frac{A^2}{g_2}\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)\frac{\partial \tilde{B}_1}{\partial x^1}+\frac{g_3}{g_1}\left(\frac{\partial}{\partial x^1}\frac{A^2}{g_3}\right)\frac{\partial \tilde{B}_1}{\partial x^3} -\frac{A^2}{g_2}\left(\frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^1}\frac{\sqrt{g}}{g_1}\right)\cdot\tilde{B}_1. \tag{5}$$

Equations (3a) and (5) form a closed system of equations for the $\tilde{B}_1$ and $\tilde{B}_3$ components.

In a homogeneous plasma this system describes two independent oscillation modes, namely an Alfvén wave and a fast magnetosonic wave, and the $\tilde{B}_3$ component is different from zero only in the second of them. In an inhomogeneous plasma such a separation is, strictly speaking, impossible; but also in this case, for illustration purposes, we shall treat the $\tilde{B}_3$ component as a perturbation in a fast magnetosound wave, and the relation between $\tilde{B}_1$ and $\tilde{B}_3$ ensuing from equation (5) will be treated as the mutual influence of the two oscillation modes attributable to the inhomogeneity of the medium.

In our previous papers (Leonovich and Mazur, 1989a,b) we have considered in detail toroidal (or more exactly, nearly toroidal) oscillations with small values of κ_2. The smallness of κ_2, or equivalently, of corresponding values of m, was understood in the following sense. From simple considerations it is easy to see that in the frequency range of long-period geomagnetic pulsations ($f < 30$ mHz, say) the magnetosphere is an opacity region for a fast magnetosound wave even for $m \sim 1$. In other words, the magnetosound field does not have an oscillatory character in space but decreases exponentially from the source localization region, and a typical scale of the decrease is L/m, where L is a typical scale of the magnetosphere. In accordance with the widely accepted viewpoint we have supposed that the magnetosound source is located either outside the magnetosphere or on its boundary, and the requirement that the magnetosound field penetrated sufficiently deep into the magnetosphere places a stringent constraint on the value of m; it virtually means $m \leqslant 10$. The magnetosound field generates, near so-called resonance magnetic shells, an Alfvén oscillation of large amplitude, whose typical spatial scale along the normal to the magnetic shell is very small (it is determined by a weak damping and by dispersion effects). Therefore, for an Alfvén wave the condition $\hat{\kappa}_1 \gg \hat{\kappa}_2$ is quite conservatively satisfied, which does determine its toroidal character. Equation (5) describes the back influence of the Alfvén wave upon the fast magnetosound wave, and this has also been analyzed in our papers. Thus, the fast magnetosound wave generated by a non-magnetospheric source, has a global character and occupies a significant part of the magnetosphere, and the Alfvén wave is concentrated in narrow layers along the resonance shells.

In this paper we intend to consider Alfvén oscillations with $\hat{\kappa}_2 \gg \hat{\kappa}_1$. This presupposes rather large m. A corresponding fast magnetosound wave generated by non-magnetospheric sources virtually cannot penetrate inside the magnetosphere. In this case we shall be treating equation (5) in the following way. Its right-hand side determined by the Alfvén wave, plays the role of the source for the left-hand side, the fast magnetosound wave. That is, we are dealing with an inverse situation, namely the field $\tilde{B}_3$ is a forced magnetosound response to the presence of an Alfvén wave.

Bearing in mind the zeroth-approximation for the poloidal mode ($m \to \infty$), one may argue that the Alfvén wave is concentrated near an identified magnetic surface. The fast magnetosound wave also cannot move far away from its source. That is, the entire MHD oscillation has a scale normal to the magnetic shell much smaller than a typical size of the magnetosphere and, as will be assumed subsequently, than the longitudinal length of the oscillation wave. In such a case on the left-hand side of equation (5) we can leave only the first two terms (and differentiation with respect to x^1 can be referred to disturbance $\tilde{B}_3$ only), and the right-hand side can retain the first term only.

As a result, we obtain

$$\frac{\partial^2 \tilde{B}_3}{\partial x^{1^2}} - \kappa_2^2 \frac{g_1}{g_2} \tilde{B}_3 = -\frac{g_1}{g_2}\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)\cdot\frac{\partial \tilde{B}_1}{\partial x^1}. \quad (6)$$

A solution of this equation is easily obtained using Green's function

$$G(x^1, x^{1'}) = -\frac{1}{2\mathscr{H}} \exp\left(-\mathscr{H}|x^1 - x^{1'}|\right),$$

$$\mathscr{H} = \kappa_2 \left(\frac{g_1}{g_2}\right)^{1/2}.$$

We have

$$\tilde{B}_3 = \frac{1}{2\mathscr{H}}\frac{g_1}{g_2}\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)\int_{-\infty}^{\infty} e^{-\mathscr{H}|x^1 - x^{1'}|}\frac{\partial \tilde{B}_1}{\partial x^{1'}}\,dx^{1'}. \quad (7)$$

Here integration with respect to $x^{1'}$ is formally extended to infinity, taking into consideration that it is actually carried out with respect to the localization region of the Alfvén wave.

From the expression (7) it is quite evident that the Alfvén wave on the surface x^1 engenders a magnetosound field which penetrates at a distance $|x^1 - x^{1'}| \sim \mathscr{H}^{-1}$. Let us make the assumption, to be justified below, that a typical scale of variation of the mode in coordinate $x^{1'}$, measured in physical units (we denote it as b) is much larger than a typical scale of Green's function. This means that

$$\hat{\kappa}_2 \mathrm{b} \gg 1, \quad (8)$$

and is equivalent to the inequality $\hat{\kappa}_2 \gg \hat{\kappa}_1$. From (7) we then have

$$\tilde{B}_3 = \frac{1}{\kappa_2^2}\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)\frac{\partial \tilde{B}_1}{\partial x^1}. \quad (9)$$

This expression is at once obtainable from equation (6) if the differential term on its left-hand side is omitted.

On substituting (9) into equation (3a), we obtain the desired differential equation for poloidal Alfvén oscillations

$$\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^2}\frac{g_2}{\sqrt{g}}A^2\frac{\partial \tilde{B}_1}{\partial x^3} + \omega^2 \tilde{B}_1 - \frac{1}{\kappa_2^2}\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}A^2\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)\frac{\partial^2 \tilde{B}_1}{\partial x^{1^2}} = 0. \quad (10)$$

This equation should be supplemented with the boundary condition on the ionosphere. Its form for the low-frequency waves concerned is rather well known (see e.g. Southwood and Hughes, 1983)

$$\left.\left(\frac{\partial \tilde{B}_1}{\partial x^3} \mp i\delta_{\pm}\frac{\omega\sqrt{g_3}}{A}\tilde{B}_1\right)\right|_{x^3 = x^3_{\pm}} = 0. \quad (11)$$

Here the "+" and "−" signs correspond to the conjugate ionospheres, and

$$\delta_{\pm} = \frac{c^2 \cos\chi_{\pm}}{4\pi A_{\pm}\Sigma_{\mathrm{P}}^{(+)}},$$

where χ is the angle between the normal to the ionosphere and the field line, Σ_{P} is integral Pedersen conductivity, and $A_{\pm} = A(x^1, x^3_{\pm})$. Some important comments on this boundary condition are given in a paper by Leonovich and Mazur (1989a). They pointed out, in particular, that the boundary between the ionosphere and the magnetosphere should be drawn at a height of about 1.5×10^3 km. This choice was dictated by the fact that below this height the Alfvén velocity increases rapidly from values of order 300 km s^{-1} in the ionospheric *F*-layer to a maximum of order 10^4 km s^{-1} on the boundary involved, and decreases slowly above it. The dimensionless parameter δ for the dayside ionosphere is rather small, $\delta \sim 10^{-2}$, while for the nightside ionosphere it is of the order of unity.

To conclude this section, we wish to make one important remark concerning the self-consistency of the approximations made. For deriving equation (10), we have used equations (3a) and (5) and have ignored equation (3b). The question arises as to whether the same equation (10) is obtained if equation (3b) is used. The relationship (4) and the equality (9) combine to give in the main order in κ_2^{-1}

$$\tilde{B}_2 = \frac{i}{\kappa_2}\frac{g_2}{g_1}\frac{\partial \tilde{B}_1}{\partial x^1}. \quad (12)$$

If (9) and (12) are substituted into (3b), we obtain the equation

$$\frac{\partial}{\partial x^1}\left(\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}A^2\frac{\partial \tilde{B}_1}{\partial x^3} + \omega^2 \tilde{B}_1\right) = 0,$$

which is, in fact, equation (10) but without the last term. There seems to be direct evidence of a contradiction; but it is easily resolvable, by considering that the approximations (9) and (12) are already insufficient for deriving the basic equation from (3b) and it is necessary to take account in these relationships of the next expansion terms in κ_2^{-1}. After that, simple but cumbersome calculations again lead to equation (10). In all the other respects the expressions (9) and (12), together with equation (10) are quite sufficient for determining the wave field of poloidal Alfvén waves in the magnetosphere.

3. THE LONGITUDINAL SPATIAL STRUCTURE OF POLOIDAL ALFVEN OSCILLATIONS

The method of solving equation (10) we are using here, is quite similar to that suggested by Leonovich and Mazur (1989a). It is based on perturbation theory relying on the presence of the small parameter κ_2^{-1}. More strictly, a dimensionless small parameter is represented by $(\hat{\kappa}_2 L)^{-1}$, where L is the field line length. Further, the parameter δ characterizing the damping in the ionosphere will be considered small.

An important role through the subsequent discussion will be played by the solution of the following auxiliary problem for eigenvalues

$$\hat{R}(\omega)H = 0; \quad \left.\frac{\partial H}{\partial x^3}\right|_{x^3=x^3_\pm} = 0. \tag{13}$$

Here $\hat{R}(\omega)$ is a differential (in coordinate x^3) operator:

$$\hat{R}(\omega) \equiv \frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}\frac{g_2}{\sqrt{g}}A^2\frac{\partial}{\partial x^3}+\omega^2.$$

The variable x^1 is involved in problem (13) as a parameter, on which the eigenvalues and the eigenfunctions depend:

$$\omega = \Omega_N(x^1), \quad H = H_N(x^1, x^3), \tag{14}$$

where $N = 1, 2, 3, \ldots$, is the harmonic number. Since this is a Hermitian problem, frequencies Ω_N are real. From general considerations follows the completeness of the system of functions H_N in the variable x^3. They can be chosen real and orthonormalized with a corresponding weight:

$$\oint \frac{\sqrt{g}}{g_1} H_N H_{N'}\,\mathrm{d}x^3 \equiv 2\int_{x^3_-}^{x^3_+} \frac{\sqrt{g}}{g_1} H_N H_{N'}\,\mathrm{d}x^3 = \delta_{NN'}.$$

Here the sign of the line integral denotes integration "forward and back" along the field line.

The problem for eigenvalues (13) may be regarded as a zeroth-approximation of perturbation theory, when $\kappa_2^{-1} = 0$ and $\delta_\pm = 0$. In other words, equations (13) describe the longitudinal structure of poloidal eigenmodes (Radoski and Carovillano, 1969). Their total spatial structure in the zeroth-approximation is defined by the relationship

$$\tilde{B}_1 = C\delta(x^1 - \bar{x}^1)H_N(x^1, x^3),$$

where C is an arbitrary constant, and $x^1 = \bar{x}^1$ represents a resonance magnetic surface. The frequency of the mode $\omega = \Omega_N(\bar{x}^1)$. This is the standing wave localized on the resonance surface which has been dealt with in the Introduction.

For the fundamental harmonics ($N \sim 1$) the problem (13) can only be solved numerically (Cummings *et al.*, 1969); but for higher harmonics ($N \gg 1$) the WKB approximation is applicable. In this case we have

$$\Omega_N(x^1) = N\Omega(x^1), \quad \Omega(x^1) = 2\pi/t_A(x^1),$$

$$H_N(x^1, x^3) = \left(\frac{2}{At_A}\right)^{1/2}\left(\frac{g_1}{g_2}\right)^{1/4} \times \cos\left(\Omega_N \int_{x^3_-}^{x^3} \frac{\sqrt{g_3}\,\mathrm{d}x^{3'}}{A}\right). \tag{15}$$

Here

$$t_A(x^1) = \oint \frac{\sqrt{g_3}\,\mathrm{d}x^3}{A}$$

is the travel time along the field line with Alfvén velocity forward and back. Note that these formulae describe the solution qualitatively correctly even for $N \sim 1$.

The dissipation in the ionosphere can be included in the same, one-dimensional in coordinate x^3, approach. For this purpose we employ perturbation theory in the parameters $\delta_\pm$. Formulas (14) give a zeroth-approximation. In the next order we put $H = \tilde{H}_N \equiv H_N + h_N$, where $h_N = h_N(x^1, x^3)$ is the first-order correction. Linearization of the boundary condition (11) yields

$$\left.\left(\frac{\partial h_N}{\partial x^3} \mp \mathrm{i}\delta_\pm \frac{\omega\sqrt{g_3}}{A} H_N\right)\right|_{x^3=x^3_\pm} = 0. \tag{16}$$

We linearize equation (13) by taking into consideration that the difference $(\omega^2 - \Omega_N^2)$ is a quantity of the first order of smallness. By multiplying the obtained relationship by $(\sqrt{g}/g_1)H_N$ and integrating along the field line, we obtain

$$\omega^2 - \Omega_N^2 = -\oint \frac{\sqrt{g}}{g_1} H_N \hat{R}(\Omega_N) h_N\,\mathrm{d}x^3.$$

The last integral is easily calculated with the use of the relationship (16):

$$\oint \frac{\sqrt{g}}{g_1} H_N \hat{R}(\Omega_N) h_N\,\mathrm{d}x^3 = 2\mathrm{i}\omega\gamma_N, \tag{17}$$

where it is designated

$$\gamma_N = \gamma_N(x^1) = \delta_+\left.\left[\left(\frac{g_2}{g_1}\right)^{1/2} AH_N^2\right]\right|_{x^3_+} + \delta_-\left.\left[\left(\frac{g_2}{g_1}\right)^{1/2} AH_N^2\right]\right|_{x^3_-}.$$

These equalities yield

$$\omega = \pm\Omega_N - i\gamma_N,$$

i.e. the quantity γ_N is the damping decrement of the poloidal mode. For $N \gg 1$, when formula (15) is applicable, we have

$$\gamma_N = \gamma(x^1) \equiv \pi^{-1}[\delta_+(x^1)+\delta_-(x^1)]\Omega(x^1).$$

In this approximation the damping decrement does not actually depend on number N.

4. THE EQUATION FOR RADIAL STRUCTURE OF POLOIDAL ALFVEN OSCILLATIONS

The equation for radial structure is obtained in the next, first order of perturbation theory. Using the completeness of the system of functions $\bar{H}_N$ in the variable x^3 (which is a consequence of the completeness of the system H_N), the desired solution will be represented as a series

$$\tilde{B}_1(x^1,x^3) = \sum_{N=1}^{\infty} F_N(x')\bar{H}(x^1,x^3), \qquad (18)$$

whose coefficients are functions of the variable x^1. Such a representation ensures in the first order the fulfilment of the boundary condition (11).

Now, the problem is reduced to finding the equation for $F_N(x^1)$. We substitute (18) into (11) and linearize it, assuming the last term in (11), the function h_N and the difference $(\omega^2-\Omega_N^2)$ to be first-order quantities. We get

$$\sum_N \left[(\omega^2-\Omega_N^2)H_N F_N + \hat{R}(\Omega_N)h_N F_N - \frac{1}{\kappa_2^2}\frac{g_1}{\sqrt{g}}\frac{\partial}{\partial x^3}A^2\frac{g_2}{\sqrt{g}}\left(\frac{\partial}{\partial x^3}\frac{g_2}{g_1}\right)H_N\frac{d^2F_N}{dx^{1^2}}\right] = 0.$$

We multiply this equality by $(\sqrt{g}/g_1)H_N$ and integrate along the field line, by using relation (17). As a result, we obtain the equation for F_N defining the radial structure of the mode

$$r_N^2\frac{d^2F_N}{dx^{1^2}} - \left[\frac{(\omega^2+i\gamma_N(x^1))^2}{\Omega_N^2(x^1)} - 1\right] = 0. \qquad (19)$$

Here it is designated

$$r_N^2 = \frac{\sigma_N^2(x^1)}{\kappa_2^2\Omega_N^2(x^1)};$$

$$\sigma_N^2 = \oint H_N\frac{\partial}{\partial x^3}\left[A^2H_N\frac{g_2}{\sqrt{g}}\frac{\partial}{\partial x^3}\left(\frac{g_2}{g_1}\right)\right]dx^3. \quad (20)$$

Equation (19) is quite similar to the equation for toroidal oscillations as obtained in a paper by Leonovich and Mazur (1989a). The first differential term in (19) corresponds to the dispersion term of the toroidal mode; therefore, it may be treated as being a transverse dispersion of poloidal Alfvén oscillations. If the applicability of the WKB approximation in coordinate x^1 is assumed, then from (19) one can obtain the equality

$$\omega = \Omega_N(1-\tfrac{1}{2}\kappa_1^2r_N^2)-i\gamma_N,$$

from which it is immediately evident that the differential term has a dispersion character. If the azimuthal number m is used as κ_2, then one may write

$$r_N^2 = \frac{R_N^2}{m^2}; \quad R_N^2 = \frac{\sigma_N^2}{\Omega_N^2}. \qquad (21)$$

From expression (20) it is evident that in a homogeneous magnetic field with straight field lines, when g_1 and g_2 are constant, the value of $r_N = 0$. This suggests that the dispersion of poloidal modes is caused by the curvature of the field lines. Let us consider this issue in greater detail.

5. THE DISPERSION CAUSED BY THE CURVATURE OF GEOMAGNETIC FIELD LINES

On each given magnetic shell it is possible to use instead of the coordinate x^3 the physical length of the field line l, whose differentials are related by the relationship $dl = \sqrt{g_3}\,dx^3$. Relationship (20) can then be represented as

$$\sigma_N^2 = 2\oint H_N\frac{\partial}{\partial l}\left(H_NA^2p^2\frac{\partial p}{\partial l}\right)dl,$$

where the quantity

$$p = \left(\frac{g_2}{g_1}\right)^{1/2}$$

is introduced. This quantity has a simple geometrical meaning. If we take a thin flux tube of a rectangular cross-section having in coordinates x^1 and x^2 the sizes dx^1 and dx^2, then its physical size will be, respectively, $\sqrt{g_1}\,dx^1$ and $\sqrt{g_2}\,dx^2$, and the ratio of these latter is

$$\frac{\sqrt{g_2}\,dx^2}{\sqrt{g_1}\,dx^1} = p\frac{dx^2}{dx^1}.$$

That is, the function p describes a variation of this ratio during the movement along the tube. It is clear that this ratio is able to vary only in a magnetic field with curved field lines.

Integration of the expression for σ_N^2 by parts yields

$$\sigma_N^2 = 4A^2H_N^2p^2\frac{\partial p}{\partial l}\Big|_{l_-}^{l_+} - \oint A^2p^2\frac{\partial p}{\partial l}\frac{\partial H_N^2}{\partial l}dl. \qquad (22)$$

For calculating this quantity when $N \gg 1$, one can use the formula of WKB approximation (15), and the thus obtained value may be used as an order-of-magnitude estimate for $N \sim 1$ as well. We rewrite the expression (15) as

$$H_N(l) = \left[\frac{2}{t_A A(l) p(l)}\right]^{1/2} \cos\left(\Omega_N \int_{l_-}^{l} \frac{dl'}{A(l')}\right). \quad (23)$$

The second term on the right-hand side of formula (22) in the WKB approximation is zero because the integrand involves a rapidly oscillating term $\partial H_N^2/\partial l$. On substituting into (22) the expression (23), we obtain

$$\sigma_N^2 = 4\frac{A(l)}{t_A}\frac{\partial p^2(l)}{\partial l}\bigg|_{l_-}^{l_+}. \quad (24)$$

In order to represent the behaviour of the function let us consider, as an example, a model of the geomagnetic field, whose field lines in the polar coordinate system (in a given meridional plane) are specified by the equation

$$r = L\cos^{\mu}\theta. \quad (25)$$

Here r is the distance from the origin of coordinates, θ is the polar angle measured from the equator $(-\pi/2 \leqslant \theta \leqslant \pi/2)$, L is the equatorial radius of a field line which can be used as the coordinate x^1, and μ is a constant. $\mu = 2$ corresponds to the case of a dipole field. When $\mu = 1$, the field lines are circles making contact at the origin of coordinates. When $\mu < 0$, the field lines go into infinity along the symmetry axis; in particular, when $\mu = -1$, they are straight lines perpendicular to the equatorial plane (see Fig. 2).

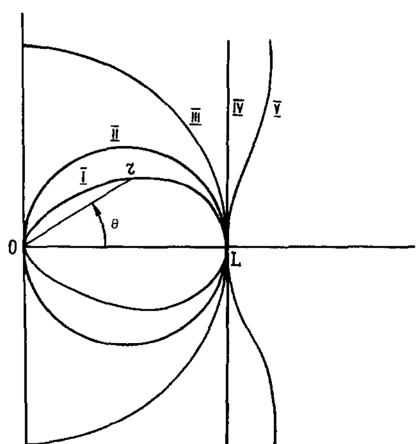

FIG. 2. PLOTS OF FIELD LINES SIMULATED BY THE FUNCTION $r = L\cos^{\mu}\theta$: I$-\mu = 2$; II$-\mu = 1$; III$-\mu = 0$; IV$-\mu = -1$; V$-\mu < -1$.

If the azimuthal angle φ is chosen as the coordinate x^2 and the quantity L is taken to represent the coordinate x^1, then for model (25) we have

$$p = L\sqrt{1+(\mu^2-1)\sin^2\theta}.$$

From this expression it is evident that, when $|\mu| > 1$, the value of p increases from the equator to the ionosphere and, consequently, $\sigma_N^2 > 0$, for $|\mu| < 1$, the value of p decreases and $\sigma_N^2 < 0$. Curiously, the value of p is constant not only for straight field lines—$\mu = -1$, but also for field lines in the form of circles—$\mu = 1$. Taking account of the relationship

$$dl = L\cos^{\mu-1}\theta \cdot \sqrt{1+(\mu^2-1)\sin^2\theta}\, d\theta$$

we obtain

$$\frac{\partial p^2}{\partial l} = \frac{2L(\mu^2-1)\sin\theta}{\cos^{\mu-1}\theta \cdot \sqrt{1+(\mu^2-1)\sin^2\theta}}.$$

In particular, for an important case of a dipole field, we have

$$\frac{\partial p^2}{\partial l} = \frac{6L\sin\theta}{\sqrt{1+3\sin^2\theta}}.$$

For this last case formula (24) yields

$$\sigma_N^2 = 24\frac{LA_*}{t_A}\sqrt{\frac{L-R_E}{L-(3/4)R_E}}, \quad (26)$$

where R_E is the Earth's radius (from the centre to the upper boundary of the ionosphere), and $A_* = A(l_\pm)$ is the Alfvén velocity at the ionosphere–magnetosphere interface. On the order of magnitude σ_N^2 is estimated by

$$\sigma_N^2 \sim \bar{A} \cdot A_*,$$

where $\bar{A}$ is a mean (along the field line) value of Alfvén velocity, and the rather large numerical factor in (26) must not confuse because the path traversed by a wave along the field line is about $6L$. On substituting (26) into the expression (21) for R_N^2 in the same WKB approximation, we get

$$R_N^2 = \frac{6}{\pi^2}\frac{LA_* t_A}{N^2 m^2}\sqrt{\frac{L-R_E}{L-(3/4)R_E}}, \quad (27)$$

of the order of magnitude

$$R_N = \left(\frac{A_*}{\bar{A}}\right)^{1/2}\frac{L}{N}.$$

It should be noted that the dispersion parameter r_N for poloidal oscillations turns out to be much greater as compared with toroidal oscillations. For these latter, it is equal to the Larmor radius of ions ρ_i or to the skin length (c/ω_{pe}), whose typical value is of order

1 km. By confining ourselves to not too large harmonics, $N = 1$–5, and assuming $L \sim 10^4$ km, one can see that the dispersion parameters for poloidal and toroidal modes compare only for extremely large values of $m \sim 10^4$. Therefore, the dispersion-induced properties are able to manifest themselves much more clearly for poloidal modes than those for toroidal modes. It should further be pointed out that if in the initial equations account is taken of effects beyond the scope of ideal magnetohydrodynamics which determine the dispersion of toroidal modes, then for poloidal modes the expression for r_N^2 will incorporate a quantity of order ρ_i^2 or $(c/\omega_{pe})^2$, i.e. r_N^2 will virtually remain unchanged.

6. THE RADIAL STRUCTURE OF POLOIDAL ALFVEN OSCILLATIONS

By virtue of the smallness of the parameter r_N^2, the solution of equation (19) is concentrated near the resonance surface defined by the equality $\omega^2 = \Omega_N^2(x^1)$, on a scale much smaller than a typical scale of variation of magnetospheric parameters. The form of the solution depends substantially on the behaviour of the function $\Omega_N(x^1)$ in the neighbourhood of the resonance surface. Two different cases are possible here, namely the function $\Omega_N(x^1)$ varies monotonically in this neighbourhood or has an extremum there.

To begin with, we consider the first case. Let $x^1 = \bar{x}^1$ be a coordinate of the resonance surface. In its vicinity, we put

$$\Omega_N(x^1) = \bar{\Omega}_N(1 - x/l_N), \quad x = x^1 - \bar{x}^1. \qquad (28)$$

Here l_N is the scale of variation of the function $\Omega_N(x^1)$. It is assumed that the function decreases with x^1, as is the case in most of the magnetosphere. Using this expansion we bring equation (19) to the form

$$r_N^2 \frac{d^2 F_N}{dx^2} - 2\left(\frac{x}{l_N} + i\frac{\gamma_N}{\bar{\Omega}_N}\right) F_N = 0.$$

It has the following (bounded at all values of x) solution:

$$F_N = c Ai\left[2\left(\frac{l_N}{2r_N}\right)^{2/3}\left(\frac{x}{l_N} + i\frac{\gamma_N}{\bar{\Omega}_N}\right)\right],$$

where $Ai(Z)$ is the Eyre function (see, for example, Antosevich, 1979), and c is an arbitrary constant. This solution decreases exponentially into a region of negative x and has the form of a standing wave (i.e. of a superposition of an arriving and escaping wave) at positive x.

A typical scale of variation of the obtained solution

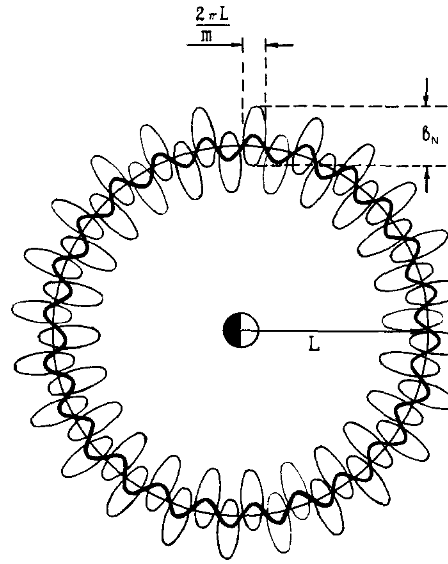

FIG. 3. THE FINE TRANSVERSE STRUCTURE OF THE POLOIDAL MODE.
The figure gives a schematic representation of the equatorial cross-section of oscillating flux tubes. The wavy curve shows the amount of displacements with respect to the magnetic shell considered.

in coordinate x is $b_N = r_N^{2/3} l_N^{1/3}$. On the order of magnitude $b_N \sim (A_*/\bar{A})^{1/3}(L^{2/3} l_N^{1/3}/N^{2/3}|m|^{2/3})$. The estimate obtained permits us to resolve the key question as to whether the mode considered is, indeed, a poloidal one. As we know, this requires that the inequality $\hat{\kappa}_1 \ll \hat{\kappa}_2$ be satisfied. Bearing in mind that $\hat{\kappa}_1 \sim b_N^{-1}$ and $\hat{\kappa}_2 \sim m/L$, we bring the required inequality to the form

$$|m| \gg m_{\min}, \quad m_{\min} = \frac{\bar{A}}{A_*}\frac{L}{l_N}N^2. \qquad (29)$$

Since $\bar{A}$ is substantially less than A_* and in most of the magnetosphere $l_N \sim L$, it is evident that for small values of N the inequality (29) is essentially reduced to the condition $m \gg 1$, whose fulfilment has been assumed from the very beginning. Thus, it is possible to draw a fundamentally important conclusion that an investigation of the fine transverse structure (see Fig. 3) proves a fundamental possibility of existence of poloidal Alfvén oscillations.

The damping of a mode in the ionosphere has a minor effect on its spatial structure if $(l_N/r_N)^{2/3}(\gamma_N/\Omega_N) \ll 1$. This inequality can be reduced to the form

$$|m| \ll \left(\frac{\bar{A}}{A_*}\right)^{1/2}\frac{L}{l_N}\frac{1}{N}\left(\frac{\Omega_N}{\gamma_N}\right)^{3/2}. \qquad (30)$$

For daytime conditions in the magnetosphere, $\gamma_N/\Omega_N \sim 10^{-2}$ and the condition (30) (when $N \sim 1$) implies $m \ll 10^3$, i.e. is not a too limiting one.

Let us now examine the second possibility that the resonance surface is located near the extremum of the function $\Omega_N(x^1)$; in this case we consider a more interesting case of the maximum. In the dayside magnetosphere the maxima $\Omega_N(x^1)$ occur on the shells $L \approx 1,3$ and on the outer edge of the plasmapause. Near the maximum one may put

$$\Omega_N^2(x^1) = \bar{\Omega}_N^2(1 - x^2/a_N^2), \quad x = x^1 - x_N^1,$$

where x_N^1 is a coordinate of the maximum, and a_N is the inhomogeneity scale. Assuming

$$\xi = x/b, \quad b = (a_N r_N)^{1/2},$$
$$\lambda = (a_N/r_N)[1 - (\omega + \mathrm{i}\gamma_N)^2/\bar{\Omega}_N^2], \tag{31}$$

we bring equation (19) to the form

$$\frac{\mathrm{d}^2 F_N}{\mathrm{d}\xi^2} + (\lambda - \xi^2)F_N = 0.$$

This is the quantum-oscillator equation. It has the following eigenvalues and eigenfunctions

$$\lambda_n = 2n+1, \quad y_n = \mathrm{e}^{-\xi^2/2}H_n(\xi), \tag{32}$$

where $n = 0, 1, 2, \ldots$ are non-negative integer numbers, and $H_n(\xi)$ represents Hermitian polynomials (Hochstrasser, 1979).

Thus, near the maximum of the function $\Omega_N(x^1)$ there exist localized (in coordinate x^1) eigenmodes $F_N(x^1) = cy_n(x/b)$. Such a phenomenon was called in our previous paper (Leonovich and Mazur, 1989a) the Alfvén resonator. The spatial scale of eigenmodes $b = b_0/|m|^{1/2}$, where

$$b_0 = (a_N R_N)^{1/2} \sim \left(\frac{A_*}{\bar{A}}\right)^{1/4}\left(\frac{La_N}{N}\right)^{1/2}.$$

The requirement $\hat{\kappa}_1 \ll \hat{\kappa}_2$ in this case leads to the inequality

$$|m| \gg m_{\min}, \quad m_{\min} = \frac{\bar{A}}{A_*}\frac{L}{a_N}N, \tag{33}$$

i.e. is virtually also satisfied when $|m| \gg 1$.

Eigenfrequencies of the Alfvén resonator are specified by the relationship ensuing from (31) and (32)

$$\Omega_{Nn} = \bar{\Omega}_N\left[1 - \frac{r_N}{a_N}(n + \tfrac{1}{2})\right] - \mathrm{i}\gamma_N.$$

In order for resonator properties to manifest themselves, the damping decrement γ_N must be much smaller than the splitting of eigenfrequencies, $\bar{\Omega}_N(r_N/a_N)$. This condition leads to the inequality

$$|m| \ll \left(\frac{A_*}{\bar{A}}\right)^{1/2}\frac{L}{a_N}\frac{1}{N}\left(\frac{\Omega_N}{\gamma_N}\right)^2, \tag{34}$$

which in the dayside magnetosphere means $|m| \ll 10^4$, i.e. is satisfied in a wide range of values of m. In this lies the essential difference between the Alfvén resonator for poloidal and toroidal modes. As far as the latter ones are concerned, owing to the far smaller dispersion, resonator properties in the real magnetosphere cannot be manifested (Leonovich and Mazur, 1989a).

7. THE AZIMUTHAL STRUCTURE OF POLOIDAL ALFVEN OSCILLATIONS

So far we have limited our attention to considering separate Fourier-harmonics in azimuthal angle φ. By adding such harmonics together, one can get quite different dependences on

$$F_N(x^1, \varphi) = \sum_m c(m) F_N^{(m)}(x^1)\,\mathrm{e}^{\mathrm{i}m\varphi}, \tag{35}$$

where $c(m)$ is the weight of the mth harmonic, and $F_N^{(m)}(x^1)$ represents the radial solutions obtained above for a given value of m. For the mode to have a poloidal character, the sum (35) must involve harmonics whose azimuthal numbers satisfy the conditions such as (29) or (33). In other words, the function $c(m)$ must be zero for $|m| \lesssim m_{\min}$. Since the allowable values of $|m| \gg 1$, in the relationship (35), without sacrificing accuracy, one may switch over from the summation to integration with respect to m. Let us show that by a suitable choice of the weighting function $c(m)$, it is possible to construct localized (in coordinate φ) solutions.

It is particularly easy to illustrate this, by considering an example of eigenmodes of the Alfvén resonator. By confining ourselves to the fundamental, zero-order eigenfunction, from (32) we have

$$F_N^{(m)}(x^1) = \exp(-x^2/2b^2) = \exp(-|m|x^2/2b_0^2).$$

We choose the weighting function in the form

$$c(m) = c\left[\exp\left(-\frac{|m|}{m_{\max}}\right) - \exp\left(-\frac{|m|}{m_{\min}}\right)\right].$$

This function is virtually zero when $|m| \ll m_{\min}$ and $|m| \gg m_{\max}$. The value of $m_{\max}$ can be chosen based on the inequality (34) and the damping in the ionosphere may then be neglected. As a result, by calculating the integral, we obtain

$$F_N(x,\varphi) = 2c\left(\frac{1}{m_{\min}} - \frac{1}{m_{\max}}\right) \times \frac{\left(\frac{1}{m_{\min}} + \frac{x^2}{2b_0^2}\right)\left(\frac{1}{m_{\max}} + \frac{x^2}{2b_0^2}\right) - \varphi^2}{\left[\left(\frac{1}{m_{\min}} + \frac{x^2}{2b_0^2}\right)^2 + \varphi^2\right]\cdot\left[\left(\frac{1}{m_{\max}} + \frac{x^2}{2b_0^2}\right)^2 + \varphi^2\right]}.$$

This solution decreases rapidly in both of the coordinates x and φ and can be considered as being localized near the chosen field line $x = 0$, $\varphi = 0$. Thus, this lends support to the widely accepted view that poloidal Alfvén oscillations are oscillations of separate field lines.

A similar result, through more cumbersome calculations, however, is obtained also for poloidal oscillations localized in the region of monotonic variation of the function $\Omega_N(x^1)$.

To conclude this section, we shall derive a differential equation in partial derivatives, describing a two-dimensional transverse structure of a poloidal Alfvén oscillation. To accomplish this, we multiply equation (19) by $\kappa_2^2\Omega_N^2$ and perform an inverse Fourier-transform in κ_2. As a result, we obtain

$$\sigma_N^2(x^1)\frac{\partial^2 F_N}{\partial x^{1^2}} + [(\omega + i\gamma_N(x^1))^2 - \Omega_N^2(x^1)]\frac{\partial^2 F_N}{\partial x^{2^2}} = 0. \quad (36)$$

If the mode is concentrated in the region of monotonic variation of the function $\Omega_N(x^1)$, where expansion (28) is applicable, then by introducing the dimensionless coordinates $\xi = x/d$ and $\eta = x^2/d$, where $d = l_N(\sigma_N^2/2\bar{\Omega}_N^2)$, equation (36) can be reduced to the form

$$\frac{\partial^2 F}{\partial \xi^2} + \xi\frac{\partial^2 F}{\partial \eta^2} = 0.$$

This is the Tricomi equation known in mathematical physics.

8. CONCLUSIONS

The main results of this study may be summarized as follows.

(1) On the basis of equations of ideal magnetic hydrodynamics we have obtained the equation for poloidal Alfvén oscillations in the axisymmetric magnetosphere which makes it possible to investigate both the longitudinal and radial structure of a perturbation field [equation (10)].

(2) The equation for longitudinal structure ensuing from it is well known. It represents a one-dimensional (in coordinate x^3) problem for eigenvalues which defines the frequency spectrum and the longitudinal dependence of the field in the form of standing waves [problem (13)].

(3) The radial structure of the field is described by the one-dimensional (in coordinate x^1) differential equation, whose coefficients are integral characteristics of magnetic shells [equation (19)]. The differential term of this equation may be treated as the transverse dispersion of poloidal Alfvén oscillations caused by the curvature of geomagnetic field lines. The solution of the radial equation is localized near the resonance magnetic surface defined by the equation $\omega = \Omega_N(x^1)$. The localization scale, though being small compared with a typical scale of variation of magnetospheric parameters is, nevertheless, sufficiently large for fulfilment of the condition $\hat{\kappa}_1 \ll \hat{\kappa}_2$ which is required for the poloidal character of the oscillations involved. Thus, an investigation of the fine transverse structure of the mode proves the very fact of the existence of poloidal Alfvén oscillations.

(4) The solution of the radial equation has been found for two essentially different possibilities of the resonance surface location, namely in the region of monotonic variation of the function $\Omega_N(x^1)$ and in the vicinity of its extremum. In the first case the oscillation is a superposition of a wave running across the magnetic shells towards the resonance surface and of a wave reflected from it. On the other side of the resonance surface there is an opacity region, where the wave field decreases exponentially. Near the maximum of the function $\Omega_N(x^1)$ there exist solutions enclosed between near-lying magnetic shells; this phenomenon was called the Alfvén resonator.

(5) By summing the Fourier-harmonics with different values of the azimuthal wave number m, we have demonstrated the existence of poloidal Alfvén oscillations localized in both transverse coordinates. They may be treated as oscillations of separate geomagnetic field lines.

The question of sources of poloidal Alfvén oscillations of the magnetosphere is beyond the scope of this paper. From the above-said (see Section 2) it follows that such a source, unlike toroidal oscillations, cannot be provided by a fast magnetosound wave of non-magnetospheric origin. It might be suggested that a triggering disturbance of poloidal oscillations can be produced by ionospheric currents of a suitable spectral composition, say, by those which are generated by internal gravity waves in the ionospheric Hall layer. After that, the triggering disturbance can be intensified to reach significant amplitudes through various magnetospheric instabilities.

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