Journal of Geophysical Research,V.106,A3, 3919-3928, 2001.

On the spectrum of magnetosonic eigenoscillations of an axisymmetric magnetosphere

A. S. Leonovich and V. A. Mazur
Institute of Solar-Terrestrial Physics, Irkutsk, Russia

Abstract

The structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere in the direction across magnetic shells was determined within the WKB approximation. The spectrum of their eigenfrequencies was investigated numerically. The model of the medium assumes a dipole magnetic field and a two-dimensional Alfven velocity distribution in the meridional plane which takes into account abrupt changes of the Alfven velocity at the plasmapause and magnetopause. It is shown that eigenfrequencies are complex valued. The imaginary part of the frequency is a damping decrement of the eigenoscillations associated with the escape of some of their energy to the solar wind. It is shown that in a dipole-like geomagnetic field in which field lines concentrated toward the origin of coordinates, the amplitude of magnetosonic oscillations increases from the magnetopause to the ionosphere. While the oscillations penetrating deep into the magnetosphere from the solar wind or generated at the magnetospheric boundary have near the magnetopause a relatively small amplitude, as the Earth is approached, their amplitude can reach a significant value for geometrical reasons.

1. Introduction

There is a rather longstanding view of the magnetosphere as a natural cavity for MHD waves. As early as the first publication of *Dungey* [1954] it was shown that Alfven oscillations of the magnetosphere represent standing (along geomagnetic field lines) waves. In succeeding years it is this branch of MHD oscillations which was most thoroughly studied because of their rather simple (nearly one-dimensional) spatial structure accessible for analytical investigation. The magnetosphere is also a cavity for magnetosonic waves. These oscillations can penetrate to the magnetosphere's interior through its boundary (magnetopause) from the solar wind, or they can be generated on this very boundary through an instability of its oscillations when the solar wind flows past the magnetosphere [*McKenzie*, 1970]. A relatively sharp boundary separating the solar wind and the magnetosphere, the magnetopause, is able to partly reflect these oscillations, confining them inside the magnetospheric cavity [*McClay*, 1970].

This property of the magnetopause needs some clarification. The chief point is that if the magnetosphere's interior involves a transparent region for magnetosonic waves, then the adjacent solar wind region, with not very high plasma velocities, also represents a transparent region for these waves [*Mann et al.*, 1999; *Lee and Kim*, 1999]. It will be recalled that the transparent region is determined by the condition of free propagation of waves inside of it in any direction, or using the notion of the wave vector $\mathbf{k}$ (structure of the wave, $\sim\exp(i\mathbf{kr})$), which in an inhomogeneous plasma should, of course, be understood in terms of the WKB approximation $\mathbf{k} = \mathbf{k}(\mathbf{r})$, it is the region inside of which $k^2 > 0$. With sufficiently large solar wind speeds, solar wind becomes a region opaque to the waves under consideration ($k^2 < 0$) [*Mann et al.*, 1999].

Thus the magnetosphere is rendered an ideal cavity for magnetosonic oscillations which are entrapped across magnetic shells inside the magnetosphere in the region whose inner boundary is formed by turning points (where $k^2 = 0$), the combination of which produces the surface separating the transparent and opaque regions, and whose outer boundary is represented by the magnetospheric boundary, the magnetopause. The oscillations that are entrapped within such a cavity form a set of standing (across magnetic shells) magnetosonic waves which can be defined as eigenoscillations of the magnetosphere. Such a situation can be realized at the flanks of the magnetotail where plasma velocities in the magnetosheath can reach significant values [*Mann et al.*, 1999].

On the other hand, in the sunward part of the magnetosphere where the velocity of plasma flow around it is significantly lower, the solar wind remains a transparent region for these waves. Even this case, however, leaves room for the formation of magnetosonic eigenoscillations of the magnetosphere similar to the above mentioned ones [*Walker*, 1998; *Mann et al.*, 1999]. As in the case considered above, the boundaries of their existence region are internal turning points and the magnetopause. Furthermore, the ability of the magnetopause to reflect magnetosonic waves can be understood only beyond the WKB approximation. When the typical wavelength ($\lambda \sim k^{-1}$) of the concerned oscillations is much larger than the typical thickness Δ of the transition region between the magnetosphere and the solar wind ($\lambda \gg \Delta$), the magnetopause becomes a barrier capable of partially reflecting these waves. The higher the difference of the values of plasma parameters at the transition through such a region, the more effective the reflection. This phenomenon is known in quantum mechanics as above-barrier reflection *Landau and Lifshitz*, [1963]. Since such reflection is not perfect, some of the magnetospheric eigenoscillations penetrate the solar wind and escape the magnetosphere. The eigenmodes are damped ones in this case. This same effect persists also in the case where a broad spectrum of magnetospheric eigenoscillations are excited through, for example, an impulsive disturbance of the magnetopause [*Freeman*, 2000].

All the above cited references employed simple plasma models in the form of two plane layers, the boundary between which represents a tangential discontinuity. In terms of such a model of the medium, it is possible to study the structure of magnetosonic eigenoscillations of the magnetosphere only in the direction across the magnetic shells. Their structure in the other two directions then represents plane waves of the form $\exp(ik_y y + ik_z z)$. Also, the values of the wave vector components can be arbitrary both in the direction along the magnetic field (k_z) and in the azimuthal direction (k_y). Since the frequencies of eigenoscillations depend on all wave vector components, it is impossible to solve (in terms of this model of the medium) the problem of the spectrum of magnetospheric eigenoscillations. Using models of the medium that have boundaries in these two directions as well (box models) leads to the fact that in these directions the eigenoscillations turn out to have the form of standing waves [*Kivelson and Southwood*, 1986; *Southwood and Kivelson*, 1986].

A limitation shared by all of the above mentioned efforts is that they use models of the medium with a magnetic field having straight field lines. The Earth's magnetic field is a magnetic dipole-like field that is deformed by solar wind plasma flow. Therefore the problem of the structure and spectrum of magnetospheric magnetosonic eigenmodes can be solved in a more adequate setting by using the model of the medium with curved geomagnetic field lines. The problems of MHD oscillations of the magnetosphere, based on such models of the medium, were solved by *Allan et al.* [1986] with a semicylinder model and by *Lee and Lysak* [1989, 1991,1994] and *Lee* [1996] with a dipole model.

The cited references employed numerical simulation techniques by which it was possible to investigate a total field of MHD oscillations in terms of a particular magnetospheric model. It was shown that the amplitude distribution of magnetosonic oscillations essentially differs from what is taking place in magnetospheric models with straight field lines. It was found that the energy of these oscillations is localized near the equatorial plane. *Leonovich and Mazur* [2000b] showed that such an amplitude distribution is caused by a corresponding boundary configuration of transparent regions of the magnetosonic eigenmodes. Unlike previous work based on a similar model of the medium, this paper considers magnetosonic oscillations in a model which includes both the magnetosphere and the solar wind region and a leaky magnetopause. Transparent regions of these oscillations near the equatorial plane are open to the solar wind. Hence magnetosonic oscillations of the solar wind can penetrate deep into the magnetosphere and vice versa some of the magnetosonic eigenoscillations of the magnetosphere can escape to the solar wind.

However, the study reported by *Leonovich and Mazur* [2000b] remains, in a sense, incomplete. They solved the problem of the longitudinal (along geomagnetic field lines) structure of the field of magnetosonic eigenoscillations and determined the configuration of their transparent regions in the meridional plane. The radial structure of the wave field of the oscillations and the spectrum of their eigenfrequencies remain uncertain. These characteristics depend on the form of boundary conditions for the waves under consideration in the direction across the magnetic shells. Such an investigation will be carried out in this paper. This paper consists of the following sections. In section 2, we obtain the WKB solution describing the structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere across geomagnetic field lines. In section 3, we determine the structure and the frequency spectrum of global modes confined within the magnetosphere by a sharp plasma density gradient on the magnetopause. Damping decrements are determined, which are associated with the escape of some of the oscillation energy to the solar wind. In section 4, we solve the problem of the transverse structure and spectrum of magnetosonic eigenoscillations of the cavity under the plasmapause. The main results of this study are discussed in section 5.

2. Transverse Structure of Magnetosonic Eigenmodes Within the WKB Approximation

We shall use a curvilinear orthogonal coordinate system (x^1, x^2, x^3) tied to geomagnetic field lines. The coordinates x^1, x^2, and x^3 determine the magnetic shell, a field line on this shell, and a point on this field line, respectively. The square of the length element in this coordinate system is given by

$$ds^2 = g_1(dx^1)^2 + g_2(dx^2)^2 + g_3(dx^3)^2,$$

where $g_i (i = 1, 2, 3)$ are diagonal components of the metric tensor. *Leonovich and Mazur* [2000b] obtained the equations for the vortical (ψ) and potential (φ) components of the MHD oscillation field of the axisymmetric magnetosphere. They form a two-dimensional vector

$$\mathbf{E}_\perp = -\nabla_\perp \varphi + [\nabla_\perp \times \boldsymbol{\psi}],$$

where $\mathbf{E}_\perp \equiv (E_1, E_2)$ is the normal part of the electric field of the oscillations ($E_3 = 0$) and $\nabla_\perp \equiv (\nabla_1, \nabla_2)$. The components of the magnetic field of MHD oscillations are related to the components of this vector by the following relations:

$$B_1 = i\frac{c}{\omega}\frac{p^{-1}}{\sqrt{g_3}}\nabla_3 E_2, \qquad B_2 = -i\frac{c}{\omega}\frac{p}{\sqrt{g_3}}\nabla_3 E_1,$$

$$B_3 = -i\frac{c}{\omega}\frac{g_3}{\sqrt{g}}\left[\nabla_1 E_2 - \nabla_2 E_1\right],$$

where $\nabla_i \equiv \partial/\partial x^i \quad (i = 1, 2, 3), \quad g = g_1 g_2 g_3$, and $p = (g_2/g_1)^{1/2}$. *Klimushkin* [1994] and *Fedorov et al.* [1998] showed that the potential field component φ describes Alfven oscillations of the magnetosphere, and the vortical component ψ describes magnetosonic oscillations. The equation describing magnetosonic eigenoscillations of the axisymmetric magnetosphere, obtained by *Leonovich and Mazur* [2000b], is of the form

$$\nabla_1 \frac{g_2}{\sqrt{g}} \nabla_1 \psi - k_2^2 \frac{g_1}{\sqrt{g}} \psi + \sqrt{\frac{g}{g_3}} \hat{L}_T \frac{g_1}{\sqrt{g}} \psi = 0, \qquad (1)$$

where

$$\hat{L}_T = \frac{1}{\sqrt{g_3}} \nabla_3 \frac{p}{\sqrt{g_3}} \nabla_3 + \frac{p\omega^2}{A^2}$$

is the longitudinal toroidal operator [see *Leonovich and Mazur*, 1993] and A is the Alfven velocity. Note that the right-hand side of equation (1) describing the influence of Alfven oscillations generated by the magnetosonic wave in the FLR on this same wave is taken to be zero. Thus the back influence of the Alfven waves on magnetosonic waves is neglected in this case. As is shown by *Leonovich and Mazur* [1989], at the transition through the region of resonance the amplitude of the magnetosonic wave changes little. This means that to some (zero-order) approximation, the back influence of the Alfven wave on the structure of magnetosonic oscillations of the magnetosphere can indeed be neglected.

To describe the structure of magnetosonic oscillations across magnetic shells, we have resorted to the WKB approximation. Of course, in the strict sense, this approximation can be used for oscillations whose wavelength in this direction is much smaller than the typical scale of a magnetospheric plasma irregularity. Yet it is known that the application of this approximation to oscillations whose wavelength is comparable with the irregularity scale yields qualitatively correct results, and even qualitatively they differ from true ones by no more than a few percent [see *Cummings et al.*, 1969]. This is true of both the structure of the oscillations under consideration and the spectrum of their eigenfrequencies. The solution can be sought in the form

$$\begin{aligned} \psi &= u(x^1) \exp[\tilde{\Phi}(x^1)][H(x^1, x^3) + h(x^1, x^3)] \\ &\quad \times \exp[ik_2 x^2 - \omega t], \qquad (2) \end{aligned}$$

where $\tilde{\Phi}$ is a large quasi-classical phase, the function $u(x^1)$ describes a slow variation of the oscillation amplitude across magnetic shells, H describes their structure along geomagnetic field lines in the main order, and h is a small correction to H in higher orders. Substitution of (2) into (1) in the main order of perturbation theory gives the equation for the function $H(x^1, x^3)$:

$$\nabla_3 \frac{p}{\sqrt{g_3}} \nabla_3 \frac{g_1}{\sqrt{g}} H + \left(\frac{\omega^2}{A^2} - k_\perp^2 \right) H = 0,$$

where $k_\perp^2 = (k_1^2/g_1 + k_2^2/g_2)$; $k_1 = \nabla_1 \tilde{\Phi}$ is a quasi-classical wave number. The solution of this equation is the eigenmodes H_n, satisfying the boundary conditions specified on the ionosphere, and corresponding eigenvalues of a quasi-classical transverse wave number k_{1n}^2. *Leonovich and Mazur* [2000b] carried out a thorough investigation of the solutions of this equation by specifying a model of the axisymmetric magnetosphere. In this paper we investigate the amplitude distribution of magnetosonic oscillations in the same model of the magnetosphere across magnetic shells. In the first order of perturbation theory, from equation (1) we obtain the following equation for the function $u(x^1)$:

$$\begin{aligned} &\sqrt{g_3} u_n \hat{L}_T \frac{g_1}{\sqrt{g}} h_n - k_{\perp n}^2 u_n h_n \\ &+ i \left(\frac{k_{1n}}{g_1} \nabla_1 u_n H_n + \frac{g_3}{\sqrt{g}} \nabla_1 \frac{g_2}{\sqrt{g}} k_{1n} u_n H_n \right) = 0. \end{aligned}$$

Upon multiplying this equation by $g_1 u_n H_n / \sqrt{g}$ and integrating along a field line between magnetoconjugate ionospheres "there and back," we get

$$\oint \nabla_1 \left(\frac{g_2}{\sqrt{g}} k_{1n} u_n^2 H_n^2 \right) \frac{dx^3}{g_2} = 0. \qquad (3)$$

Here it is taken into consideration that the integral of terms proportional to h_n becomes zero because of the Hermitian nature of the operator $\hat{L}_T$. Equation (3) may be rewritten as

$$a_{1n} \nabla_1 (k_{1n} u_n^2) = a_{2n} k_{1n} u_n^2, \qquad (4)$$

where

$$a_{1n} = \oint \frac{H_n^2}{\sqrt{g}} dx^3, \qquad a_{2n} = \oint \nabla_1 \left(\frac{g_2 H_n^2}{\sqrt{g}} \right) \frac{dx^3}{g_2}.$$

By normalizing the function H_n in such a way that $a_{1n} = 1$ and designating $\tilde{k}_{1n} \equiv a_{2n}(x^1)/2$, we write the solution (4) as

$$u_n(x^1) = \frac{C}{\sqrt{|k_{1n}|}} \exp \left(- \int \tilde{k}_{1n} dx^1 \right),$$

where C is an arbitrary constant. Hence a general solution of (1) may be represented as

$$\begin{aligned} \Psi = \frac{C}{\sqrt{|k_{1n}|}} \; &\exp \left[i \int (k_{1n} + i\tilde{k}_{1n}) dx^1 + ik_2 x^2 - i\omega t \right] \\ &\times H_n(x^1, x^3). \qquad (5) \end{aligned}$$

The particular form of this solution depends on boundary conditions in coordinate x^1. In the next two sections we shall consider two problems corresponding to different boundary conditions.

3. Global Modes

Let us consider the problem of the magnetosonic wave incident on the magnetosphere from the solar wind ($x^1 > x^1_{\rm mp}$, where $x^1_{\rm mp}$ is the radial coordinate of the magnetopause). It will be recalled that in our accepted model of the medium we neglect the solar wind plasma motion which we intend to take into account in our future work. Thus a linear setting of the problem is a perfect two-dimensional analog for the problem which was solved in Walker's one-dimensional problem [*Walker*, 1998]. The distribution of magnetospheric plasma parameters across magnetic shells in the model of the medium used in this paper is presented in Figure 1. Such a distribution is characteristic for the dayside part of a moderately disturbed magnetosphere. Outside the magnetosphere, the solution (5) may be represented as

$$\Psi_n = \bar{\Psi}_n H_n(x^1, x^3) \exp\left(im\phi - i\omega t\right),$$

where m is the azimuthal wave number, ϕ is the azimuthal angle, and

$$\bar{\Psi}_n = k_{1n}^{-1/2} \exp\left(-\int \tilde{k}_{1n} dx^1\right) \times \left[C_{1n} \exp\left(-i\int k_{1n} dx^1\right) + C_{2n} \exp\left(i\int k_{1n} dx^1\right)\right], \tag{6}$$

where C_{1n} and C_{2n} are the amplitudes of the incident and reflected waves, respectively. Note that the generic exponential term in (6) represents a geometrical factor describing the variation in amplitude of the concerned waves as they propagate in a magnetic field with field lines that are concentrated toward the origin of coordinates. As is shown by *Leonovich and Mazur* [2000b], inside the magnetosphere there is a magnetic shell ($x^1 = x^1_n$) which separates the transparent region ($x^1 > x^1_n$) and the opaque region ($x^1 < x^1_n$) for the longitudinal eigenmode with number n and with fixed values of the azimuthal wave number m and the oscillation frequency ω.

Let the solution in the opaque region be represented as

$$\bar{\Psi}_n = \frac{C}{\sqrt{|k_{1n}|}} \exp\left[\int_{x^1_n}^{x^1} (|k_{1n}| - \tilde{k}_{1n}) dx^{1\prime}\right] \quad (x^1 < x^1_n).$$

Using the Swan method [see *Landau and Lifshitz*, 1963], we bypass the turning point $x^1 = x^1_n$ in the plane of a complex x^1 and obtain in the transparent region the solution of the form

$$\bar{\Psi}_n = \frac{C}{\sqrt{|k_{1n}|}} \exp\left(\int_{x^1_n}^{x^1} \tilde{k}_{1n} dx^{1\prime}\right) \times \sin\left(\int_{x^1_n}^{x^1} k_{1n} dx^{1\prime} + \frac{\pi}{4}\right) \quad (x^1 > x^1_n). \tag{7}$$

In a dipole-like geomagnetic field, the geometrical factor in (7) is (x^1_n/x^1); i.e., it varies according to a simple power law. The boundary condition on the magnetopause can be obtained immediately from equation (1). Since the problem of our interest neglects the motion of plasma in the solar wind region, this equation describes magnetospheric oscillations both inside the magnetosphere and in the solar wind. Upon integrating it across the magnetopause over an infinitely small interval, we obtain the matching condition $(\nabla_1 \Psi_n)_+ = (\nabla_1 \Psi_n)_-$. A double integration over this same interval yields a second matching condition $(\Psi_n)_+ = (\Psi_n)_-$. Here the plus and minus signs imply that these quantities are taken at the magnetopause, on the side of the solar wind (plus sign) and the magnetosphere (minus sign). Matching logarithmic derivatives of the solutions (6) and (7) on the magnetospheric boundary $x^1 = x^1_{\rm mp}$ yields

$$k^-_{1n} \cot\left(\int_{x^1_n}^{x^1_{mp}} k_{1n} dx^1 + \frac{\pi}{4}\right) = -ik^+_{1n} \frac{C_{1n} - C_{2n}}{C_{1n} + C_{2n}}, \tag{8}$$

where $k^\pm_{1n} \equiv k_{1n}(x^1_\pm)$ stand for the values of the transverse wave number to the right and to the left of the boundary ($x^1_\pm = \lim_{\varepsilon \to 0} (x^1_{\rm mp} \pm \varepsilon)$). In this problem the magnetopause represents a boundary on which the Alfven velocity changes abruptly from the values typical of the magnetosphere to those characteristic for the solar wind.

Figure 2 presents the wave field structure of magnetosonic oscillations across magnetic shells calculated for the following values of the problem parameters $m = 1, n = 3$, and $\omega = 0.024$ Hz. In the neighborhood of the turning point where the WKB approximation is inapplicable, the oscillation structure is described by the Airy function $Ai[(x^1_n - x^1)/\lambda_n]$, where $\lambda_n = (\nabla_1 k^2_{1n})^{-1/3}|_{x^1 = x^1_n}$ is the typical transverse scale of the oscillations near the turning point. In this example, the oscillations under consideration are remote from any eigenoscillations of the magnetosphere. In the transparent region inside the magnetosphere (between two vertical dashed lines) the wave field produces no harmonic structure. An interesting feature of this plot is the fact that the oscillation amplitude inside the magnetosphere turns out to be significantly larger in comparison with the solar wind, and because of the geometrical factor, it increases from the magnetopause to the ionosphere. Furthermore, in the solar wind the wave field is produced by the combination of the incident and reflected waves of equal amplitude (shown by the arrows).

If the reflection factor is now represented by the relation $R_n = C_{2n}/C_{1n}$, then from (8) we obtain for this factor

$$R_n = \frac{1 + i(k_{1n}^-/k_{1n}^+)\cot\left(\displaystyle\int_{x_n^1}^{x_{\rm mp}^1} k_{1n}dx^1 + \frac{\pi}{4}\right)}{1 - i(k_{1n}^-/k_{1n}^+)\cot\left(\displaystyle\int_{x_n^1}^{x_{\rm mp}^1} k_{1n}dx^1 + \frac{\pi}{4}\right)}. \quad (9)$$

In a one-dimensional inhomogeneous model of the medium, this factor goes entirely into a similar factor obtained by *Walker* [1998]. It is known that if the frequency of the incident wave coincides with one of the cavity's eigenfrequencies (with the frequency of the global modes in this case), then the reflection factor, thus introduced, becomes infinite because the resonance denominator becomes zero. This should be regarded as the presence of eigensolutions for (1) in the absence of a magnetosonic wave incident on the magnetosphere ($C_{1n} = 0, C_{2n} \neq 0$). Or, within the framework of the problem formulated above, this can be given the following physical explanation. If the frequency of the incident magnetosonic wave on the magnetosphere coincides with one of the resonance eigenfrequencies of the magnetosphere, then in order for the amplitude of the magnetosonic eigenmode in the magnetosphere to be finite, the amplitude of the incident wave must be infinitely small. Otherwise the incident wave with a finite amplitude will excite over an infinite time interval (the oscillations being monochromatic) magnetosonic oscillations of the magnetosphere to an infinitely large amplitude. This picture is true, of course, until the linear approximation is applicable for the waves under consideration. It is these solutions which correspond to global modes, magnetosonic eigenoscillations of the magnetosphere. Since in this case $C_{2n} \neq 0$, the magnetosphere is not a perfect cavity, and some of the energy of these eigenoscillations escapes to the solar wind. By setting the denominator of (9) to zero, we obtain the following quantization condition:

$$\int_{x_n^1}^{x_{mp}^1} k_{1n}dx^1 = \pi(j - \frac{1}{4}) + \frac{i}{2}\ln\frac{k_{1n}^+ + k_{1n}^-}{k_{1n}^+ - k_{1n}^-}, \quad (10)$$

where $j = 1, 2, ...$ is the transverse quantum number. Since $k_{1n} \equiv k_{1n}(\omega)$, equation (10) represents an equation for oscillation eigenfrequencies ω_{mnj} which are determined by three quantum numbers: m, azimuthal; n, longitudinal; and j, transverse.

Figure 3 presents the wave field structure of the eigenharmonic of magnetosonic oscillations of the magnetosphere corresponding to the following set of wave numbers: $m = 1, n = 1, j = 3$. It is evident that the transparent region develops a harmonic structure of wave field. The wave field outside the magnetosphere represents an escaping magnetosonic wave (only the real part of the field is shown as a matter of convenience). In order for the amplitude of magnetosonic eigenoscillations of the magnetosphere to be finite, the incident wave amplitude in the setting of the problem presented above should be considered infinitely small. In this case, the more adequate setting of the problem is, of course, the one involving the structure of magnetosonic eigenoscillations of the magnetosphere, in which the incident wave on the magnetosphere is lacking altogether. However, the final result, the structure, and the spectrum of eigenoscillations in each of such problems are identical. An interesting property of this plot is the fact that the turning point of these oscillations lies sufficiently close to the ionosphere, and because of the geometrical factor, the oscillation amplitude near the ionosphere turns out to be much larger than that in the solar wind. One would therefore expect that magnetosonic oscillations of a significant amplitude can be recorded on the ground in near-equatorial regions. From the form of equation (10) it follows that the eigenfrequencies are complex-valued $\omega_{mnj} = \text{Re}(\omega_{mnj}) + i\text{Im}(\omega_{mnj})$, where $\text{Re}(\omega_{mnj})$ is the real part of the frequency, and $\text{Im}(\omega_{mnj})$ is the damping decrement associated with the escape of some of the oscillation energy to the solar wind. If the jump of parameters on the magnetopause is very large ($k_{1n}^+ \gg k_{1n}^-$), then the logarithm in (10) becomes zero and the magnetosphere becomes a perfect cavity ($\text{Im}(\omega_{mnj}) = 0$).

Figur

Let us solve numerically equation (10) for eigenfrequencies ω_{mnj} by using a model distribution of parameters for the moderately disturbed dayside magnetosphere. This model was detailed by *Leonovich and Mazur* [2000b], and Figure 1 presents the corresponding dependencies of the equatorial distribution of the Alfven velocity $A(L)$ and of the main period of Alfven eigenoscillations of the magnetosphere $t_A(L)$ on the parameter of the magnetic shell L (MacIlwain parameter). Table 1 presents the frequencies of magnetosonic eigenoscillations of the magnetosphere for the first three values of quantum numbers m, n, j.

The upper part of each cell presents the values of eigenfrequencies calculated numerically from the model outlined above. The under part of the cell (except for $m = 0$) presents the frequencies calculated by the asymptotic formula

$$\omega_{mnj} \simeq A_0 \left[\frac{m}{r_0} + \frac{\xi_j m^{1/3}}{2a_0^{2/3} r_0^{1/3}} + \frac{(2n+1)}{2L_0} \right] - \frac{i}{2} \frac{A_0 A_1}{a_0 \sqrt{A_0^2 - A_1^2}} \quad (11)$$

obtained by *Leonovich and Mazur* [2000a] for large values of the azimuthal wave number $m \gg n, j$. Here $A_0 \approx 500\,\mathrm{km/s}$ is a minimum equatorial value of the Alfven velocity inside the magnetosphere ahead of the magnetopause; $A_1 \approx 50\,\mathrm{km/s}$ is the value of the Alfven velocity in the solar wind immediately behind the magnetopause; $r_0 \approx 10\,R_E$ is the radial coordinate of the magnetopause; a_0 is a typical transverse (L_0, longitudinal) scale of variation of A inside the magnetosphere ($a_0 \sim L_0 \sim r_0/3$); and $\xi_j \approx [3\pi(j-1/4)/2]^{2/3}$ is the jth zero of the Airy function $Ai(\xi)$ ($\xi_1 \simeq 2.3, \xi_2 \simeq 4, \xi_3 \simeq 5.6, ...$). To avoid misunderstanding, we wish to note that what formula (11) describes are magnetosonic (compressional) oscillations of the magnetosphere. The wavelength of such oscillations in an azimuthal direction is significantly smaller than the typical wavelength of these oscillations in the direction along field lines and across the magnetic shells. However, these oscillations should not be confused with the poloidal Alfvenic oscillations of the magnetosphere with $m \gg 1$ which were investigated by *Radoski* [1967] and *Leonovich and Mazur* [1993]. They are described by equations of a different form and have quite different properties.

The eigenfrequency values presented in Table 1 make it possible to determine the validity range of the estimating formula (11) for leading harmonics of magnetosonic oscillations of the magnetosphere. It is evident that the frequencies calculated numerically and obtained by formula (11) can differ more than two times. The eigenfrequency of the oscillations under consideration increases with increasing quantum numbers both in the numerical solution and in the estimating formula (11). Formula (11) can be used in a qualitative estimate of the global modes frequencies when $m \gtrsim n, j$. Note that the Q factor of the oscillations under consideration is sufficiently high ($\mathrm{Re}(\omega_{mnj}) \gg \mathrm{Im}(\omega_{mnj})$), which gives grounds to expect that these oscillations can be recorded in the magnetosphere with a sufficient amplitude.

Note that the frequency of the lowest-frequency eigenoscillations is considerably above that of the oscillations which are observed by geostationary satellites in the dayside magnetosphere $\omega \sim 10^{-3}$(rad/s) [see *Harrold and Samson*, 1992; *Samson and Rankin*, 1994; *Lessard et al.*, 1999]. Hence global modes in the dayside magnetosphere calculated in terms of the model stated above obviously cannot be such low-frequency oscillations. To estimate the frequency of the lowest-frequency harmonic of magnetosonic eigenoscillations of the magnetosphere, one can use the expression $f \sim \bar{A}/L$, where $\bar{A}$ is the mean value of the Alfven velocity in the magnetosphere and L is the typical scale of the magnetospheric cavity. In the model of the medium used in this study, $\bar{A} \approx 500$ km/s, and $L = 10\,R_E \approx 6.4 \times 10^3$ km which gives $f \sim 8$ mHz ($\sim 4.8 \times 10^{-2}$ rad/s). The lowest-frequency oscillations with a stable spectrum, observed in the magnetosphere, have the frequency $f \sim 1\,\mathrm{mHz}$. If the typical scale of the magnetosphere is represented by the largest possible transverse scale of the magnetosphere $L \sim 15\,R_E$, then to ensure the observed frequency $f \sim 1$ mHz, it is necessary that $\bar{A} \approx 100$ km/s. Such low values of the mean Alfven velocity in the magnetosphere are unlikely.

4. Cavity Under the Plasmapause

Guglielmi [1970, 1972] for the first time showed that near the plasmapause there is a transparent region for magnetosonic oscillations bounded across the magnetic shells on two sides by turning points. Hence there are conditions for the existence of magnetosonic eigenoscillations confined inside such a cavity. Subsequently, the structure of these eigenmodes was studied in different models of magnetospheric plasma distribution by *Zhu and Kivelson* [1989], *Lee* [1996], *Fedorov et al.* [1998], and *Leonovich and Mazur* [2000b]. Let us determine the transverse structure and the frequency spectrum of the eigenoscillations of this cavity in terms of the magnetospheric model described in the preceding section.

Let x_{n1}^1 denote the coordinate of the magnetic shell separating the cavity's transparent region under the plasmapause and the inner (nearer to the ionosphere) opaque region, and let x^1_{n2} refer to the coordinate of the shell separating the transparent region and the outer opaque region. The solution inside the transparent region satisfying the boundary condition when $x^1 > x^1_{n1}$ is given by (7), where we must substitute $x^1_n \to x^1_{n1}$. A similar formula for the solution satisfying the boundary condition when $x^1 < x^1_{n2}$ may be written by substituting in (7) the lower and upper limits of integration for x^1 and x^1_{n2}, respectively, and by adding $\pi/2$ to the phase. For the oscillation eigenmode these solutions must both coincide, which is possible only when the quantization condition is satisfied:

$$\int_{x^1_{n1}}^{x^1_{n2}} k_{1n} dx^1 = \pi(j - \frac{1}{2}), \qquad (12)$$

where $j = 1, 2, \ldots$. This equation determines the cavity's eigenfrequencies ω_{mnj} in the zero-order approximation when the opaque regions bounding the cavity across the magnetic shells can be considered infinitely extended.

However, this statement is not universally true under real magnetospheric conditions. The point is that the outer opaque region separating the cavity from the outer magnetosphere represents a barrier of a finite height and width. This leads to a partial escape of the energy from the cavity to the outer magnetosphere; i.e., the eigenoscillation frequencies of such a cavity are complex valued. The imaginary part of the frequency represents a damping decrement of the oscillations associated with the escape of some of their energy through the barrier. As was shown by *Leonovich and Mazur* [2000a], the value of this decrement depends on the values of the quantum numbers m, n, and j which determine the spectrum of eigenfrequencies ω_{mnj}. At large values of the azimuthal wave number $m \gg 1$ this decrement is small ($\mathrm{Im}(\omega_{mnj}) \ll \mathrm{Re}(\omega_{mnj})$). In this case, the potential well in which the cavity's eigenmode is localized is deep and the escape of energy from it is small. When $m \sim 1$, the value of the decrement becomes comparable with the eigenfrequency of the mode ($\mathrm{Im}(\omega_{mnj}) \sim \mathrm{Re}(\omega_{mnj})$), which implies a decrease in the depth of the potential well and an increase in the escape of the oscillation energy to the outer magnetosphere.

Figure 4 presents the structure of eigenoscillations of the cavity under the plasmapause for the following values of quantum numbers: $m = 1, n = 3, j = 1$. It is evident that the wave field decreases to zero inside the opaque region adjacent to the ionosphere. Inside the second opaque region (between the second and third vertical dashed lines in Figure 4), an exponential decrease proceeds only to a certain finite value. This is due to the finiteness of the width and height of the barrier. Farther away is the outer transparent region propagating to the magnetopause and further out to the solar wind. For comparison, the figure presents the structure of the magnetosonic wave, for which the transparent region (cavity) under the magnetopause is absent. Thus a total structure of wave field is determined by the structure of the solution in the cavity and in the outer magnetospheric region. Therefore a total spectrum of eigenfrequencies of magnetosonic oscillations of the magnetosphere will represent a combination of the eigenfrequencies of the outer magnetosphere and of the cavity under the plasmapause [see *Lee and Kim*, 1999]. In this paper we confine ourselves to investigating the frequency spectrum of the eigenoscillations of such a cavity by neglecting their damping associated with the escape through the barrier.

Let us solve numerically equation (12) for the cavity's eigenfrequencies. The frequency spectrum of several leading harmonics of the eigenoscillations is presented in Table 2. Note that not all of the cells in this table are filled, since at fixed values of the quantum numbers n and j the potential well becomes deep enough for the existence of the eigenmode, beginning with a certain value of $m = \bar{m}_{nj}$. For instance, we have $\bar{m}_{00} = 2$, $\bar{m}_{10} = 7$, $\bar{m}_{20} = 13, \ldots$. When $m < \bar{m}_{nj}$ the external points for the oscillations under consideration disappear, and the cavity for them is absent. As in Table 1, the upper part of each cell presents the eigenfrequencies ω_{mnj} calculated numerically by formula (12), and the lower part presents the frequencies obtained by the asymptotic formula (11), in which $A_0 \approx 400\,\mathrm{km/s}$, $r_0 \approx 4\,R_E$, $a_0 \sim L_0 \sim R_E$.

As here, $m \gtrsim n, j$; the frequencies calculated numerically and obtained by the asymptotic formula do not differ by more than a 30%. Formula (11) can be used for quantitative estimation of the eigenoscillation frequencies of the cavity located under the plasmapause. Note that at the same values of the azimuthal m, longitudinal n, and transverse j wave numbers, the structure of the eigenoscillations in the cavity under the plasmapause and of the global modes is different because they are localized in regions having a different size. This leads to the fact that the eigenfrequencies of such oscillations ω_{mnj} are essentially different.

5. Conclusion

The main results of this study may be summarized as follows:

1. We have obtained equations (5)-(7) describing the field structure of magnetosonic eigenoscillations of the axisymmetric magnetosphere across magnetic shells in the WKB approximation.

2. It has been shown that the amplitude of magnetosonic oscillations in a dipole-like magnetic field, in which the field lines are concentrated toward the origin of coordinates, increases in the direction from the magnetopause to the ionosphere because of geometrical factor. This furnishes a means of observing, in near-equatorial regions on the ground, magnetosonic oscillations of a significant amplitude, even if near the magnetopause they have only a moderate amplitude.

3. The expression (9) was obtained for the reflection factor of the magnetosonic wave incident from the solar wind on the axisymmetric magnetosphere.

4. We have determined the frequency spectrum of magnetosonic eigenoscillations of the axisymmetric magnetosphere (global modes) with proper account of the escape of some of their energy to the solar wind. It has been demonstrated that the eigenoscillation frequencies calculated numerically and obtained by the asymptotic formula (11) can differ by more than a factor of 2. In both cases the dependence of the eigenfrequencies on the values of quantum numbers (azimuthal m, longitudinal n, and transverse j) is qualitatively similar.

5. We have solved the problem of the frequency spectrum of magnetosonic eigenoscillations within the cavity under the plasmapause. It has been shown that the spectrum of these oscillations, calculated numerically for the axisymmetric model of the magnetosphere, is in qualitative agreement with that obtained by the asymptotic formula (11). The existence conditions of such oscillations depend heavily on the values of the quantum numbers m, n, and j. At fixed values of n and j, the cavity can accommodate only oscillations with $m \geq \bar{m}_{nj}$, where $\bar{m}_{nj}$ is a minimum value of the azimuthal wave number at which the eigenmode exists in the cavity.

Acknowledgments. We are grateful to V. G. Mikhalkovsky for his assistance in preparing the English version of the manuscript and for typing the text.

Janet G. Luhmann thanks Dong-Hun Lee and another referee for their assistance in evaluating this paper.

References

Allan, W., S. P. White, and E. M. Poulter, Impulse-excited hydromagnetic cavity and field line resonance in the magnetosphere, *Planet. Space Sci., 34,* 371-380, 1986.

Cummings, W. D., R. L. O'Sullivan, and P. J. Coleman, Standing Alfven waves in the magnetosphere, *J. Geophys. Res., 74,* 778-786, 1969.

Dungey, J. W., Electrodynamics of the outer atmospheres, *Ionos. Sci. Rep. 69,* Ionos. Res. Lab., Cambridge, Pa., 1954.

Fedorov, E., M. Mazur, and V. Pilipenko, MHD wave conversion in plasma waveguides, *J. Geophys. Res., 103,* 26,595-26,605, 1998.

Freeman, M. P., Effect of magnetopause leakage on the magnetospheric cavity modes, *J. Geophys. Res., 105,* 5463-5470, 2000.

Guglielmi, A. V., The ring trap for low-frequency waves in the Earth's magnetosphere (in Russian), *Pis'ma Zh. Eksp. Teor. Fiz., 12,* 35-38, 1970.

Guglielmi, A. V., The magnetosonic channel under the plasmapause (in Russian), *Geomagn. Aeron., 12,* 147-154, 1972.

Harrold, B. G., and J. C. Samson, Standing ULF modes of the magnetosphere: A theory, *Geophys. Res. Lett., 19,* 1811-1814, 1992.

Kivelson, M. G., and D. J. Southwood, Coupling of global magnetospheric MHD eigenmodes to field line resonances, *J. Geophys. Res., 91,* 4345-4351, 1986.

Klimushkin, D. Yu., Method of description of the Alfvén and magnetosonic branches of inhomogeneous plasma oscillations, *Plasma Phys. Rep., 20,* 280–286, 1994.

Landau, L. D., and E. M. Lifshitz, *Theoretical Physics* (in Russian), vol. 3, Nauka, Moscow, 1963.

Lee, D.-H., Dynamics of MHD wave propagation in the low-latitude magnetosphere, *J. Geophys. Res., 101,* 15,371-15,386, 1996.

Lee, D.-H., and K. Kim, Compressional MHD waves in the magnetosphere: A new approach, *J. Geophys. Res., 104,* 12,379-12,385, 1999.

Lee, D.-H., and R. L. Lysak, Magnetospheric ULF wave coupling in the dipole model: The impulsive excitation, *J. Geophys. Res., 94,* 17,097-17,109, 1989.

Lee, D.-H., and R. L. Lysak, Monochromatic ULF wave excitation in the dipole magnetosphere, *J. Geophys. Res., 96,* 5811-5823, 1991.

Lee, D.-H., and R. L. Lysak, Numerical studies on ULF wave structures in the dipole model, in *Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves,* Geophys. Monogr. Ser., vol. 81, edited by M. J. Engebretson, K. Takahashi, and M. Scholer, pp. 293-297, AGU, Washington, D.C., 1994.

Leonovich, A. S., and V. A. Mazur, Resonance excitation of standing Alfven waves in an axisymmetric magnetosphere (monochromatic oscillations), *Planet. Space Sci., 37,* 1095-1108, 1989.

Leonovich, A. S., and V. A. Mazur, A theory of transverse small-scale standing Alfven waves in an axially symmetric magnetosphere, *Planet. Space Sci., 41,* 697- 717, 1993.

Leonovich, A. S., and V. A. Mazur, Toward the theory of magnetospheric magnetosonic eigenoscillations: Simple theoretical models, *J. Geophys. Res., 105,* 21,251-21,260, 2000a.

Leonovich, A. S., and V. A. Mazur, Structure of magnetosonic eigenoscillations of an axisymmetric magnetosphere, *J. Geophys. Res., 105,* in press, 2000b.

Lessard, M. R., M. K. Hudson, J. C. Samson, and J. R. Wygant, Simultaneous satellite and ground-based observations of a discretely driven field line resonances, *J. Geophys. Res., 104,* 12,361-12,371, 1999.

Mann, I. R., A. N. Wright, K. L. Mills, and V. M. Nakariakov, Excitation of magnetospheric waveguide modes by magnetosheath flows, *J. Geophys. Res., 104,* 333-353, 1999.

McClay, J. F., On resonant modes of a cavity and the dynamical properties of micropulsations, *Planet. Space Sci., 18,* 1673-1682, 1970.

McKenzie, J. F., Hydromagnetic wave interaction with the magnetopause and the bow shock, *Planet. Space Sci., 18,* 1-23, 1970.

Radoski, H. R., Highly asymmetric MHD resonances: The guided poloidal mode, *J. Geophys. Res., 72,* 4026-4033, 1967.

Samson, J. C., and R. Rankin, The coupling of solar wind energy to MHD cavity modes, waveguide modes, and field line resonances in the Earth's magnetosphere, in *Solar Wind Sources of Magnetospheric Ultra-Low-Frequency Waves,* Geophys. Monogr. Ser., vol. 81, edited by M. J. Engebretson, K. Takahashi, and M. Scholer, pp. 253-264, AGU, Washington, D.C., 1994.

Southwood, D. J., and M. G. Kivelson, The effect of parallel inhomogeneity of magnetospheric hydromagnetic wave coupling, *J. Geophys. Res., 91,* 6871-6877, 1986.

Walker, A. D. M., Excitation of magnetohydrodynamic cavities in the magnetosphere, *J. Atmos. Sol. Terr. Phys., 60,* 1279-1293, 1998.

Zhu, X., and M. G. Kivelson, Global mode ULF pulsations in a magnetosphere with a nonmonotonic Alfven velocity profile, *J. Geophys. Res., 94,* 1479-1485, 1989.

A. S. Leonovich and V. A. Mazur, Institute of Solar-Terrestrial Physics, Irkutsk 33, P.O. Box 4026, 664033, Russia. (leon@iszf.irk.ru; vmazur@iszf.irk.ru)

Received June 13, 2000; revised September 26, 2000; accepted September 26, 2000.

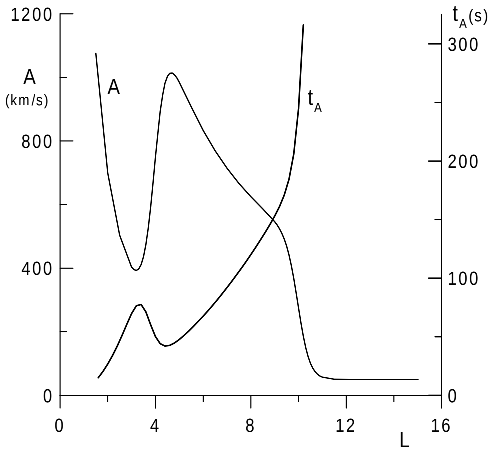

Figure 1. Equatorial dependence of the Alfven velocity $A(L, 0)$ and of the main period of Alfven eigenoscillations $t_A(L)$ on the parameter of the magnetic shell L in the axisymmetric magnetospheric model used in numerical calculations.

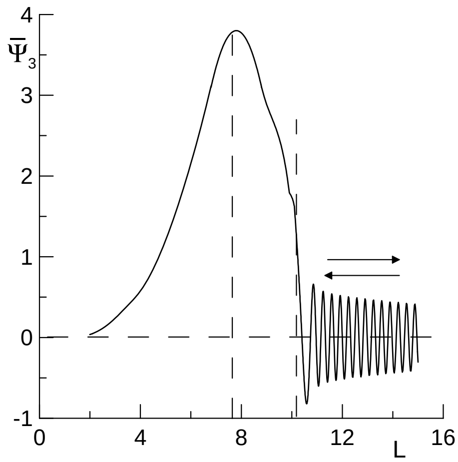

Figure 2. Structure of the magnetosonic oscillation field when the wave is incident from the solar wind and is reflected from the magnetosphere in the case where the wave frequency is far away from the eigenoscillation frequency of the magnetosphere. Arrows in the figure intimate that the magnetosonic oscillation field outside the magnetosphere is caused by the combination of the incident and reflected waves. Vertical dashed lines show the position of the turning point ($L = 8.2$) and of the magnetopause ($L = 10$).

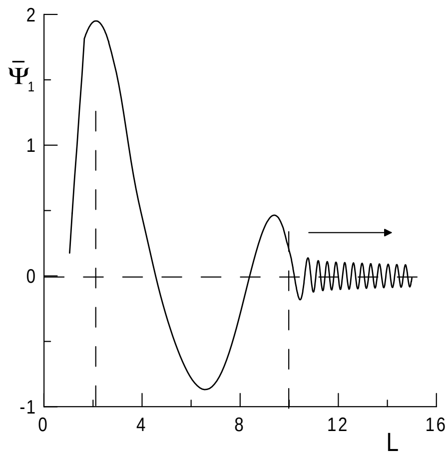

Figure 3. Field structure of magnetosonic eigenoscillations of the magnetosphere for the eigenmode with the following set of wave numbers: $m = 1, n = 1, j = 3$. The arrow intimates that the field outside the magnetosphere is caused by the field of the escaping magnetosonic wave (only the real part of the field is shown). Vertical dashed lines show the position of the turning point ($L = 2.1$) and of the magnetopause ($L = 10$).

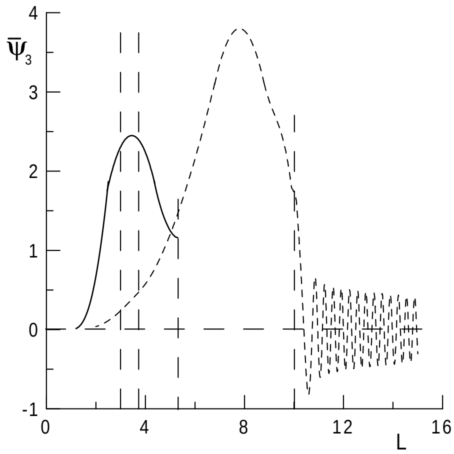

Figure 4. Wave field structure of magnetosonic eigenoscillations of the cavity under the plasmapause for the eigenmode with the following set of wave numbers: $m = 1, n = 3, j = 1$ (solid line). For comparison, the short-dashed line shows the wave structure of magnetosonic waves with no cavity under the plasmapause for them. Vertical dashed lines show the position of the turning points ($L = 2.8, 3.8, 5.6$) and of the magnetopause ($L = 10$).

Table 1. Frequencies of Global Modes
[Re(ω_{mnj})(rad/s), Im(ω_{mnj})(s^{-1})]$\times 10^{-2}$

	n		
m	0	1	2
		j=1	
0	(4.58, −0.12)	(9.41, −0.12)	(13.76, −0.14)
1	(4.99, −0.16) (3.88, −0.12)	(9.66, −0.16) (6.28, −0.12)	(14.0, −0.15) (8.68, −0.12)
2	(5.68, −0.17) (6.19, −0.12)	(10.15, −0.17) (8.59, −0.12)	(14.4, −0.15) (11.0, −0.12)
		j=2	
0	(9.1, −0.1)	(17.4, −0.14)	(23, −0.12)
1	(10.8, −0.15) (5.3, −0.12)	(17.6, −0.15) (7.7, −0.12)	(23.1, −0.15) (10.1, −0.12)
2	(11.8, −0.16) (7.0, −0.12)	(18, −0.16) (9.4, −0.12)	(23.4, −0.14) 11.8, −0.12)
		j=3	
0	(13.3, −0.13)	(23.3, −0.14)	(30.2, −0.16)
1	(13.5, −0.15) (6.6, −0.12)	(24.0, −0.15) (9.0, −0.12)	(30.3, −0.16) (11.4, −0.12)
2	(14.1, −0.18) (8.5, −0.12)	(24.6, −0.16) (10.9, −0.12)	(30.7, −0.15) (13.3, −0.12)

Table 2. Frequencies of the Cavity's Magnetosonic Eigenmodes Under the Plasmapause: ω_{mnj}(rad/s)

m	n: 0	n: 1	n: 2
	j=1		
0	...	...	0.32
1	.../...	0.23/0.124	0.327/0.231
2	0.136/0.155	0.238/0.261	0.334/0.353
	j=2		
3	...	...	0.473
5	.../...	0.39/0.305	0.5/0.41
7	0.317/0.343	0.42/0.456	0.527/0.558
	j=3		
9	...	...	0.662
11	.../...	0.586/0.637	0.694/0.733
13	0.523/0.583	0.623/0.677	0.728/0.774