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Alfvenic and magnetosonic resonances in a nonisothermal plasma

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Abstract
We investigate the problem of the incidence and reflexion of a monochromatic magnetosonic wave at the transition plasma layer with two resonant surfaces—for the Alfven and slow magnetosonic oscillations. A qualitative analysis of the solution to this problem is performed. It is shown that absorption of the incident wave energy at the transition layer increases substantially when there is a resonant surface for slow magnetosonic oscillations. In the neighbourhood of this resonant surface, the energy of an incident wave is totally absorbed, resulting in additional plasma heating. There is a range of frequencies and wavelengths for which the absorption coefficient is 100%. Our numerical calculations have shown that the energy absorption coefficient for an incident fast magnetosonic wave in plasma with $\beta \sim 1$ exceeds its counterpart in 'cold' plasma or in a plasma configuration with one resonant surface, for Alfven waves only.

1. Introduction

MHD oscillations play an important part in many processes in various plasma media [1–3]. In homogeneous plasma there are three independent branches of MHD oscillations—the Alfven waves, the fast and slow magnetic sound (FMS and SMS). In an inhomogeneous plasma these oscillations become inseparable and exist as a united field of MHD oscillations, whose dispersion properties in some regions may be like those of one of the branches of MHD oscillations in a homogeneous plasma. Separating them from each other is possible only for a few special cases. In theoretical papers dealing with MHD oscillations in inhomogeneous plasma, the transition between regions with oscillations with different dispersion is described in terms of interaction of different MHD oscillation branches [4, 5]. This coupling has a resonant nature and occurs at a small spatial scale.

The Alfven resonance is the best known of such processes [6, 7]. In this process a monochromatic FMS wave propagating in an inhomogeneous plasma excites an Alfven wave at a resonant shell where the local frequency of Alfven eigen oscillations coincides with the frequency of the FMS wave. Part of the energy transported by the FMS wave is absorbed by the plasma in the neighbourhood of the resonant magnetic shell, leading to plasma heating. An interesting feature of such a resonant interaction is that, for a small decrement of Alfven waves, the FMS wave energy loss does not depend on the dissipation mechanism involved: whether it is the local electrical resistance of the plasma [8] or the resistance at the ends of magnetic field lines [9], or Alfven waves moving away from the resonant region across the magnetic shells due to their small transverse dispersion [10]. The process of Alfven resonance is applied to explaining some types of global magnetospheric oscillations [11], the heating of solar coronal plasma [12, 13] and is also offered as a mechanism of plasma heating in nuclear fusion devices [1, 14].

Another, lesser known, type of resonant interaction is the magnetosonic resonance, in which a magnetosonic wave drives the SMS oscillations on the resonant shell [15, 16]. The SMS, similar to Alfven waves, when propagating in inhomogeneous plasma, is easily canalized in the direction along the magnetic field. This produces a possibility for their resonant interaction with the FMS, the same as in the Alfven resonance case.

Generally, the above types of resonant interactions of MHD waves are studied separately [19, 20]. However, a considerable part of real plasma configurations under study involves both types of resonance and a correct understanding of the processes requires them to be examined together. The problem of the two above-mentioned resonant Alfven and SMS oscillations excited by a magnetosonic wave incident on the transition plasma layer is treated in [15]. That work exhibits a correct qualitative approach to solving the energy absorption problem for magnetosonic waves incident on the transition layer. The conclusion is that the energy absorption coefficient increases substantially for the incident waves, in the presence of a resonant surface for SMS oscillations. However, quantitative applicability of the results obtained in that paper is restricted by a certain range of the background plasma ion and electron temperatures. That paper used a method which takes into account local electrical resistance in the plasma, in a narrow neighbourhood of resonant surfaces in order to pass through the region of resonant coupling, both for the Alfven and SMS waves. The rank of the equations to be solved increases, which allows a mathematically correct solution in the regions of resonant interaction. The method is justified for the Alfven waves, since they usually experience a weak decay. In this case, as was noted above, any physical processes resulting in the escape of wave energy from the resonance region or its dissipation may be taken into account in order to regularize the solution singularity in the neighbourhood of a resonant surface. As long as these effects are small, they do not alter the integral energy absorption coefficient of resonant oscillations.

The situation with SMS oscillations is different. In plasma with $\beta \lesssim 1$ at $T_e \sim T_i$ or $T_e < T_i$ (where T_e, T_i are the temperatures of plasma electrons and ions, respectively) the speed of SMS waves is close to that of the ion sound of the background plasma. This results in strong dissipation of SMS waves due to their collisionless interaction with background plasma ions (Landau's damping [17]). The only exception is strongly nonisothermal plasma with $T_e \gg T_i$ [18]. In such a plasma the speed at which the SMS waves travel is determined by the plasma electron, rather than the ion temperature. This velocity is sufficiently far from the ion sound velocity, allowing the SMS waves to propagate in such plasmas without experiencing any substantial dissipation. As was noted above, the wave energy absorption coefficient in this case is never altered by including whatever dissipation mechanism of resonant oscillations with small decrements. In plasma with $T_e \sim T_i$ or $T_e < T_i$ collisionless dissipation of oscillations is high, therefore the absorption coefficient changes essentially. Therefore employing the decrement for SMS waves with the Landau damping would be more correct in these cases for calculating the energy dissipation near the resonant surfaces.

There is one more feature related to the transition layer model used in [15]. The model consists of two homogeneous half-spaces joined by the transition layer with a linear profile. Constructing a complete solution for the field of waves incident on and reflected by the transition layer consists of finding analytical solutions in each of the homogeneous half-spaces, a numerical solution in the transition layer, followed by their matching. Using such a model, as will be evident in the results of this paper, does not allow an essentially important result to be obtained, namely, the existence of a particular range of frequencies and wave vectors of magnetosonic waves incident on the transition layer, the energy of which is completely absorbed in the neighbourhood of the resonance surface for SMS waves. This conclusion follows from the analytical solution of the problem in a model with a smoothly varying transition layer in which the reflected wave is absent. In the model in [15] the reflected wave inevitably appears in matching the solutions obtained in the transition layer and in the regions of propagation of the incident and reflected magnetosonic waves. This is an essential property of any models with parameters the derivative of which has a sudden jump at one or more points.

In this paper we will consider a plasma configuration in the form of a transition layer with an Alfven speed profile of the form $\tanh(x)$ where both types of resonances exist. Besides, we consider a nonisothermal plasma, where temperatures of ions and electrons can differ considerably. The problem is reduced to inspecting the process of incidence and reflexion from such a transition layer of the monochromatic magnetosonic wave. Part of the incident wave energy is absorbed in the neighbourhood of resonant shells, resulting in plasma heating.

It is necessary to note that a theoretical description of the resonant interaction of MHD oscillations usually involves a set of ideal MHD equations. In the frame of this approach, it is impossible to consider the MHD oscillation dissipation due to resonant interaction between waves and plasma particles. Such interaction is described adequately in the kinetic approach. Unfortunately, it is difficult enough for this approach to describe the interaction of different branches of MHD waves in an inhomogeneous plasma. This paper suggests a mixed scheme for calculating the MHD oscillation field to be applied in the following manner. The structure of the field of low-frequency MHD oscillations far from the resonant surfaces is described by a set of two-fluid MHD equations.

To regularize the singularities in the resulting solution near the resonant surfaces, an effective decrement is introduced as an imaginary additive to the oscillation frequency. The decrement of Alfven waves is assumed to be small enough in the neighbourhood of the resonant surface, so that the form of the resulting solution is practically unaffected by the specific way it is expressed. Unlike the Alfven oscillations, the decrement of resonant SMS oscillations can be comparable to their frequency. Its specific form determines the structure of the solution not only in the neighbourhood of but also far from the resonant surface. We determine the value of the decrement by solving the local kinetic equation for homogeneous plasmas with parameters corresponding to the resonant surface for SMS waves. At the same time, the decrement is introduced into MHD equations taking into account the SMS oscillation field localization on the scale defined by the solution of these equations.

This paper is structured as follows. The model of the medium is presented and the basic equations obtained describing the field of MHD oscillations in section 2. In the first part of section 3 the solution of the problem is investigated qualitatively. In order to find a solution in the neighbourhood of resonant surfaces and turning points the coefficients of the equation describing the MHD oscillations are linearized. Far from these surfaces the thus-found solution coincides with the WKB one. In the second part of section 3 the local damping decrement of SMS waves in homogeneous nonisothermal plasma is obtained. In numerical calculations the decrement is formally introduced into the equations as an imaginary part of the frequency

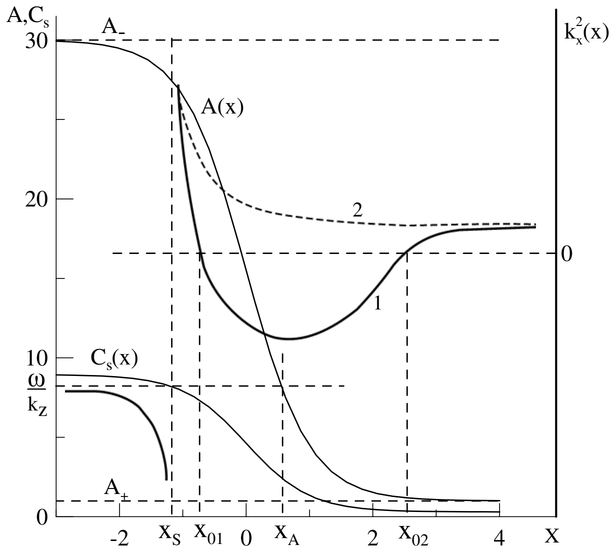

Figure 1. Profiles of Alfven speed $A(x)$, velocity of SMS waves $C_s(x)$ and square of the wave vector WKB component $k_x^2(x)$ across the transition layer.

of oscillations under study with regard to their localization near the magnetosonic resonant surface. In section 4 the solution is numerically investigated for oscillations to which the WKB approximation is inapplicable. The main results of the paper are summarized in the section 5.

2. Model of the medium and basic equations

We introduce a system of Cartesian coordinates (x, y, z) for solving the problem we wish to solve. We consider a plasma configuration in which the magnetic field is directed along the z axis, and the plasma parameters vary in the x direction. The y axis completes a right-hand set. Figure 1 presents the characteristic distribution of the Alfven speed and the velocity of SMS waves in the plasma configuration in question.

To describe the MHD oscillation field we will use the system of MHD equations of the form

$$\rho \frac{\mathrm{d}\bar{\boldsymbol{v}}}{\mathrm{d}t} = -\nabla \bar{P} + \frac{1}{4\pi}[\mathrm{curl}\,\bar{\boldsymbol{B}} \times \bar{\boldsymbol{B}}], \tag{1a}$$

$$\frac{\partial \bar{\boldsymbol{B}}}{\partial t} = \mathrm{curl}[\bar{\boldsymbol{v}} \times \bar{\boldsymbol{B}}], \tag{1b}$$

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla(\rho \bar{\boldsymbol{v}}) = 0, \tag{1c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\bar{P}}{\bar{\rho}^{\gamma}} = 0, \tag{1d}$$

where $\bar{\boldsymbol{B}}$, $\bar{\boldsymbol{v}}$ are the vectors of the magnetic field and velocity of plasma motion, $\bar{\rho} = \bar{n}(m_\mathrm{e}+m_\mathrm{i})$, $\bar{P} = \bar{n}(T_\mathrm{e}+T_\mathrm{i})$ are the density and pressure ($\bar{n}$ is the concentration) of a two-component plasma, $\gamma = 5/3$ is the adiabatic constant. We will assume that the wave-related disturbance is weak enough to permit the initial set of equations to be linearized. We use subscripts zero for the parameters of the unperturbed plasma, while the wave-related parameters will have no indices ($\bar{\boldsymbol{B}} = \boldsymbol{B}_0 + \boldsymbol{B}$, $\bar{\boldsymbol{v}} = \boldsymbol{v}_0 + \boldsymbol{v}$, $\bar{\rho} = \rho_0 + \rho$, $\bar{P} = P_0 + P$). We will assume the background plasma to be immobile ($\boldsymbol{v}_0 = 0$). In the zero approximation the x component of (1a) gives the equilibrium condition of a plasma configuration in steady state ($\partial/\partial t = 0$):

$$P_0 + \frac{B_0^2}{8\pi} = \text{const}. \tag{2}$$

Denote by $v_x = \partial\zeta/\partial t$ the x component of the plasma velocity vector in the wave, where ζ is the displacement of the plasma element. We will consider a simple harmonic wave, which in the y and z directions is a plane wave of the form $\exp(\mathrm{i}k_y y + \mathrm{i}k_z z - \mathrm{i}\omega t)$, where k_y, k_z are the corresponding components of the wave vector, ω is the wave frequency. Linearizing the set of equations (1a) and (1d) and expressing the other components of the oscillation field through ζ, we obtain

$$v_x = -\mathrm{i}\omega\zeta, \qquad v_y = -\frac{k_y}{\omega}\frac{1}{K_s^2}\left(A^2 + \frac{K_A^2 S^2}{\chi_S^2}\right)\frac{\partial\zeta}{\partial x}, \qquad v_z = -\frac{k_z K_A^2 S^2}{\omega\chi_S^2}\frac{\partial\zeta}{\partial x}, \tag{3a}$$

$$B_x = -\mathrm{i}k_z B_0\zeta, \qquad B_y = -\frac{k_z B_0}{\omega}v_y, \qquad B_z - \frac{K_A^2 B_0}{\chi_s^2}\left(1 - \frac{k_z^2 S^2}{\omega^2}\right)\frac{\partial\zeta}{\partial x}, \tag{3b}$$

$$P = -\gamma P_0 \frac{K_A^2}{\chi_S^2}\frac{\partial\zeta}{\partial x}, \tag{3c}$$

where

$$K_A^2 = 1 - \frac{k_z^2 A^2}{\omega^2}, \qquad K_s^2 = K_A^2 - \frac{k_y^2 A^2}{\omega^2},$$
$$\chi_S^2 = 1 - \frac{k_y^2 + k_z^2}{\omega^2}\left(A^2 + S^2 - \frac{k_z^2 A^2 S^2}{\omega^2}\right).$$

$A = B_0/\sqrt{4\pi\rho_0}$ is the Alfven speed, $S = \sqrt{\gamma P_0/\rho_0}$ is the sound speed in plasma. For the displacement ζ we have the equation

$$\frac{\partial}{\partial x}\frac{\rho_0\Omega^2}{k_x^2}\frac{\partial\zeta}{\partial x} + \rho_0\Omega^2\zeta = 0, \tag{4}$$

where $\Omega^2 = \omega^2 - k_z^2 A^2$,

$$k_x^2 = -k_y^2 - k_z^2 + \frac{\omega^4}{\omega^2(A^2 + S^2) - k_z^2 A^2 S^2}. \tag{5}$$

It is evident from (4) that k_x^2 is the square of the wave vector x-component in the WKB approximation when the solution may be represented as $\zeta \sim \exp(\mathrm{i}\int k_x\,\mathrm{d}x)$.

The behaviour of the $k_x^2(x)$ function is important for a correct statement of the problem. We wish to explore a process of the incidence and reflexion of a magnetosonic wave on a smoothly varying transition layer in which there are two resonant surfaces—for the Alfven and SMS waves. The solution of the problem should be the solution of (4), which is a superposition of incident and reflected finite-amplitude waves as $x \to \pm\infty$. The magnitude of $A_+ \equiv A(\infty)$ should be chosen such that $k_x^2(\infty) > 0$ for the chosen magnitude of ω. In this case there is a region of transparency for magnetosonic waves as $x \to \infty$. Two variants of distribution of the $k_x^2(x)$ function are possible, labelled as 1 and 2 in figure 1. Analysis of (5) reveals that for a monotonic increase in $A(x)$ when the x coordinate varies from $+\infty$ to $-\infty$ the $k_x^2(x)$ function passes through zero twice at the points we will denote as x_{01}, x_{02}, between which the opacity region (where $k_x^2(x) < 0$) is located. Such a behaviour of $k_x^2(x)$ is illustrated by curve 1 in figure 1 and corresponds to the case of an FMS wave incident on the transition layer.

There are also two singular points in equation (4) in which the coefficient attached to the higher derivative tends to zero. One of them—the point of the Alfven resonance x_A, defined by the equality $\Omega^2(x_A) = 0$—is located in the opacity region in the interval (x_{01}, x_{02}). The second one is the point of magnetosonic resonance x_S, where the denominator in the expression (5) tends to zero, which yields the local dispersion equation for SMS waves as $|k_x^2| \to \infty$: $\omega^2 = k_z^2 C_s^2(x_S)$, where $C_s^2 = A^2 S^2/(A^2 + S^2)$. Point x_S is left of the turning point x_{01}, and the region of transparency for SMS waves is located between them. To the left of x_S is the opacity region reaching $-\infty$ in the x direction.

The behaviour of $k_x^2(x)$ is qualitatively shown in figure 1. The value of $A_- \equiv A(-\infty)$ should be such that there exist a point of magnetosonic resonance x_S. Part of the energy of the incident wave is absorbed in the neighbourhood of two resonant surfaces $x = x_A$ and $x = x_S$, therefore the reflected wave has a smaller amplitude than has the incident wave. The difference between the energy of the incident and reflected waves is spent on heating the plasma in the neighbourhoods of the resonant surfaces. The absorption coefficient, defined as the ratio of this difference to the energy of the incident wave, depends on the plasma parameters on the resonant surfaces. In the following sections we explore in detail the structure of the field of MHD oscillations under study and the dependence of the absorption coefficient on the values of the wave vector components k_y, k_z, the ratio of the plasma electron to ion temperature T_e/T_i and the β parameter. Note that the exact expression for the $\beta = 8\pi P_0/B_0^2$ parameter, which is the ratio of gas-kinetic pressure of plasma to magnetic pressure, accurate to the factor close to unity, coincides with the squared plasma sound velocity to squared Alfven speed ratio. In the subsequent calculations, this ratio will be denoted as $\beta^* = S^2/A^2 = 4\pi\gamma P_0/B_0^2$.

Curve 2 in figure 1 corresponds to the case when the region of transparency for SMS waves extends up to ∞, and the resonant surface for the Alfven wave is absent. By analysing expression (5) it is possible to define the two ranges of wave field and plasma parameters corresponding to curves 1 and 2 in figure 1, for which $k_x^2(\infty) > 0$:

$$\frac{\omega^2}{k_z^2 A_+^2} > \omega_{A1}^2, \qquad \frac{\beta^*}{1+\beta^*} < \frac{\omega^2}{k_z^2 A_+^2} < \omega_{A2}^2,$$

where $\omega_{A1}^2, \omega_{A2}^2$ are the two roots of the biquadratic equation

$$\omega_A^4 - (1 + k_y^2/k_z^2)[\omega_A^2(1+\beta^*) - \beta^*] = 0,$$

corresponding to solutions with the plus and minus signs before the radical. The first of these describes an FMS wave, and the second an SMS wave incident on the transition layer. Note that the second range is rather narrow. We have $\omega_{A2}^2 \approx [\beta^*/(1+\beta^*) + k_z\beta^{*2}/(1+\beta^*)^3\sqrt{k_y^2 + k_z^2}]$ as $\beta^* \to 0$ or $k_y^2/k_z^2 \to \infty$. As $k_y^2/k_z^2 \to 0$ we have $\omega_{A2}^2 \approx \beta^*$. Besides, there is another limitation on the parameters of the SMS wave incident on the transition layer, determined by the form of curve 2 in figure 1. For this curve the condition $(k_x^2)' < 0$ is satisfied in the regions $x > x_S$, which gives the limitation $\omega^2/k_z^2 A^2(x) < 2\beta^*/(1+\beta^*)$. A variant of the $k_x^2(x)$ distribution is also possible when a resonant surface is present for the Alfven wave, but is absent for SMS waves.

3. A qualitative investigation of the MHD oscillation field structure

To understand qualitatively the wave field structure we will solve the problem of the incidence and reflexion of a magnetosonic wave the parameters of which permit us to apply the WKB approximation far from the turning points and resonant surfaces. We will use the linearization of coefficients in (4) to study the solutions in the neighbourhood of these points, subsequently matching these solutions to the WKB ones. Let us consider, as a more common case, the incidence of the FMS wave on the transition layer having resonant surfaces for both the Alfven and SMS oscillations. In the process, we will note especially the features of a solution for the case of an SMS wave incident on the transition layer.

3.1. Solution near and between the singular and the turning points

3.1.1. Solution for $x \to -\infty$. In the opacity region left of x_S the WKB solution of (4) satisfying the boundary conditions has the form

$$\zeta = C_1\sqrt{\frac{|k_x|}{\rho_0\Omega^2}}\exp\left(\int_{x_S}^{x}|k_x|\,\mathrm{d}x'\right), \tag{6}$$

where C_1 is an arbitrary constant.

3.1.2. Solution near the resonant surface $x = x_S$. We linearize the coefficient attached to the higher derivative in (4), representing $k_x^{-2} \approx a_s^2\xi_s$, where $\xi_s = (x-x_S)/a_s$, $a_s = (\partial k_x^{-2}/\partial x)_{x=x_S}$ is the characteristic scale of the k_x^{-2} variation near $x = x_S$. Equation (4) may then be rewritten as

$$\frac{\partial}{\partial\xi_s}\xi_s\frac{\partial\zeta}{\partial\xi_s} + \zeta = 0. \tag{7}$$

near $x = x_S$. Its solution matched to (6) is

$$\zeta = C_2K_0(2\sqrt{-\xi_s}), \tag{8}$$

where $K_0(z)$ is the modified Bessel function. Using its asymptotic representation as $\xi_s \to -\infty$, we find a relation between the integration constants: $C_2 = 2C_1/(\Omega_s\sqrt{\pi\rho_{0s}a_s})$, where the subscript s indicates that the values of parameters are taken at $x = x_S$. In order to pass to the region $\xi_s > 0$ the singularity in solution (8) should be regularized. To this effect we will formally introduce a damping decrement γ_s for SMS waves near the resonant surface $x = x_S$, making a substitution, $\omega \to \omega - \mathrm{i}\gamma_s$, in the denominator of (5). A specific expression for γ_s will be obtained in section 3.2. Then solution (8) undergoes the substitution $\xi_s \to \xi_s - \mathrm{i}\varepsilon_s$ where $\varepsilon_s = \gamma_s/\omega$, and for $\xi_s > 0$ it has the form

$$\zeta = -\mathrm{i}\frac{C_2\pi}{2}H_0^{(2)}(2\sqrt{\xi_s + \mathrm{i}\varepsilon_s}), \tag{9}$$

where $H_0^{(2)}(z)$ is the Hankel function of the second kind, which, as $\xi_s \to \infty$, has an asymptotic representation, $H_0^{(2)}(2\sqrt{\xi_s}) \approx \pi^{-1/2}\xi_s^{-1/4}\exp(-\mathrm{i}2\sqrt{\xi_s} + \mathrm{i}\pi/4)$. When $\xi_s \to 0$ the solution (8) and (9) is

$$\zeta = -\frac{C_2}{2}\ln(-\xi_s + \mathrm{i}\varepsilon_s).$$

3.1.3. A WKB solution in the region of transparency $x_S < x < x_{01}$. This may be represented as

$$\zeta = C_3\sqrt{\frac{k_x}{\rho_0\Omega^2}}\exp\left(-\mathrm{i}\int_{x_{01}}^{x}k_x\,\mathrm{d}x'\right), \tag{10}$$

where the constant $C_3 = C_2\Omega_s\sqrt{\pi\rho_{0s}a_s}\exp(-\mathrm{i}\int_{x_{01}}^{x_S}k_x\,\mathrm{d}x - \mathrm{i}\pi/4)$ is determined by matching with (9). It is interesting to note that the solution (10) describes the wave impinging on the resonant surface $x = x_S$, but the reflected wave is absent. This means that the incident wave is absorbed completely in the neighbourhood of the resonant surface, irrespective of the dissipation mechanism. A similar effect takes place for the Alfven waves in a plasma with a curvilinear magnetic field [21, 22].

Note that if the region of transparency for SMS waves extends up to ∞ (which corresponds to curve 2 for $k_x^2(x)$ in figure 1), the SMS wave impinging on the transition layer is absorbed completely in the neighbourhood of the resonant shell. A numerical solution of the equations describing this process in [15] has shown that in this case the absorption coefficient of the waves impinging on the transition layer is maximum (~90%). However, the model medium used in that work, consisting of three regions with different distributions of plasma parameters along the x coordinate, has not allowed the conclusion about a 100% absorption of the incident wave. When matching the solutions obtained with this model in different regions the reflected wave is inevitable.

3.1.4. Solution in the neighbourhood of the turning point $x = x_{01}$. Differentiating (4) with respect to x, we will enter the designation $u = (1/k_x^2)\partial\zeta/\partial x$ and, linearizing $k_x^2 \approx -\xi_1/a_1^2$ close to $x = x_{01}$ (where $a_1^{-3} = -(\partial k_x^2/\partial x)_{x=x_{01}}$, $\xi_1 = (x - x_{01})/a_1$), we obtain the equation

$$\frac{\partial^2 u}{\partial \xi_1^2} - \xi_1 u = 0. \tag{11}$$

Its solution matched with (10) has the form

$$u = C_4(Ai(\xi_1) + \mathrm{i}Bi(\xi_1)), \tag{12}$$

where $Ai(z)$, $Bi(z)$ are Airy's functions. Matching this solution with (10) gives a relation of the constants $C_4 = C_3\sqrt{\pi a_1/\rho_{01}\Omega_1^2}$.

3.1.5. WKB solution in the opacity region $x_{01} < x < x_A$. We will present it in the form

$$\zeta = C_5\sqrt{\frac{|k_x|}{\rho_0\Omega^2}}\exp\left(\int_{x_{01}}^{x}|k_x|\,\mathrm{d}x'\right). \tag{13}$$

Matching it with (12) relates the constants of the solution $C_5 = \mathrm{i}C_3$.

3.1.6. Solution near the resonant surface $x = x_A$. As $x \to x_A$ we have $k_x^2 \approx -k_y^2$. We linearize $\Omega^2 \approx k_z^2 A^2(x_A)[(x - x_A)/a_A - 2\mathrm{i}\gamma_A/k_z A(x_A)]$ close to $x = x_A$, where $a_A = (\partial \ln(A^2)/\partial x)^{-1}_{x=x_A}$ is the characteristic scale of variation of $A(x)$, γ_A is the formally introduced decrement of the Alfven waves ($\omega \to \omega - \mathrm{i}\gamma_A$). We have the equation

$$\frac{\partial}{\partial \xi_A}(\xi_A - \mathrm{i}\varepsilon_A)\frac{\partial \zeta}{\partial \xi_A} - (\xi_A - \mathrm{i}\varepsilon_A)\zeta = 0, \tag{14}$$

where $\xi_A = k_y(x - x_A)$, $\varepsilon_A = k_y a_A \gamma_A/k_z A(x_A) \ll 1$. The solution of (14) matched with (13) is

$$\zeta = C_6 K_0(-\xi_A + \mathrm{i}\varepsilon_A), \tag{15}$$

where $K_0(z)$ is the modified Bessel function. When $x \to x_A$ we have

$$\zeta = -C_6 \ln(-\xi_A + \mathrm{i}\varepsilon_A).$$

Matching (15) with solution (13) gives a relation of the constants $C_6 = \mathrm{i}C_5 \mathrm{e}^{\psi_2}\sqrt{2k_y/\pi\rho_{0A}k_z^2 A^2(x_A)}$ where $\psi_2 = \int_{x_{01}}^{x_A}|k_x|\,\mathrm{d}x$, and the subscript A denotes the parameters at $x = x_A$.

3.1.7. The WKB solution in the opacity regions $x_A < x < x_{02}$. This has the form

$$\zeta = C_7\sqrt{\frac{|k_x|}{\rho_0\Omega^2}}\exp\left(\int_{x_{02}}^{x}|k_x|\,\mathrm{d}x'\right). \tag{16}$$

Matching it with (15) gives $C_7 = -C_5 \mathrm{e}^{\psi_2}$.

3.1.8. Solution in the neighbourhood of turning point $x = x_{02}$. For the function $u = (1/k_x^2)\partial\zeta/\partial x$ we have an equation similar to (11) where it is necessary to replace ξ_1 by $-\xi_2$ (where $\xi_2 = (x - x_{02})/a_2$, $a_2^{-3} = (\partial k_x^2/\partial x)_{x=x_{02}}$). Its solution matched with (16) is

$$u = C_8 Ai(-\xi_2), \tag{17}$$

where $C_8 = 2C_7\sqrt{\pi a_2/\rho_{02}\Omega_2^2}$.

3.1.9. WKB solution in the region of transparency $x > x_{02}$. We represent it in the form

$$\zeta = \sqrt{\frac{k_x}{\rho_0\Omega^2}}\left[C_\mathrm{i}\exp\left(-\mathrm{i}\int_{x_{02}}^{x}k_x\,\mathrm{d}x'\right) + C_\mathrm{r}\exp\left(\mathrm{i}\int_{x_{02}}^{x}k_x\,\mathrm{d}x'\right)\right], \tag{18}$$

where C_i is the amplitude of the incident magnetosonic wave and C_r is the amplitude of the reflected magnetosonic wave. Matching (18) with (17) gives $C_\mathrm{r} = -C_\mathrm{i} = -2C_7$. Note that in the orders of WKB approximation in question the corrections related to absorption of the oscillation energy near the resonant surfaces are not taken into account, therefore the amplitude of the incident wave is equal to the amplitude of the reflected wave. As we will see in the following section, if the characteristic value of the wave vector is such that $k_y\Delta; k_z\Delta \sim 1$, this approximation becomes too rough and the field of MHD oscillations can only be computed correctly by numerical methods.

3.2. The local decrement of SMS waves

Let us consider a homogeneous plasma the parameters of which are equal to the parameters of the transition layer under consideration at the point of magnetosonic resonance $x = x_S$. The dispersion equation for low-frequency oscillations of plasma with a Maxwellian distribution of particles over velocities (see [18]) is

$$1 + \sum_{\alpha=\mathrm{i,e}}\frac{\omega_{\mathrm{p}\alpha}^2}{k^2 v_\alpha^2}\left[1 + \mathrm{i}\sqrt{\pi}z_0^\alpha \mathrm{e}^{x_\alpha}\sum_{n=-\infty}^{\infty}I_n(x_\alpha)w(z_n^\alpha)\right] = 0, \tag{19}$$

where summation is with respect to the kind of the particles (the α index denotes the ions and electrons of plasma) and to the cyclotron harmonics (the n index). The notation here are k is the wave vector module, $x_\alpha = k_\perp^2\rho_\alpha^2$, $\rho_\alpha = v_\alpha/\omega_\alpha$ is the Larmor radius, $\omega_\alpha = eB_0/m_\alpha c$ is the cyclotron frequency, $\omega_{\mathrm{p}\alpha} = \sqrt{4\pi n_\alpha e^2/m_\alpha}$ is the plasma frequency and $v_\alpha = \sqrt{T_\alpha/m_\alpha}$ is the thermal velocity of particles of the α kind, $z_n^\alpha = (\omega - n\omega_\alpha)/\sqrt{2}k_z v_\alpha$. The modified Bessel function $I_n(x_\alpha)$ with small values of the argument (we will assume the condition $|k_\perp\rho_\alpha| << 1$ to be satisfied) is approximately represented as $I_n(x_\alpha) \approx (x_\alpha/2)^n/n!$. Function $w(z)$ is the probability integral having the following asymptotic representations (see [23]):

$$w(z) = \mathrm{e}^{-z^2}\left(1 + \frac{2\mathrm{i}}{\sqrt{\pi}}\int_0^z \mathrm{e}^{t^2}\,\mathrm{d}t\right) \approx \begin{cases} 1 - z^2 + 2\mathrm{i}z/\sqrt{\pi}, & |z| \ll 1,\\ \exp(-z^2) + \mathrm{i}/\sqrt{\pi}z, & |z| \gg 1.\end{cases}$$

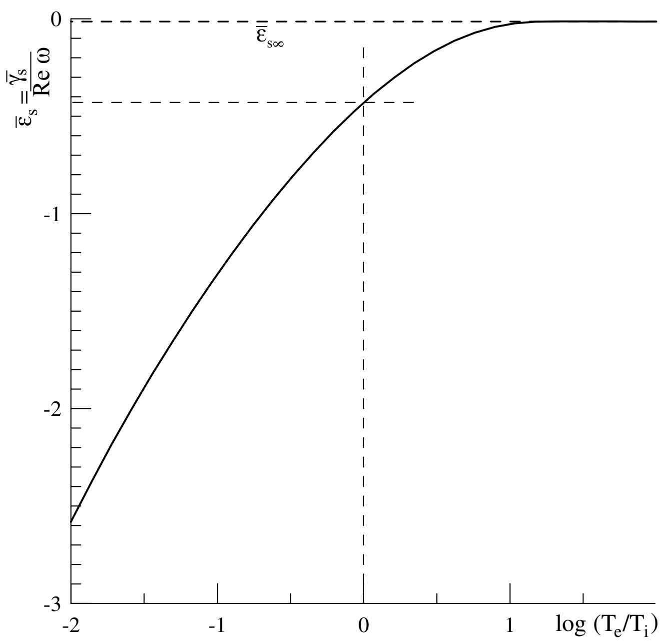

Figure 2. Ratio of the decrement of SMS oscillations $\bar{\gamma}_s \equiv \mathrm{Im}\,\omega$ to their frequency $\mathrm{Re}\,\omega$ versus the plasma nonisothermality parameter ($\log T_e/T_i$).

In the known extreme case (see [18]) $v_i \ll |\omega/k_z| \ll v_e$, it is possible to confine ourselves to the zero harmonics in the sum with respect to n in (19) and write down the dispersion equation approximately as

$$\frac{\omega_{\mathrm{pi}}^2}{k^2}\left(\frac{1}{v_s^2}(1+\mathrm{i}\sqrt{\pi}z_e^0) - \frac{k_z^2}{\omega^2}\right) \approx 0,$$

where $\omega_{\mathrm{pi}}^2/v_s^2 = \omega_{\mathrm{pe}}^2/v_e^2$, $v_s = \sqrt{T_e/m_i}$. In the zero-order perturbation theory the solution of this equation gives, for the extreme case $T_e \gg T_i$, the dispersion equation for SMS waves: $\omega^2 = k_z^2 v_s^2$. Taking into account the next order of the perturbation theory, we obtain the dispersion equation including the oscillation energy absorption

$$\omega^2 = k_z^2 v_s^2 \left(1 - \mathrm{i}\sqrt{\frac{\pi m_e}{2m_i}}\right).$$

In this extreme case ($T_e/T_i \to \infty$) the value of interest is $\bar{\varepsilon}_s = \bar{\gamma}_s/\mathrm{Re}(\omega) \equiv \bar{\varepsilon}_{s\infty} = -\sqrt{\pi m_e/2m_i}/2 \approx -0.015$. The total solution of (19) calculated numerically in the range $10^{-2} < T_e/T_i < 10^2$ is presented in figure 2. The calculated curve $\bar{\varepsilon}_s(T_e/T_i)$ has a universal form in a wide enough range of variation of the plasma parameters ($1\,\mathrm{nT} \leqslant B_0 \leqslant 10\,\mathrm{T}$; $1\,\mathrm{km\,s^{-1}} \leqslant A \leqslant 10^4\,\mathrm{km\,s^{-1}}$; $10^{-2} \leqslant \beta \leqslant 1$).

It is necessary, however, to note that unlike the Alfven oscillations, the damping decrement of SMS oscillations for $T_e/T_i \geqslant 1$ is rather large ($|\bar{\varepsilon}_s| \sim 1$). Therefore the regularizing factor ε_s in the denominator of (5), should be localized near $x = x_S$ on such a scale where the oscillations in question can be treated as SMS waves. Obviously, this scale is specified by the size of the transparency region for the SMS waves $\Delta_s = x_S - x_{01}$. If the linear expansion for the Alfven speed of the form $A^2(x) \approx A_S^2[1-(x-x_S)/a_s]$ is applicable near x_S, x_{01}, we have $\Delta_s \approx k_z^2 a_s \beta^*/[(k_z^2+k_y^2)(1+\beta^*)^2 - 2k_z^2\beta^*]$. For small magnitudes $\beta^* \ll 1$ this scale

$\Delta_s \approx a_s\beta^* k_z^2/(k_z^2 + k_y^2)$ is much smaller than the scale $a_s = (\partial \ln(A^2(x))/\partial x)^{-1}_{x=x_S}$, and when $\beta^* \sim 1$ they are comparable. To localize the decrement of SMS oscillations near $x = x_S$ we will make a substitution $\omega \to \omega - \mathrm{i}\gamma_s$ in the denominator of (5), where the model expression

$$\gamma_s = -\bar{\gamma}_s \exp[-(x - x_S)^2/\Delta_s^2] \tag{20}$$

will be chosen for the decrement γ_s, or, similarly, $\varepsilon_s = -\bar{\varepsilon}_s \exp(-(x - x_S)^2/\Delta_s^2)$ used in the subsequent numerical calculations. Obviously, such an approach cannot be used for the case of SMS waves incident on the transition layer. These waves will decay strongly in their entire region of existence. This problem requires a special statement for being solved, which is beyond the scope of this research.

4. Numerical calculation results and their discussion

Let us describe the Alfven speed profile $A(x)$ as a function of the form

$$A(x) = \frac{1}{2}\left[A_- + A_+ + (A_- - A_+)\tanh\left(\frac{x}{\Delta}\right)\right], \tag{21}$$

where $A_\pm$ is the Alfven speed as $x \to \pm\infty$, Δ is the typical thickness of the transition layer. In the chosen model of the Alfven speed $A_+ = 1$ and $\Delta = 1$ (see figure 1). The ratio $A_-/A_+ = 30$ is chosen large enough for both types—Alfven and SMS waves—to be able to exist in such a system of resonant surfaces under a considerable spread of the plasma temperature values (from $\beta^* = 0.01$ to $\beta^* \sim 1$).

In numerical calculations below, the x coordinate is measured in units of thickness of the transition layer Δ. Accordingly, the values of the wave vector components are measured in units of $2\pi/\Delta$. The magnetic field is supposed to be homogeneous, $\boldsymbol{B}_0 = \text{const}$, which according to (2) means $\beta = 8\pi P_0/B_0^2 = \text{const}$. The values of frequency ω and the wave vector parallel component k_z were chosen such that the resonant surfaces for the Alfven and SMS waves exist simultaneously in the system, in accordance with (2), and the transparency region for the FMS exists when $x \to \infty$: $A_+ < \omega/k_z < C_s(-\infty)$. In the following calculations the value $\omega/k_z = A(x_A) = 2.5A_+$ is fixed, which means an invariable position of the Alfven resonant point x_A.

The structure of the derivative $\partial\zeta/\partial x$ for $\beta^* = 0.5$ and rather small values of the Alfven and SMS wave absorption coefficients $\varepsilon_A = 0.01$, $\varepsilon_s = 0.01$ is presented in figure 3. The resonant structure of oscillations near the resonant surfaces $x = x_A$ and $x = x_S$ is well pronounced. Decomposition of the oscillation field into the incident and reflected waves in the WKB approximation, in accordance with (18), is also presented in the region $x > x_{02}$. Note that the amplitude of the incident and reflected waves in this calculation differs noticeably, due to absorption of the oscillation energy near the resonant surfaces. The oscillation amplitude is normalized to the amplitude of the incident wave as $x \to \infty$.

The distribution of the magnetic field components of oscillations having an average unit amplitude when $x \to \infty$ are given in figure 4. The same values of the calculated parameters are used here as in figure 3. The main component of the wave magnetic field on the resonant surface for Alfven waves $x = x_A$ is B_y, and on the resonant surface for SMS waves $x = x_S$ is B_z. Similar relations are also valid for the velocity field components. Thus, the resonant SMS oscillations are often referred to as longitudinal (relative to magnetic field direction $\boldsymbol{B}_0$), the resonant Alfven oscillations as azimuthal (or toroidal). This terminology has arisen from axisymmetric model investigations where the y coordinate corresponds to the azimuthal coordinate (for example to the azimuthal angle).

A distinctive feature of resonant Alfven oscillations is the change of the hodograph rotation direction of the transverse magnetic field vector $\boldsymbol{B}_\perp = (B_y, B_x)$ as we pass through the resonant

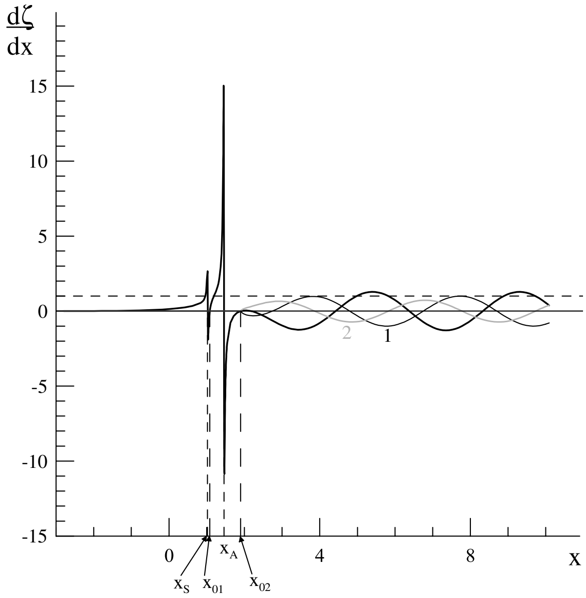

Figure 3. Numerical solution of equation (4) (bold line) and its decomposition into the incident (curve 1) and reflected (curve 2) waves in the WKB approximation (18) when $x > x_{02}$.

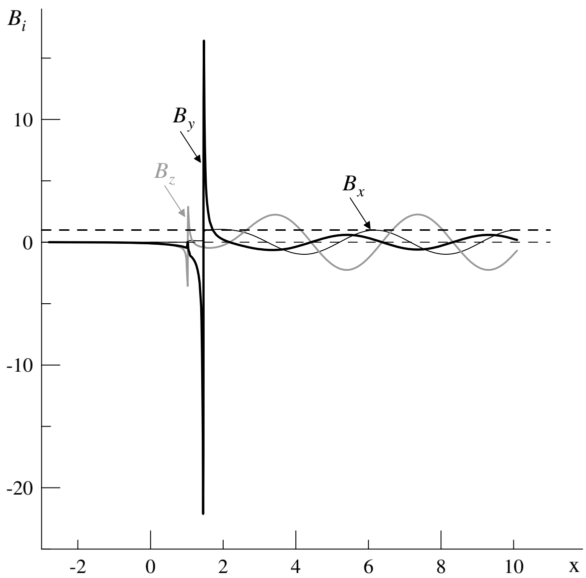

Figure 4. Distribution of magnetic field components of MHD oscillations $\boldsymbol{B} = (B_x, B_y, B_z)$ across the transition layer; corresponds to the solution shown in figure 3.

surface. It follows from the sign change of $\partial\zeta/\partial x$. In a case with small decrements $\gamma_A, \gamma_s \ll \omega$ this rule is valid when we pass through each resonant surface $x = x_A$ and $x = x_S$. We will look, however, at what happens when decrements γ_A and γ_s are not too small. Figure 5 demonstrates the distribution of $\partial\zeta/\partial x$, calculated for $\varepsilon_A = \gamma_A/\omega = 0.1$ and three values of

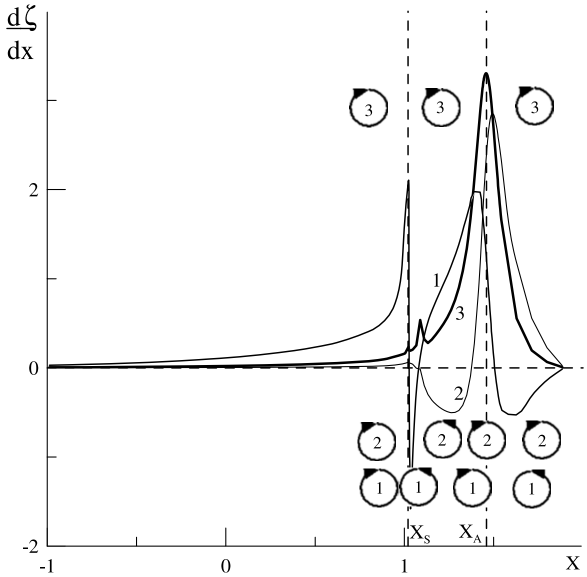

Figure 5. Rotation of the polarization hodograph of resonant MHD oscillations in the neighbourhood of resonant surfaces x_A and x_S for different dissipation levels in the Alfven ($\varepsilon_A = 0.1$) and SMS oscillations. Numbers 1, 2 and 3 correspond to different dissipation rates of the SMS oscillations: 1—$|\bar{\varepsilon}_s| = 0.01$, 2—$|\bar{\varepsilon}_s| = 0.1$, 3—$|\bar{\varepsilon}_s| = 1$.

$\varepsilon_s = \gamma_s/\omega = 0.01, 0.1, 1$. Here the behaviour of the hodographs is conventionally presented in the plane (B_y, B_x). For small $\varepsilon_s = 0.01$ ($T_e/T_i \gg 1$) the behaviour of the hodograph is as expected. When ε_s increases to 0.1 ($T_e \sim T_i$) the points where the hodograph rotation direction changes shift away from the resonant surfaces about as far as the distance between them. With ε_s increasing further to 1 ($T_e/T_i \approx 0.1$) the hodograph rotation direction does not change at all. This example demonstrates that the presence of strongly damped resonant SMS oscillations in the system can change the behaviour of the field components essentially, even in the neighbourhood of the resonant surface for the Alfven waves.

Let us consider the dependence of the absorption coefficient for the FMS waves incident on the transition layer, defined as

$$D = 1 - \frac{C_r^2}{C_i^2},$$

on the characteristic values of the wave vector components k_y, k_z, and the β^* and ε_s parameters (or, which is the same, on the ratio T_e/T_i).

Figure 6 displays the distribution of $D(\lambda)$, where $\lambda = k_y[\Delta_2/(k_z^2 + k_y^2)]^{1/3}$ is a dimensionless parameter, $\Delta_2 = |\partial \ln(A^2)/\partial x|_{x=x_{02}}$ is the characteristic scale of variation of $A^2(x)$ at the turning point x_{02}. The Alfven wave dissipation was chosen to be extremely small ($\varepsilon_A = 10^{-6}$) in these calculations so that coefficient D would not depend on its magnitude. Grey colour represents the distribution $D(\lambda)$ in a cold plasma for $k_z\Delta = 1$, as well as for $k_z\Delta \to \infty$. The last case is the maximum extreme case for a cold plasma. All other distributions $D(\lambda)$ at finite magnitudes of parameter $k_z\Delta$ in models with a smooth transition layer in a cold plasma lie below this curve.

Also presented is the distribution of $D(\lambda)$ for plasma with finite β^* for $k_z\Delta = 1$ in the presence of two resonant surfaces—for the Alfven and SMS waves—in the system. For each of

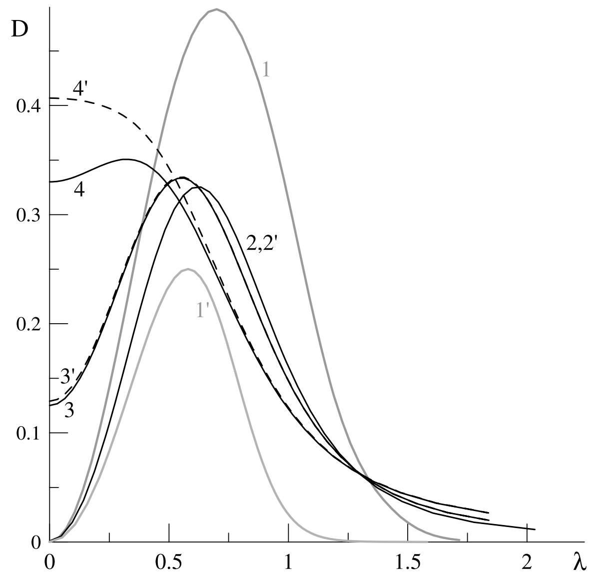

Figure 6. Energy absorption coefficient of FMS wave incident on the transition layer (21) against dimensionless parameter $\lambda = k_y[\Delta_2/(k_z^2+k_y^2)]^{1/3}$. Curves $D(\lambda)$ corresponding to a 'cold' plasma model ($\beta^* = 0$) for $k_z\Delta = \infty$ and $k_z\Delta = 1$ are labelled as 1 and 1′, respectively. For plasma configurations with $\beta^* = 0.01,\ 0.1,\ 1$ curves $D(\lambda)$ are constructed for $k_z\Delta = 1$ and two rates of SMS oscillation damping: numbers 2, 3, 4 (for $\varepsilon_s = 0.01$) and 2′, 3′, 4′ (for $\varepsilon_s = 1$).

$\beta^* = 0.01, 0.3, 1$ two curves $D(\lambda)$ corresponding to two limiting levels of the SMS oscillation dissipation studied in this paper ($\varepsilon_s = 10^{-2}$ and $\varepsilon_s = 1$) are calculated.

The main difference between the oscillation energy absorption in plasma with finite β^* and in cold plasmas is that $D(0)$ is nonzero. It is explained by the onset of the dissipation mechanism of resonant SMS oscillations, which, unlike the Alfven, do not disappear when $k_y = 0$. The efficiency of this dissipation grows with β^*, which may be explained as follows. The higher the β^*, the further the turning point x_{01} is located from the resonant surface x_S, and the closer to the Alfven resonant surface x_A. As a result the amplitude and, hence, the energy of the oscillations which have passed through the opacity region $x_{01} < x < x_A$ increases. As we saw in the previous section, all this energy is absorbed in the neighbourhood of the resonant surface x_S, which causes the absorption coefficient D to increase. In the above examples the value of D also increases when the dissipation level ε_s increases, which is explained by expansion of the region of existence of resonant SMS oscillations, where they are absorbed.

Figure 7 shows a similar calculation for plasma with finite $\beta^* = 0.3$, 1 and different values of parameter $k_z\Delta = 0.1, 1, 3.5$. The upper and lower magnitude of the last parameter are close to the extremes for which both resonant surfaces x_A and x_S are present in the system. The value of $D(0)$ increases considerably with $k_z\Delta$. Notably, the maximum of D can considerably exceed its limit in a cold plasma. This conclusion may be essential for problems related to plasma heating in the solar corona and nuclear fusion devices. The presence of a resonant surface for SMS waves leads to total absorption of the oscillation energy that reaches it, and the absorption maximum shifts towards small values of the azimuthal wave vector component k_y and, on the contrary, large values of the parallel component k_z. It is interesting to note that for $\beta^* = 1$ and $k_z\Delta = 3.5$, unlike all the other cases we considered, the value of D is greater for a smaller value of SMS oscillation dissipation (for $\varepsilon_s = 0.01$, instead of $\varepsilon_s = 1$). This

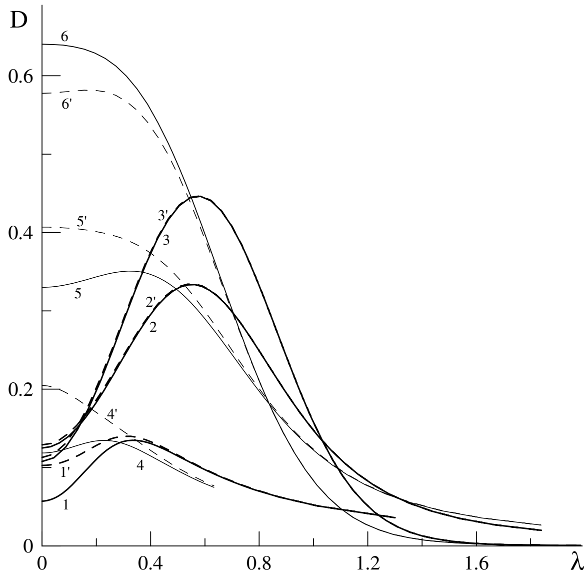

Figure 7. Absorption coefficient D as a function of λ for plasma configurations with $\beta^* = 0.3$ for $k_z\Delta = 0.1,\ 1,\ 3.5$ (curves 1, 2, 3 for $\varepsilon_s = 0.01$ and curves $1'$, $2'$, $3'$ for $\varepsilon_s = 1$) and with $\beta^* = 1$ for $k_z\Delta = 0.1,\ 1,\ 3.5$ (curves 4, 5, 6 for $\varepsilon_s = 0.01$ and curves $4'$, $5'$, $6'$ for $\varepsilon_s = 1$).

may be explained by the fact that the effect of the earlier-introduced localization of decrement γ_s comes into play (see (20)), restricting the integral growth of absorbed energy due to the resonant region expansion.

4.1. Possible application fields

Let us discuss briefly the possible application fields for the theory developed in this paper. Since the resonant interaction of MHD oscillations considered herein is characterized by very high values of the dissipation coefficients, the most obvious applications are those related to plasma heating.

The problem of coronal heating by MHD waves excited at the photospheric level has been discussed widely enough [24]. MHD waves propagating from the photosphere to the solar corona are observable in reality [25]. The main problem relating to coronal heating by MHD waves consists of determining an effective energy dissipation mechanism for these oscillations. For this purpose, the mechanisms of viscous interaction or finite electrical resistance are generally used [26]. The characteristic scales of the dissipation region may be considerable, while heating power is usually insufficient to explain the observable temperature in the corona ($\sim 2 \times 10^6$ K). Plasma flowing out of the photosphere is heated most intensely in the narrow enough ($\sim 10^3$ km) transition layer [27], the same layer where a strong plasma density (hence Alfven speed) gradient takes place. This means that the density of resonant surfaces, where such a broadband source of MHD waves as the photosphere excites localized resonant Alfven and SMS oscillations, is extremely high in the transition layer. Almost all energy of these oscillations is spent on plasma heating, which can explain the strong gradient of plasma temperature in the transition layer. Such a mechanism of resonant heating of the corona by MHD waves was suggested in [15]. However, since photospheric plasma is collisional, the ion and electron velocity distributions are close to the Boltzmann distribution with $T_e \sim T_i$.

This means that it is necessary to use a dissipation mechanism linked to collisionless Landau damping on plasma ions, as is done in this paper, in order to correctly describe the resonant SMS waves. Detailed calculation is required of the thermal balance in a specific plasma configuration corresponding to the corona. It should be kept in mind that when the Sun gravitation is included, a cutoff frequency appears below which the SMS waves have reflexion surfaces that are below the transition layer, which prevents these waves from reaching the corona [25]. There are no such limitations for FMS waves capable of exciting the resonant Alfven and SMS oscillations in the transition layer.

The resonant mechanism of plasma heating may be of use in research involving plasma heating in laboratory appliances. Such a mechanism, related to resonant Alfven waves, was suggested in [8]. It is peculiar in that it appears only for oscillations with $k_y \neq 0$. For an axisymmetric plasma tube this means that excitation must necessarily involve the asymmetric mode of oscillations. Disturbing a plasma configuration by such modes may result in lost stability. In the above problem, with two resonant surfaces, or with one resonant surface for SMS oscillations only, oscillation absorption in the symmetric mode ($k_y = 0$) is most effective for $\beta^* \sim 1$. Moreover, total energy absorption of oscillations by plasma ions may be expected for certain values of frequencies and parallel wave numbers, when SMS oscillations can propagate freely up to the resonant surface.

As to the Earth's magnetosphere, the mechanism of plasma heating discussed here may be invoked for explaining the strong difference between the plasma temperature in the internal ($\sim 10^3$ K) and external ($\sim 10^5$ K) magnetosphere [28]. The resonant surfaces for Alfven and SMS waves are concentrated in the external magnetosphere adjoining the magnetopause [16]. The mechanism of magnetosonic resonance may be employed to try to solve yet another problem. Even though the magnetospheric plasma is clearly separated from the solar wind plasma by the magnetopause, energy and impulsive moment transfer is known to exist from the solar wind into the magnetosphere. Plasma moves inside the magnetosphere, forming two convective cells in geomagnetic tail lobes [28]. The mechanism of 'quasi-viscous coupling' is generally used for explaining this motion, understood as momentum transfer from the solar wind into the magnetosphere. The mechanism of this coupling is not quite clear. It is known, however, that there is a rather intensive flux of FMS waves penetrating from the solar wind into the magnetosphere [29]. The problem is in 'utilizing', at least partially, the energy and impulse transferred by these waves. For each FMS wave there is a turning surface in the magnetosphere. If the FMS wave propagating in the magnetosphere interacts weakly with the magnetospheric plasma, the wave energy is reflected back into the solar wind almost completely. Fortunately plasma in the external magnetosphere is hot enough ($\beta^* \sim 1$) and strongly nonisothermal ($T_{\rm i} \gg T_{\rm e}$). As we have seen from the results of this work, this creates conditions for the existence of strongly dissipative resonant SMS oscillations in the magnetosphere. This allows one to hope that a considerable portion of the energy and impulse of magnetosonic waves penetrating into the magnetosphere is transferred to background plasma ions, providing a high temperature in the external magnetosphere and magnetospheric convection.

5. Conclusion

Let us formulate the main results of this paper.

1. The problem about the incidence and reflexion of magnetosonic waves from the plasma transition layer is solved. A qualitative analysis of the solution obtained by linearizing the coefficients in equation (4) near its singular points and turning points, and by the WKB method between these points has been carried out. To regularize the singularities

near the resonant surfaces, effective decrements are introduced for the Alfven and SMS waves as imaginary additives to the oscillation frequency. The dissipation of the Alfven oscillations is assumed to be small enough for the value of the oscillation energy absorbed in the neighbourhood of the Alfven resonant surface not to depend on the decrement. In contrast, the decrement of the SMS oscillations can be rather large. Its value is determined by the ratio of the plasma ion to electron temperatures near the resonant surface for the SMS.

2. It is shown that dissipation of the oscillation energy due to Landau's collisionless dissipation increases considerably when two types of resonant surfaces—for the Alfven and SMS waves—are present in the system under consideration. The oscillations reaching the resonant surface for SMS waves are absorbed completely in its neighbourhood, leading to increased plasma heating.
3. Numerical integration of equation (4) was carried out for the oscillations whose wavelength in the (y, z) plane is comparable to the thickness of the transition layer Δ and for which the WKB approximation is inapplicable. It was shown that, if the dissipation of oscillations near the resonant surfaces is not too small, the presence in the system of strongly decaying resonant SMS oscillations changes the wave field substantially. When $\varepsilon_s = \gamma_s/\omega \sim 1$ this influence extends up to the resonant surface for the Alfven waves. Specifically, the hodograph rotation direction is not reversed for monochromatic oscillations when we pass through the resonant surface, the phenomenon which is generally employed to identify the presence of such a surface in a specific plasma configuration.
4. We have studied the influence of various wave field and plasma layer parameters on the incident FMS wave absorption. It was shown that, with increasing β^*, the value of the absorbed oscillation energy increases considerably in the regions with small azimuthal wave numbers ($k_y \to 0$). This is caused by a growing amplitude of oscillations reaching the resonant surface for SMS waves, in the neighbourhood of which they are completely absorbed. The higher the magnitude of the wave vector parallel component k_z, the higher the maximum value of the absorption coefficient D. The maximum energy absorption of oscillations impinging on the transition layer can considerably exceed that which takes place in a plasma configuration with a single resonant surface—for the Alfven waves.
5. It is shown that when the plasma layer and incident magnetosonic wave parameters are such that the transparency region for SMS waves extends up to ∞ the incident wave energy is totally absorbed in the neighbourhood of the resonant surface. However, the origin of SMS waves in this case must be at a finite distance from the transition layer since they dissipate strongly in the entire region of their existence. The amplitude of the SMS oscillations reaching the resonance surface should be sufficient for the resonant oscillations to stand out against the background.

Acknowledgments

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References

[1] Tataronis J A and Grossmann W 1976 *Nucl. Fusion* **16** 667
[2] Allan W, White S P and Poulter E M 1985 *Geophys. Res. Lett.* **12** 287
[3] Sakurai T, Goossens M and Hollweg J V 1990 *Sol. Phys.* **133** 227

[4] Chen L and Hasegawa A 1974 *J. Geophys. Res.* **79** 1024
[5] Southwood D J 1974 *Planet. Space Sci.* **22** 483
[6] Inhester B 1987 *J. Geophys. Res.* **92** 4751
[7] Rankin R, Kabin K and Merchand R 2006 *Adv. Space Res.* **38** 1720
[8] Chen L and Hasegawa 1974 *Phys. Fluids* **17** 1399
[9] Leonovich A S and Mazur V A 1991 *Planet. Space Sci.* **39** 529
[10] Goertz C K 1984 *Planet Space Sci.* **32** 1381
[11] Cheng C Z and Zaharia S 2003 *J. Geophys. Res.* **108** SMP 1-1
[12] Osterbrock D E 1961 *Astrophys. J.* **134** 347
[13] Olfman L 2005 *Space Sci. Rev.* **120** 67
[14] Karney C F F, Perkins F W and Sun Y-C 1979 *Phys. Rev. Lett.* **42** 1621
[15] Čadež V M, CsikÁ, Erdélyi R and Goossens M 1997 *Astron. Astrophys.* **326** 1241
[16] Leonovich A S, Kozlov D A and Pilipenko V A 2006 *Ann. Geophys.* **24** 2277
[17] Lifshitz E M and Pitaevskii L P 1981 *Physical Kinetics* (*Course of Theoretical Physics* vol 10) (Oxford: Pergamon)
[18] Akhiezer A I *et al* 1979 *Plasma Electrodynamics* (Moscow: Nauka) p 237 (in Russian)
[19] Vdovin V L *et al* 1971 *ZhETF Pis.* **14** 228
[20] Arregui, Oliver R and Ballester J L 2003 *Astron. Astrophys.* **402** 1129
[21] Leonovich A S and Mazur V A 1993 *Planet. Space Sci.* **41** 697
[22] Leonovich A S and Mazur V A 1995 *Planet. Space Sci.* **43** 885
[23] Abramowitz M and Stegun I A 1965 *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (New York: Dover)
[24] Erdelyi R 2004 *Astron. Geophys.* **45** 4.34
[25] Nakariakov V M *et al* 2004 *Astron. Astrophys.* **362** 1151
[26] Aschwanden M J 2004 *Physics of the Solar Corona: An Introduction* (Chichester, UK: Publishing Ltd., Berlin: Springer Verlag)
[27] Peter H 2001 *Astron. Astrophys.* **374** 1108
[28] Hargreaves J K 1979 *The Upper Atmosphere and Solar-Terrestrial Relations* (New York: Van Nostrand-Reinhold)
[29] Leonovich A S, Mishin V V and Cao J B 2003 *Ann. Geophys.* **21** 1083