Penetration to the Earth's surface of standing Alfven waves excited by external currents in the ionosphere

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Abstract

The problem of boundary conditions for monochromatic Alfven waves, excited in the magnetosphere by external currents in the ionospheric E-layer, is solved analytically. Waves with large azimuthal wave numbers $m \gg 1$ are considered. In our calculations, we used a model for the horizontally-homogeneous ionosphere with an arbitrary inclination of geomagnetic field lines and a realistic height distribution of Alfven velocity and conductivity tensor components. A relationship of such Alfven waves on the upper ionospheric boundary with electromagnetic oscillations on the ground was detected, and the spatial structure of these oscillations determined.

1 Introduction

While penetrating from the magnetosphere to the ground through the ionosphere and atmosphere, the electromagnetic field of Alfven waves is subjected to substantial changes. A significant number of theoretical and experimental publications have addressed the problem of ascertaining a relationship between the field of these waves and electromagnetic oscillations on the ground. In this paper, we will confine ourselves to theoretical aspects of this problem. Therefore, we will not concern ourselves with questions relating to in-situ observations of geomagnetic pulsations. We will briefly run through publications devoted to a theoretical treatment of the penetration of the Alfven waves field from the magnetosphere to the ground. Ionospheric effects in the Alfven wave penetration process in different frequency ranges manifests itself differently. A central element of this process is the fact that they act to drive currents within the ionospheric conducting layer. These currents are associated with magnetosound waves which make the main contribution to pulsations induced on the ground. For the highest-frequency pulsations out of these ($f \sim 10^{-1} - 1$ Hz), the ionospheric F2-layer includes a region where magnetosonic waves can propagate through waveguide along the ionosphere. This waveguide and the pulsations induced on the ground were taken up in *Greifinger and Greifinger*(1968),*Greifinger*(1972), *Rudenko et al.*(1985).

In the lower-frequency range ($f \sim 10^{-2} - 10^{-1} Hz$) in the ionospheric E-layer, conditions for exciting a sort of ionospheric whistler are created. This phenomenon was for the first time investigated in *Sorokin and Fedorovich*(1082) and in greater detail in *Mazur*(1987). In the same frequency range in the upper ionosphere (F2-layer and higher), conditions for partial suppression of the Alfven waves can be produced in the direction along geomagnetic field lines *Belyaev et al.*(1990).

The simplest situation applies for the lowest- frequency Alfven oscillations of the magnetosphere ($f \sim 10^{-3} - 10^{-2} Hz$). The wavelength of such oscillations along geomagnetic field lines is much larger than all typical scales of the ionosphere in that direction. In this connection, the ionosphere can be regarded mathematically as a thin layer. This makes it possible to advance rather greatly in the analytical study of the penetration process of the Alfven waves from the magnetosphere to the ground. One of publications that pioneered the construction of such a theory is *Hughes*(1974) which addresses the issue of the penetration to the ground of waves with $k_\perp \ll k_\|$, where $k_\|$ and $k_\perp$ are, respectively, the field-aligned (along the geomagnetic field) and transverse components of the wave vector of the Alfven wave in the magnetosphere (which was taken to be homogeneous).A further extension of the theory to the case $k_\perp \gg k_\|$ was undertaken in *Hughes and Southwood*(1976a), *Hughes and Southwood*(1976b). In this case the authors considered the case when the geomagnetic field is normal to the terrestrial surface, unlike *Hughes*(1974) where it was thought of as being oblique.

Subsequently, the theory was further developed by workers who considered the propagation of Alfven waves through the horizontally- inhomogeneous ionosphere. The inhomogeneous ionosphere was modelled in *Maltsev et al.*(1984), *Ellis and Southwood*(1983), *Polyakov*(1988) as a plane on which lies a bounded region, whose conductivity differs from the conductivity of the remainder of the ionosphere. Also, the conductivity both inside and outside that region was considered homogeneous. An alternative type of inhomogeneous ionosphere was examined in *Glassmeier*(1983), *Glassmeier*(1984), with its conductivity changing continuously. It was shown that in the case of the penetration from the magnetosphere to the ground the rotation of the polarization ellipse of the electromagnetic oscillation field can differ greatly from $\pi/4$, as it followed from the horizontally-homogeneous ionospheric model.

A further step forward in the study of the penetration of the Alfven oscillation field to the ground through the horizontally- homogeneous ionosphere was made in *Leonovich and Mazur*(1991) (hereinafter referred to as Paper 1). Paper 1 addressed the passage of the waves with $k_\perp \gg k_\parallel$ in the case of an arbitrary inclination of the geomagnetic field lines to the terrestrial surface and an arbitrary height distribution of the Alfven velocity and the conductivity tensor components. On the around, using harmonics with different k_t (t being an arbitrary direction in the horizontal plane) this made it possible to compose analytically a complete packet of oscillations, corresponding to the field of a monochromatic Alfven wave on the upper boundary of the ionosphere.

Almost in all of the cited papers, the ionosphere was treated as a region that distorts the field of Alfven waves incident from the magnetosphere. Also, the various physical processes occurring both inside and outside the magnetosphere, are regarded as sources for the waves (see reviews *Yumoto*(1988),*Pilipenko*(1994)). The sole exception is a paper *Maltsev et al.*(1976) (see also a monograph *Lyatsky and Maltsev*(1983)) where the source for Alfven waves is taken to be a modulation of ionospheric conductivity in the presence of an external electric field. And use was made of a model of an optically-thin ionosphere with a vertical geomagnetic field.

In a paper *Leonovich and Mazur*(1993) (hereinafter referred to as Paper 2), a theory of transversally-small-scale standing Alfven waves (in the homogeneous magnetosphere, the analog to them is provided by the case $k_\perp \gg k_\parallel$) was constructed for a model of the axisymmetric magnetosphere. It was assumed that the source for these waves is provided by external currents in the ionosphere that are driven by the neutral component in motion. These motions can be associated with, for example, internal gravity and acoustic-gravity waves or with neutral winds in the lower ionosphere.

Paper 2 employed, without the derivation, a boundary condition for Alfven waves on the upper boundary of the ionosphere, including external currents in the ionosphere. In this paper, this boundary condition is obtained analytically by generalizing the findings reported in Paper 1 to the case of the existence of external currents in the ionosphere. In addition, here we have investigated the spatial structure of the field of monochromatic electromagnetic oscillations in the ionosphere and atmosphere which are associated with Alfven waves in the magnetosphere driven by these external currents. The same model as used in Paper 1 was used in modelling the ionosphere. This paper is organized as follows. Section 2 presents a model of the medium, the coordinate systems used and the input equations. Section 3 gives a description of the oscillation field in isotropically- conducting layers: the Earth and the atmosphere. In Section 4, we describe the oscillation field in anisotropically-conducting layers: the ionosphere and the magnetosphere. In Section 5, we will obtain boundary conditions on the upper boundary of the ionosphere for Alfven waves that are driven in the magnetosphere by external currents in the ionosphere. In Section 6, for the case of the terrestrial surface, Fourier-harmonics with different values of k_t will be used to compose analytically a packet of oscillations, corresponding to the field of a standing Alfven wave with $m \gg 1$ on the upper boundary of the ionosphere. In Section 7, we discuss the results obtained from general formulas. And the main results of this study are formulated in the Conclusions.

2 A model of the medium, and input equations

The height distribution of the Alfven velocity and the conductivity tensor components in our chosen model of the medium are described in detail in Paper 1. Fig.1b shows their typical height profiles. Note that the upper boundary of the ionosphere, as shown in Paper 1, should be considered to be represented by the height $(1.5 \div 2) \cdot 10^3 km$ where the height behavior of the Alfven velocity varies from a rapid growth in the upper ionosphere to a slow decline in the magnetosphere. Note that such a choice of the upper boundary of the ionosphere is justified only for harmonics of standing Alfven waves with $N \sim 1$.

Electromagnetic monochromatic oscillations with frequency ω in the presence of external currents in the medium obey Maxwell's equations of the form

$$\mathrm{curl}\,\mathbf{E} = ik_0\mathbf{B},$$

$$\mathrm{curl}\,\mathbf{B} = -ik_0\mathbf{E} + \frac{4\pi}{c}\left[\mathbf{j} + \mathbf{j}^{ext}\right], \tag{1}$$

where $k_0 = \omega/c$, $\mathbf{E}$ and $\mathbf{B}$ are, respectively, the disturbed electric and the magnetic fields; and $\mathbf{j}^{ext}$ is external current unassociated with oscillations. For the low-frequency oscillations of our interest, conductivity current is representable as

$$\mathbf{j} = \sigma_{\|}\mathbf{E}_{\|} + \sigma_{\perp}\mathbf{E}_{\perp} + \sigma_{H}\left[\frac{\mathbf{B}_0}{B_0}\mathbf{E}_{\perp}\right],$$

where $\mathbf{B}_0$ is the geomagnetic field vector; $E_{\|}$ and $\mathbf{E}_{\perp}$ are, respectively, the longitudinal (along $\mathbf{B}_0$) and the transverse components of a disturbed electric field; and $\sigma_{\|}, \sigma_{\perp}$ and σ_H are, respectively, the longitudinal, transverse and Hall conductivities of the medium. Note that current $\mathbf{j}$ includes a frequency-dependent displacement current.

For solving the system of equations (1), we choose the following coordinate systems. In the ground region, when investigating the electromagnetic field of our interest, the terrestrial surface can to the desired accuracy be considered plane. Let us introduce a Cartesian coordinate system (x, y, z), in which we let the x axis be directed along the magnetic meridian from south- to northward, the y axis - along the parallel from west- to eastward, and the axis z - upward along the normal toward the terrestrial surface (see Fig. 1a). The components of a perturbed electromagnetic field are functions of coordinates and time (say, the component $B_x = B_x(x, y, z, t)$). Presuming the medium to be stationary these components are representable as a Fourier-series expansion in terms of harmonics with a certain frequency ω :

$$B_x(x, y, z, t) = \int_{-\infty}^{\infty} \tilde{B}_x(x, y, z, \omega)\exp(-i\omega t)\, d\omega.$$

Taking into account the horizontal homogeneity of the medium it is possible to carry out a similar Fourier-series expansion in terms of spatial harmonics with specified values oz the components of the wave vector $\mathbf{k}_t = (k_x, k_y)$:

$$\tilde{B}_x(x, y, z, \omega) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \; \bar{B}_x(k_x, k_y, z, \omega) \exp(ik_x x + ik_y y). \qquad (2)$$

It is these Fourier-harmonics which we will be using in subsequent calculations; for brevity sake, we will not write out their dependence on the arguments k_x, k_y and ω.

When solving equations (1) in isotropic media, namely in the Earth and in the atmosphere, we take advantage of the coordinate system (t, b, z) rotated with respect to the system (x, y, z) about the z axis. In this case we let the t axis be directed along the horizontal wave vector $\mathbf{k}_t$, and the b axis in the same horizontal plane normal to the t axis. The relationships

$$\bar{B}_t = (k_x/k_t)\bar{B}_x + (k_y/k_t)\bar{B}_y,$$
$$\bar{B}_b = -(k_y/k_t)\bar{B}_x + (k_x/k_t)\bar{B}_y$$

occur between the horizontal components of the wave's perturbed magnetic field vector, as well as similar relationships for the components of a perturbed electric field.

For anisotropic media (the ionosphere and the magnetosphere), use is made of the coordinate system (n, y, l) rotated with respect to the system (x, y, z) about the y axis by an angle χ so that the l axis is directed along $\mathbf{B}_0$ (see Fig.1a). The n axis lies in the meridional plane and is directed normal to the axes l and y. In this coordinate system

$$B_n = B_x \cos\chi + B_z \sin\chi, \;\; B_l = -B_x \sin\chi + B_z \cos\chi$$

and similarly for the components of the electric field.

The difference of the problem to be solved in this paper from that solved in Paper 1 is the presence of external currents in the ionospheric conducting layer. In this connection, where such differences are unimportant, we will be making use of the Paper 1 results.

3 The electromagnetic oscillation field in the Earth and in the atmosphere

Since in isotropic media (the Earth any the atmosphere), with our statement of the problem, no external currents are present, we employ the solutions obtained in Paper 1. Within the Earth's thickness, solutions for the components of a perturbed electromagnetic field have the form

$$\bar{E}_b(z) = \bar{E}_b(0) \exp(k_g z), \;\; \bar{B}_b(z) = \bar{B}_b(0) \exp(k_g z),$$

where the point $z = 0$ corresponds to the Earth-atmosphere boundary, while $k_g = \sqrt{k_t^2 - ik_0\kappa_g}$, $\kappa_g = 4\pi\sigma_g/c$ and σ_g is the Earth's conductivity. The other components of the electromagnetic field in isotropic layers are expressable as

$$\bar{E}_t = -\frac{1}{\kappa - ik_0}\frac{\partial \bar{B}_b}{\partial z}, \ \ \bar{E}_z = i\frac{k_t}{\kappa - ik_0}\bar{B}_b,$$

$$\bar{B}_t = \frac{i}{k_0}\frac{\partial \bar{E}_b}{\partial z}, \ \ \bar{B}_z = \frac{k_t}{k_0}\bar{E}_b. \qquad (3)$$

For solutions in the Earth, here one should put $\kappa = \kappa_g$. For reasons of the Earth's high conductivity ($k_0 k_g \gg k_t^2 \gg k_0^2$), for the sake of simplicity in subsequent calculations we will be using the limit $\kappa \to \infty$ which yields $E_t(0) = E_b(0) = 0$. Results for finite values of κ_g were reported in Paper 1. In this case it is supposed that within the Earth there are no external telluric currents capable generate oscillations with an amplitude comparable with the oscillation amplitude excited by external currents in the ionosphere.

Atmospheric conductivity σ_a is much less than the Earth's conductivity σ_g; hence throughout the atmospheric thickness $(0 < z < H)$ the inequality $k_t^2 \gg k_0\kappa_a$ holds, where $\kappa_a = 4\pi\sigma_a/c$. As a result, the solutions for the components E_t and E_b are of the form

$$\bar{B}_t(z) = B_t(0)\cosh(k_t z), \quad \bar{E}_b(z) = -i\frac{k_0}{k_t}B_t(0)\sinh(k_t z).$$

Thus, when $z = H$, we have

$$\bar{B}_t(H) = i\frac{k_t}{k_0}E_b(H)\coth(k_t H). \qquad (4)$$

This equation is used as a boundary condition for solving the problem in the ionosphere. Expressions resulting for the field components E_t and B_b are more unwieldy (see Paper 1). Since they do not play any special role in the penetration of the oscillation field from the ionosphere to the Earth, we will not write them out. Note, however, that to an accuracy of our interest, when $z = H$, it may be assumed that

$$\bar{B}_b(H) = 0.$$

This equation is used as a second boundary condition for solving the problem in the ionosphere.

4 The electromagnetic oscillation field in the ionosphere

The system of equations (1) in the ionosphere is written in the simplest form in the coordinate system (n, y, l). If we introduce a four-component column-vector

$$\alpha = \begin{pmatrix} \bar{E}_n \\ \bar{E}_y \\ \bar{B}_y \\ \bar{B}_n \end{pmatrix},$$

then the system of equations (1) is representable as

$$-i\frac{\partial \alpha}{\partial z} = \hat{Q}\alpha + \hat{q}\alpha + ig. \qquad (5)$$

Here the matrix $\hat{Q}$ is composed of the components of a horizontal wave vector $k_t = (k_x, k_y)$, and the matrix $\hat{q}$ consists of the components $\kappa_{P,H} = 4\pi\sigma_{P,H}/c$. These matrices have the same form as in Paper1, and the components of the matrix $\hat{Q}$ are much larger than those of the matrix $\hat{q}$. The system of equations (5) differs from that investigated in Paper 1 by the presence of a column-vector of extraneous currents

$$g = \frac{4\pi}{c\cos\chi}\begin{pmatrix} 0 \\ 0 \\ j_n^{ext} \\ -j_y^{ext} \end{pmatrix}.$$

As will be evident from subsequent calculations, the components of this vector are much smaller compared to the vector $\hat{Q}\cdot\alpha$. It is therefore possible to seek the solution of the system (5) by the perturbation method. To a zero-order approximation, we have a system of equations

$$-i\frac{\partial\alpha^{(0)}}{\partial z} = \hat{Q}\alpha^{(0)} \tag{6}$$

whose solutions have the form

$$\alpha^{(0)}(z) = \psi\exp(ik_z z). \tag{7}$$

Substituting (7) into (6) gives a system of algebraic equations for k_z, whose solutions have the form

$$k_z^{(1)} = k_z^{(2)} = k_x\tan\chi \equiv k_{zA}, \quad k_z^{(3)} = ik_t \equiv k_{zF},$$

$$k_z^{(4)} = -ik_t \equiv k_{zF}^*.$$

The roots $k_z^{(1)}$ and $k_z^{(2)}$ correspond to the Alfven wave, and $k_z^{(3)}$ and $k_z^{(4)}$ correspond to the magnetosonic wave with frequency $\omega = 0$. In calculations to follow, it will also be necessary to have transverse components of the wave vector constituents for the Alfven

$$k_{nA} \equiv k_x/\cos\chi, \quad k_{\perp A} = \sqrt{k_{nA}^2 + k_y^2}$$

and magnetosonic

$$k_{nF} = k_x\cos\chi + ik_t\sin\chi, \quad k_{\perp F} = k_t\cos\chi + ik_x\sin\chi,$$

waves.

To each root $k_z^{(i)}$ $(i = 1, 2, 3, 4)$ there corresponds a columnvector of the coefficients ψ^i, defined by equations (6), which relate amplitudes of different components of the electric field in each of the waves. The complete solution of the system (6) is a an arbitrary combination of linearly-independent vectors $\psi^i\exp(ik_z^{(i)}z)$. Since in the space of 4-vectors the set ψ^i forms a complete system, the solution of equations (7) may be sought in the form

$$\alpha(z) = \psi^i F_i(z) \equiv \hat{\psi}F(z), \tag{8}$$

where $F(z)$ is a column-vector of the desired coefficients, and the matrix $\hat{\psi}$ is composed of the column-vectors ψ^j (see Paper 1).

On substituting the solution (8) and multiplying it from the left by $\hat{\psi}^{-1}$, a matrix inverse to $\hat{\psi}$ we obtain a system of equations for the coefficients $F(z)$:

$$-\frac{\partial F}{\partial z} = \hat{\Lambda} F + \hat{P} + r. \tag{9}$$

The expressions for the matrices $\hat{\Lambda}$ and $\hat{P}$ may be found in Paper 1, and the column-vector r has the form

$$r = \frac{4\pi\omega}{c^2} \begin{pmatrix} -i(j_n \tan\chi)/(k_{\perp A}\cos\chi) \\ i(k_{nA} j_n + k_y j_y)/(k_{\perp A}^2 \cos\chi) \\ -(k_y j_n - k_{nF} j_y)/(2k_{\perp F} k_t) \\ (k_y j_n - k_{nF}^* j_y)/(2k_{\perp F}^* k_t) \end{pmatrix}.$$

Because in a zero-order approximation $F_i(z)$ satisfy (8), we will seek them in the form

$$F_i(z) = f_i(z)\exp(ik_z^{(i)}(z-H)),$$

where, for the sake of convenience, the phase is reckoned from the boundary $z = H$. Equations (9) should be supplemented with boundary conditions on the lower boundary of the ionosphere $(z = H)$. Boundary conditions for the functions $f_i(z)$ obtainable from (3) and (4) have the form

$$f_2(H) = 0, \tag{10}$$

$$(1 + \cot(k_t H)) f_1(H) + \\ (1 - \cot(k_t H)) f_4(H) = 0. \tag{11}$$

In the upper ionosphere and in the magnetosphere, a natural condition is the absence of solutions, whose amplitude grows when $z \to \infty$. This leads to a boundary condition on the upper boundary of the ionosphere

$$f_4(z_A) = 0. \tag{12}$$

It will become evident from subsequent calculations that the inequality

$$|f_1| \gg |f_2|, |f_3|, |f_4|.$$

is satisfied throughout the ionospheric thickness. This makes it possible, in the first order of perturbation theory, to leave on the right-hand side of equations (10) only the terms proportional to f_1. As a result, the first pair of equations (9) splits off from the other two and has the form

$$f_1' = i\frac{k_{\perp A}}{\cos\chi} f_2 - \\ i\frac{k_2 \tan\chi}{k_{\perp A}\cos\chi}\left(\frac{k_{nA}}{k_{\perp A}}\kappa_{PH} - \frac{k_y}{k_{\perp A}}\kappa_H\right) f_1 + \\ \frac{4\pi k_0 \tan\chi}{ck_{\perp A}\cos\chi} j_n \exp(-ik_{zA}(z-H)), \tag{13}$$

$$f_2' = -\frac{k_0}{k_{\perp A}\cos\chi}\kappa_{PH} f_1 - \frac{4\pi k_1}{ck_{\perp A}^2\cos\chi}(k_{nA}j_n + k_y j_y)\exp(-ik_{zA}(z-H)), \quad (14)$$

where the prime denotes the derivative d/dz. As will be shown, the function $f_1(z)$ changes little within the interval $H \leq z \leq z_A$, so we may put

$$f \equiv f_1(z) \approx f_1(H). \quad (15)$$

Let us introduce the designations

$$X_{P,H}(z) = \frac{4\pi}{c}\int_H^z \sigma_{P,H}(z')dz',$$

$$K_{P,H} \equiv X_{P,H}(H+\Delta),$$

$$\bar{J}_{n,y}(z) = \frac{4\pi}{c}\int_H^z \bar{j}_{n,y}(z')\exp(-ik_{zA}(z'-H))dz',$$

$$\bar{I}_{n,y} \equiv \bar{J}_{n,y}(H+\Delta),$$

where Δ is the ionospheric conductive layer thickness.

Upon integrating equation (14) with the boundary condition (10), we obtain

$$f_2(z) = -f\frac{k_0}{k_{\perp A}\cos\chi}X_P(z) - \frac{k_0}{k_{\perp A}^2\cos\chi}(k_{nA}\bar{J}_n(z) + k_y\bar{J}_y(z)).$$

In the upper ionosphere $(z \to \infty)$, we have

$$f_2 = -f\frac{k_0}{k_{\perp A}\cos\chi}K_N - \frac{k_0}{k_{\perp A}^2\cos\chi}(k_{nA}\bar{I}_n + k_y\bar{I}_l). \quad (16)$$

In order for the condition $|f_2| \ll |f_1|$ to be satisfied, it is necessary that each of the terms on the right-hand side of (16) should be much smaller than $|f|$. As far as the terms proportional to currents in the ionosphere are concerned, the condition for their smallness will be formulated in the next Section. For the smallness of the first term, it is essential that the inequality

$$\frac{k_0 K_P}{k_{\perp A}} \ll 1. \quad (17)$$

is satisfied. It permits the term proportional to f_1 to be omitted in equation (13). After that, upon integrating (13), we obtain

$$f_1(z) = f\left[1 - y\frac{k_0}{\cos^2\chi}\int_H^z X_P(z')dz'\right] - \frac{k_0\sin\chi}{k_{\perp A}\cos^2\chi}\bar{J}_n(z). \quad (18)$$

As will be evident from the next Section's results, the last term in this equation is small. Hence, in order for the condition (15) to be satisfied, it is necessary that

$$k_0 K_P \Delta \ll 1. \tag{19}$$

In Paper 1 it was shown that the conditions (17) and (19) are equivalent to the requirement for the smallness of the frequency of the oscillations involved, ω , compared to the whistler frequency in the ionosphere $\omega_{PH} = c^2/4\pi\Sigma_P\Delta$. It was also shown that for the transverse conductivity to be totally neglected in the upper ionosphere (as done in the present study), it is necessary that the frequency ω should be much lower than the eigen-frequency of an ionospheric Alfven cavity

$$\omega \int_{H+\Delta}^{z_A} \frac{dz}{A(z)} \ll 1.$$

With a knowledge of the expression for $f_1(z)$, it is easy to integrate the second pair of equations (9), too. We will not write them here because it turns out that terms related to external currents in the ionosphere can be totally neglected in them. In this connection, corresponding solutions are fully equivalent to those obtained in Paper 1. In addition, is follows from Paper 1, in the upper ionosphere only the solutions of $F_1(z)$ and $F_2(z)$ are essentially nonzero, and the complete solution of the system (5) has the form

$$\alpha = \psi^1 F_1 + \psi^2 F_2 = \begin{pmatrix} F_1\ k_{nA}/k_{\perp A} \\ \\ F_1\ k_y/k_{\perp A} \\ \\ F_2\ k_{nA}/k_0 \\ \\ -F_2\ k_y/k_0 \end{pmatrix}. \tag{20}$$

This solution involves only components related to the Alfven wave in the magnetosphere. The magnetosonic wave's electromagnetic field generated in the E-layer, does not penetrate the upper ionosphere because the ionosphere is an opacity region for it. To an accuracy of our interest, the solutions of $F_1(z)$ and $F_2(z)$ are representable as

$$F_1(z) = \left[f - i\frac{k_0(z-H)}{\cos^2\chi}(fK_P + \bar{I}_\perp)\right] \times \exp(ik_x(z-H)\tan\chi), \tag{21}$$

$$F_2(z) = -\frac{k_0}{k_{\perp A}\cos\chi}\left[(fK_P + \bar{I}_\perp)\right] \times \exp(ik_x(z-H)\tan\chi), \tag{22}$$

where

$$\bar{I}_\perp = \frac{k_{nA}\bar{I}_n + k_y\bar{I}_y}{k_{\perp A}}.$$

Using the closure condition for currents div $\mathbf{j} = 0$ we find:

$$\bar{I}_\perp = \frac{4\pi}{ck_{\perp A}} j_\parallel^*,$$

where $j_\parallel^*$ is the field-aligned current density on the upper boundary of the ionospheric conducting layer $(z = H + \Delta)$.

It follows from solutions (20-22) that transverse components of a disturbed magnetic field may be written as

$$\begin{aligned} \bar{B}_y(z) &= B_A \frac{k_{nA}}{k_{\perp A}} \exp(ik_x(z-H)\tan\chi), \\ \bar{B}_n(z) &= -B_A \frac{k_y}{k_{\perp A}} \exp(ik_x(z-H)\tan\chi), \end{aligned} \tag{23}$$

where

$$B_A = -\frac{fK_P + \bar{I}_\perp}{\cos\chi}$$

is the Alfven oscillation amplitude in the upper ionosphere. The last equality may be rewritten as

$$f = -\frac{B_A \cos\chi + \bar{I}_\perp}{K_P}. \tag{24}$$

5 Boundary conditions for magnetospheric Alfven waves on the ionosphere

Using solutions (23) it is possible to obtain boundary conditions for Alfven waves on the upper boundary of the ionosphere $(z = z_A)$. Note that the derivative of the wave's field components along a field line, once written in the coordinate system (x, y, z), has the form

$$\frac{\partial}{\partial l} = \cos\chi \frac{\partial}{\partial z} + ik_x \sin\chi.$$

Using for the derivatives with respect to z of equations (13,14), in view of the expressions (16), (18) and (24), to an accuracy of our interest we obtain

$$\left[\frac{\partial \bar{B}_{n,y}}{\partial l} + i\frac{\omega}{A^2}\frac{c}{4\pi\Sigma_P}\left(\bar{B}_{n,y}\cos\chi \pm \frac{4\pi}{c}\frac{k_{y,nA}}{k_{\perp A}^2}\bar{j}_\parallel \exp(ik_x(z-H)\tan\chi)\right)\right]_{z=z_A} = 0, \tag{25}$$

where the upper and lower signs correspond, respectively, to the indices n and y. Note that terms which are larger in magnitude than those left in parentheses, are omitted in (25) because they do not describe any new physical effects but give only a small correction to the frequency of a standing Alfven wave in the magnetosphere. The terms that are retained, describe the source of Alfven waves (external currents) and their dissipation in the ionosphere. Using Maxwell's equations (1) we now write analogous boundary conditions for transverse components of the wave's electric field

$$\left[\bar{E}_{n,y} + i\frac{c^2\cos\chi}{4\pi\omega\Sigma_P}\frac{\partial\bar{E}_{n,y}}{\partial l} - \right. \tag{26}$$

$$\left. i\frac{k_{nA,y}}{k_{\perp A}^2}\frac{\bar{j}_\parallel}{\Sigma_P}\exp(ik_x(z-H)\tan\chi)\right]_{z=z_A} = 0,$$

By application of the inverse Fourier-transform (2) it is possible in (35) and (22) to switch to boundary conditions for a monochromatic Alfven wave in the magnetosphere

$$\left.\frac{\partial\tilde{B}_{n,y}}{\partial l}\right|_{l=l_*} = i\frac{v_*\omega}{A_*^2}\left[\tilde{B}_{n,y} \mp \frac{4\pi}{c}\nabla_{y,n}\tilde{J}_\parallel\right]_{l=l_*}, \tag{27}$$

$$\left.\tilde{E}_{n,y}\right|_{l=l_*} = \left[-i\frac{v_*}{\omega}\frac{\partial\tilde{E}_{n,y}}{\partial l} + \frac{\nabla_{n,y}\tilde{J}_\parallel}{V}\right]_{l=l_*}. \tag{28}$$

Here the index $*$ stands for quantities taken on the upper boundary of the ionosphere $(z = z_A)$, with the designations

$$\nabla_{y,n} \equiv \frac{\partial}{\partial n}, \frac{\partial}{\partial y}; \quad v = \frac{c^2\cos\chi}{4\pi\Sigma_P}; \quad V = \frac{\Sigma_P}{\cos\chi}.$$

The function $\tilde{J}_\parallel$ represents the solution of the equation

$$\triangle_\perp\tilde{J}_\parallel = \tilde{j}_\parallel(x - x_A, y, z_A, \omega),$$

where $\triangle_\perp = \nabla_n^2 + \nabla_y^2$ is the transverse Laplacian operator, $x_A = (z_A - H - \Delta)\tan\chi$.

Boundary conditions of the form (28) were used in Paper 2 in solving the problem of the spatial structure of a transversally-small-scale $(m \gg 1)$ monochromatic Alfven wave in the magnetosphere. Solutions obtained permit us to write the amplitude of such a pave excited by external currents in the E-layer, on the upper boundary of the ionosphere

$$B_a = \frac{2c}{\omega t_A A_0}\frac{k_y l_N}{\lambda_{PN}}\frac{\tilde{J}_\parallel^*}{V}, \tag{29}$$

where A_0 is a value of the Alfven velocity on the magnetic shells under consideration in the equatorial plane, and t_A is the transit time with the Alfven velocity along a field line between the magnetically-conjugate ionospheres. In addition, two intrinsic scales are involved in (29): l_N - a typical transverse scale of the ionospheric plasma inhomogeneity, and λ_{PN} - a typical transverse Alfven wavelength in the neighborhood of a resonance magnetic shell where it is generated. Explicit expressions for these parameters may be found in Paper 2. In regards to order-of-magnitude estimations, we wish to note that

$$k_y l_N \sim m,$$

where $m \gg 1$ is azimuthal wave number,

$$\omega t_A \sim N,$$

where N is the harmonic number of standing Alfven waves in the magnetosphere (for the fundamental harmonics of our interest, $N \sim 1$), and

$\lambda_{PN} \sim l_N^{1/3}/k_y^{2/3}$.

Substituting (29) into (24) and then into (16) and (18) we find that the terms of these equations that are proportional to currents, are small if

$$\frac{m^{2/3}}{N}\frac{c^2}{A_0\Sigma_P} \gg 1. \tag{30}$$

For typical values of the parameters $A_0 \sim 10^3 km/s$ and $\Sigma_P \sim 10S \sim 10^8 km/s$ the condition (30) reduces to the requirement that $m^{2/3} \gg N$ which is satisfied according to the condition of the problem.

6 Electromagnetic oscillations, induced on the terrestrial surface by standing Alfven waves in the magnetosphere

The condition (30) permits neglecting a direct effect of external currents in the ionosphere on electromagnetic oscillations on the terrestrial surface. This is because the Alfven waves are ”tied” to a particular magnetic resonance shell. As time progresses, the wave ”accumulate” on it an amplitude such that the amplitude of electromagnetic oscillations induced by them on the terrestrial surface becomes considerably larger compared to oscillations directly associated with currents. As a result, on the terrestrial surface it becomes possible to use the formulas obtained in Paper 1:

$$\bar{B}_x(0) = \bar{B}_y(z_A)\bar{R}(k_x,k_y)\cos\chi,$$

$$\tag{31}$$

$$\bar{B}_y(0) = -\bar{B}_n(z_A)\bar{R}(k_x,k_y),$$

where

$$\bar{R}(k_x,k_y) = \frac{1}{\Sigma_P}\int_0^\infty\left(\sigma_H(z) - i\frac{k_y}{k_t}\sigma_P(z)\sin\chi\right)\times \exp(-k_t z + ik_x(z-z_A)\tan\chi)dz.$$

Formulas (31) establish a link between the Alfven oscillation field on the upper boundary of the ionosphere $(z = z_A)$ and electromagnetic oscillations on the terrestrial surface $(z = 0)$. Expressions for the other components of the field may be found in Paper 1. It follows from the results reported in Paper 2 that a transversally-small-scale Alfven wave is localized across magnetic shells in the interval $\Delta x_N \equiv x_{TN} - x_{PN} \ll l_N$. Here x_{TN} and x_{PN} are coordinates of the magnetic resonance shells, on which the wave's frequency ω coincides, respectively, with the poloidal $\omega = \Omega_{PN}(x_{PN})$ and toroidal $\omega = \Omega_{PN}(x_{PN})$ eigenfrequencies of magnetospheric Alfven oscillations. The wave is excited in the neighborhood of the poloidal magnetic shell $(x = x_{PN})$ and propagates toward the toroidal shell $(x = x_{TN})$ where it is totally absorbed dup to Joule dissipation in the ionosphere. Note that both poloidal (near $x = x_{PN}$) and toroidal (near $x = x_{TN}$) oscillations are present in the standing Alfven waves with $m \gg 1$ under consideration. The correlation of their amplitudes and their spatial structure are quite definite and are given in what follows.

The spatial structure of the transverse components of the field of the given Alfven wave on the upper boundary of the ionosphere $(z = z_A,$ or $l = l_*)$ is representable as (see Paper 2):

when $|x - x_{PN}| \ll \Delta x_N$

$$\begin{aligned} \tilde{B}_{nN}(x, k_y, l_*, \omega) &= B_A G(\xi_{PN}(x, \omega)), \\ \tilde{B}_{yN}(x, k_y, l_*, \omega) &= i\frac{B_A}{k_y \lambda_{PN} \cos\chi} G'(\xi_{PN}(x, \omega)), \end{aligned} \tag{32}$$

when $x - x_{PN} \gg \lambda_{PN}$ and $x_{TN} - x \gg \lambda_{TN}$

$$\begin{aligned} &\tilde{B}_{nN}(x, k_y, l_*, \omega) = \\ &\quad -\sqrt{\pi} B_A \left(\frac{v^*_{PN}}{v^*_N} \frac{k_y^2 \cos^2\chi}{k_{xN}^2 + k_y^2 \cos^2\chi} \right)^{1/2} \times \\ &\quad \exp(i\Psi_N(x) - \Gamma_N(x) + i\pi/4), \\ &\tilde{B}_{yN}(x, k_y, l_*, \omega) = -\frac{k_{xN}}{k_y \cos\chi} \tilde{B}_{nN}(x, k_y, l_*, \omega), \end{aligned} \tag{33}$$

and when $|x - x_{TN}| \ll \Delta x_N$

$$\begin{aligned} &\tilde{B}_{nN}(x, r_y, l_*, \omega) = \\ &\quad -2ik_y \lambda_{PN} \cos\chi B_A \; g(\xi_{TN}(x, \omega)) \times \\ &\quad \exp(i\bar{\Psi}_N - \bar{\Gamma}_N), \\ &\tilde{B}_{yN}(x, k_y, l_*, \omega) = 2\frac{\lambda_{PN}}{\lambda_{TN}} B_A \; g'(\xi_{TN}(x, \omega)) \times \\ &\quad \exp(i\bar{\Psi}_N - \bar{\Gamma}_N). \end{aligned} \tag{34}$$

Not that in these expressions the overscribed tilde denotes a separate Fourier-harmonic in the expansion of the oscillations not only in terms of frequencies ω but also azimuthal wave numbers k_y . In the expressions (32-34), λ_{PN} and λ_{TN} stand for a typical wavelength across magnetic shells, respectively, near the poloidal and toroidal resonance shells. The functions $G(\xi)$ and $G'(\xi)$, and $g(\xi)$ and $g'(\xi)$ describe the wave's field structure in coordinate x near these shells and have the following integral representations

$$\begin{aligned} G(\xi) &= -\int_0^\infty \exp(is\xi - is^3/3) ds, \quad G'(\xi) = \frac{\partial G}{\partial \xi}, \\ g(\xi) &= \frac{1}{2}\int_0^\infty s^{-1} \exp(is\xi - i/s) ds, \quad g'(\xi) = \frac{\partial g}{\partial \xi}. \end{aligned} \tag{35}$$

The arguments of these functions in (32) and (34) have the form

$$\xi_{PN}(x,\omega) = \frac{x - x_{PN}(\omega)}{\lambda_{PN}} + i\epsilon_{PN},$$

$$\xi_{TN}(x,\omega) = \frac{x - x_{TN}(\omega)}{\lambda_{TN}} + i\epsilon_{TN},$$

where $\epsilon_{(P,T)N} = \gamma_{(P,T)N} l_N / \Omega_{(P,T)N} \lambda_{(P,T)N}$, and $\gamma_{(P,T)N}$ are values of the damping decrement of the waves on the ionosphere near corresponding resonance surfaces (the decrement is thought of being small:$\gamma_{(P,T)N} \ll \Omega_{(P,T),N}$, see Paper 2).

The expressions (33) describe the wave's field in the region between the resonance surfaces. In these formulas, $k_{x,N}(x,\omega)$ is the component of the horizontal wave vector in coordinate x in the WKB approximation. The function $k_{x,N}(x,\omega)$ is defined by a considerably complicated expression (see Paper 2); however, its behavior is quite well simulated by the expression

$$k_{x,N}(x,\omega) = k_y \left(\frac{x - x_{PN}(\omega)}{x_{TN}(\omega) - x} \right)^{1/2}. \tag{36}$$

The function $v_N^x(x,\omega)$ is a component of the group velocity on coordinate x and is defined as

$$v_N^x(x,\omega) = \frac{\partial k_{xN}(x,\omega)}{\partial \omega},$$

and v_{PN}^x is a typical value of $v_N^x(x,\omega)$ near the poloidal resonance surface where we may write

$$v_N^x(x,\omega) \approx v_{PN}^x \sqrt{\frac{x - x_{PN}(\omega)}{\lambda_{PN}}}.$$

The function

$$\Psi_N(x) = \int_{X_{PN}}^{X} k_{xN}(x',\omega) dx'$$

is phase run-up, and the function

$$\Gamma_N(x) = \int_{X_{PN}}^{X} \frac{\gamma_N(x',\omega)}{v_N^x(x',\omega)} dx'$$

is an integral decrement, both of which are acquired by the wave as it propagates across the magnetic shells from the poloidal resonance surface to the point x inside the interval Δx_N. Accordingly, $\bar{\Psi} = \Psi(x_{TN})$ is a total phase run-up on the interval Δx_N , and $\bar{\Gamma} = \Gamma(\bar{x})$ is an integral decrement as the wave travels from $x = x_{PN}$ to $x = \bar{x}$, an arbitrary point located near the toroidal surface: $|\bar{x} - x_{TN}| \ll \Delta x_N$.

To determine the structure of the oscillation field on the ground, it is necessary to Fourier-expand the expressions (32-34) in terms of wave numbers k_x . Next, using coupling formulas (31) it is essential to carry out an inverse Fourier-transform of the form (2) of the oscillation field on the terrestrial surface. Through integral representations (35), we find the Fourier-transforms of the functions g and G in the form

$$\bar{G}(l_*) = -i2\pi\lambda_{PN}\theta(k_x) \times \\ \exp(-i\frac{(k_x\lambda_{PN})^3}{3} - ik_x x_{PN} - \epsilon_{PN} k_x \lambda_{PN}),$$

$$\bar{G}'(l_*) = -ik_x\lambda_{PN}\bar{G}(l_*),$$

(37)

$$\bar{g}(l_*) = \pi k_x^{-1}\theta(k_x) \times$$
$$\exp(-\frac{i}{k_x\lambda_{TN}} - ik_x x_{TN} - \epsilon_{TN}k_x\lambda_{TN}),$$
$$\bar{g}'(l_*) = ik_x\lambda_{TN}\bar{g}(l_*).$$

To carry out a similar expansion of functions (33), we take advantage of the fact that their exponent involves a large quasiclassical phase $|\Psi(x)| \gg 1$. When evaluating integrals of the form

$$\bar{B} = \int_{-\infty}^{\infty} \varphi(x)\exp(i\Psi(x) - ik_x x + i\frac{\pi}{4})dx,$$

where $\varphi(x)$ is the corresponding pre-exponent, this makes it possible to use the stationary-phase method. Upon setting the first derivative of the exponent equal to zero, we obtain the equation that defines the saddle-point $\bar{x}$:

$$\left.\frac{\partial\Psi}{\partial x}\right|_{x=\bar{x}} = k_{xN}(\bar{x},\omega) = k_x. \quad (38)$$

The second derivative of phase at the saddle-point has the form

$$\left.\frac{\partial^2\Psi}{\partial x^2}\right|_{x=\bar{x}} = \left.\frac{k_{xN}(x,\omega)}{\partial x}\right|_{x=\bar{x}} = -\frac{\omega}{2l_N}\frac{1}{v_N^x(\bar{x},\omega)}.$$

In this equality, we rely on the fact that the wave involved is localized in the interval $\Delta x_N \ll l_N$. The dependence $k_{xN}(x,\omega)$ may be written in a general form as (see Paper 2)

$$k_{xN}(x,\omega) \equiv k_{xN}(x - x_{PN}(\omega)).$$

From this it follows that

$$\frac{\partial k_{xN}}{\partial\omega} = -\frac{\partial x_{PN}}{\partial\omega}\frac{\partial k_{xN}}{\partial x}.$$

Because the wave is localized within a narrow interval of magnetic shells, we can use for $x_{PN}(\omega)$ a linear approximation

$$x_{PN}(\omega) \approx x - 2\frac{\omega - \Omega_{PN}(x)}{\omega}l_N,$$

which does yield the result obtained. Using standard formulas from the stationary-phase method *Budden*(1961) we get

$$\bar{B}_{nN}(l_*) = -2i\pi B_A\theta(k_x)\frac{k_y\lambda_{PN}\cos\chi}{\sqrt{k_{xN}^2 + k_y^2\cos^2\chi}} \times$$
$$\exp(i\Psi(\bar{x},\omega) - ik_x\bar{x} - \Gamma(\bar{x},\omega)),$$

(39)

$$\bar{B}_{yN}(l_*) = -\frac{k_x}{k_y\cos\chi}\bar{B}_{nN}(l_*).$$

In the case of an inverse Fourier-transform (2) on the terrestrial surface the functions $\bar{G}$ and $\bar{g}$ again convolves to G and g (as also do $\bar{G}'$ and $\bar{g}'$ to G' and g'); as a result, we have

when $|x - x_{PN}| \ll \Delta x_N$

$$\begin{aligned}\tilde{B}_{xN}(0) &= i\frac{B_A}{k_y\lambda_{PN}\Sigma_P} \times \\ &\int\limits_0^\infty \left[\sigma_H(z) - i\frac{k_y}{|k_y|}\sigma_P(z)\sin\chi\right] \times \\ &G'(\tilde{\xi}_{PN}(x_z,\omega))\, e^{-k_y z}dz, \\ \tilde{B}_{yN}(0) &= -\frac{B_A}{\Sigma_P}\int\limits_0^\infty \left[\sigma_H(z) - i\frac{k_y}{|k_y|}\sigma_P(z)\sin\chi\right] \times \\ &G(\tilde{\xi}_{PN}(x_z,\omega))\, e^{-k_y z}dz,\end{aligned} \tag{40}$$

and when $|x - x_{TN}| \ll \Delta x_N$

$$\begin{aligned}\tilde{B}_{xN}(0) &= 2\frac{\lambda_{PN}}{\lambda_{TN}}\frac{B_A\cos\chi}{\Sigma_P}\exp(i\bar{\Psi}_N - \bar{\Gamma}_N) \times \\ &\int\limits_0^\infty \sigma_H(z)\ g'(\tilde{\xi}_{TN}(x_z,\omega))dz, \\ \tilde{B}_{yN}(0) &= 2ik_y\lambda_{PN}\frac{B_A\cos\chi}{\Sigma_P}\exp(i\bar{\Psi}_N - \bar{\Gamma}_N) \times \\ &\int\limits_0^\infty \sigma_H(z)g(\tilde{\xi}_{TN}(x_z,\omega))dz.\end{aligned} \tag{41}$$

Here $x_z = x + (z - z_A)\tan\chi$ is the coordinate x projected along a field line from the height z onto the upper boundary of the ionosphere (see Fig. 2). The arguments of the functions $g(\xi)$ and $G(\xi)$ have the form

$$\tilde{\xi}_{PN}(x_z,\omega) = \frac{x_z - x_{PN}}{\lambda_{PN}} + i\epsilon_{PN},$$

$$\tilde{\xi}_{TN}(x_z,\omega) = \frac{x_z - x_{TN}}{\lambda_{TN}} + i\left(\epsilon_{TN} + \frac{z}{\lambda_{TN}}\right).$$

When deriving the expressions (40), allowance was made for the fact that the main contribution to the Fourier-integral of the functions G and G' is made by harmonics with $k_x \ll k_y$, and in (31) we can put $k_t \approx k_y$. Similarly, when calculating (41), it was kept in mind that the main contribution to the Fourier-integral of the functions g and g' is made by harmonics with $k_x \gg k_y$, and in (31) one may put $k_t \approx k_x$.

When carrying out an inverse Fourier-transform of functions (39) on the terrestrial surface, we again make use of a large quasiclassical phase $|\Psi(\bar{x}(k_x),\omega)| \gg 1$ involved in the exponent. To evaluate integrals of the form

$$\begin{aligned}\tilde{B} &= \int\limits_0^\infty \tilde{\varphi}(k_x) \times \\ &\exp(i\Psi(\bar{x}(k_x)) - ik_x(x_z - \bar{x}(k_x)) - k_t z)dk_x\end{aligned}$$

we are using the stationary-phase method. Here $k_t = (k_x^2 + k_y^2)^{1/2}$, and $\bar{x}(k_x)$ is defined by equation (38). Upon setting the first derivative of the exponent to zero, we obtain the equation

$$x_z - \bar{x}(\bar{k}_x) + i\frac{\bar{k}_x}{\bar{k}_t}z = 0,$$

that defines the saddle-point $\bar{k}_x$ (according to (38), $\bar{k}_x = k_{xN}(\bar{x}(\bar{k}_x, \omega)\ )$. This equation can be considered to be an equation defining $\bar{\bar{x}} = \bar{x}(\bar{k}_x)$ and can be rewritten as

$$x_z - \bar{\bar{x}} + i\frac{k_{xN}(\bar{\bar{x}}, \omega)}{\bar{k}_t}z = 0,$$

where $\bar{k}_t = (k_{xN}^2(\bar{\bar{x}}, \omega) + k_y^2)^{1/2}$. The second derivative of the exponent at the saddle-point is

$$\frac{2l_N}{\omega}\bar{v}_N^x \equiv \frac{2l_N}{\omega}v_N^x(\bar{\bar{x}}, \omega) + i\frac{k_y^2}{\bar{k}_t^3}z.$$

Using standard formulas from the stationary phase method *Budden*(1961) on the terrestrial surface for $x_z - x_{PN} \gg \lambda_{PN}$ and $x_{TN} - x_z \gg \lambda_{TN}$ we obtain:

$$\begin{aligned}\tilde{B}_{nN}(0) = \pi\frac{B_A}{\Sigma_P}\int_0^\infty \left[\sigma_H(z) - i\frac{k_y}{\bar{k}_t}\sigma_P(z)\sin\chi\right] \times \\ \left(\frac{v_{PN}^x}{|\bar{v}_N^x|}\frac{\bar{k}_{xN}^2\cos^2\chi}{\bar{k}_{xN}^2 + k_y^2\cos^2\chi}\right)^{1/2} \times \\ \exp\left(i\Psi(\bar{x}) - \Gamma(\bar{x}) - \frac{k_y^2 z}{\bar{k}_t} + i\left(\frac{\pi}{4} - \frac{\arg\,\bar{v}_N^x}{2}\right)\right) dz, \\ \tilde{B}_y(0) = \sqrt{\pi}\frac{B_A}{\Sigma_P}\int_0^\infty \left[\sigma_H(z) - i\frac{k_y}{\bar{k}_t}\sigma_P(z)\sin\chi\right] \times \\ \left(\frac{v_{PN}^x}{|\bar{v}_N^x|}\frac{\bar{k}_y^2\cos^2\chi}{\bar{k}_{xN}^2 + k_y^2\cos^2\chi}\right)^{1/2} \times \\ \exp\left(i\Psi(\bar{x}) - \Gamma(\bar{x}) - \frac{k_y^2 z}{\bar{k}_t} + i\left(\frac{\pi}{4} - \frac{\arg\,\bar{v}_N^x}{2}\right)\right) dz.\end{aligned} \tag{42}$$

Formulas (40-42) do solve entirely the problem of the penetration of the field of monochromatic transversally-small-scale Alfven oscillations of the magnetosphere on the terrestrial surface. These formulas can be written in a most straightforward manner if the functions $\sigma_{P,H}(z)$ are localized (at height H, say) on a scale much smaller than a typical vertical scale of the wave in the ionosphere. Mathematically, the ionosphere can then be thought of as being a thin layer, and all functions (excepting $\sigma_{P,H}(z)$) at the point $z = H$ can be factored outside the integral sign. As a result, we obtain

when $|x_H - x_{PN}| \ll \Delta x_N$

$$\begin{aligned}\tilde{B}_{xN}(0) &= i\frac{\bar{B}}{k_y\lambda_{PN}}\, G'(\tilde{\xi}_{PN})\exp(-k_y H), \\ \tilde{B}_{yN}(0) &= -\bar{B}\, G(\tilde{\xi}_{PN})\exp(-k_y H),\end{aligned} \tag{43}$$

when $x_H - x_{PN} \gg \lambda_{PN}$ and $x_{TN} - x_H \gg \lambda_{TN}$

$$\tilde{B}_{xN}(0) = \sqrt{\pi}\bar{B}\left(\frac{v_{PN}}{|\bar{v}^x_N|}\frac{\bar{k}^2_{xN}\cos^2\chi}{\bar{k}^2_{xN}+k^2_y\cos^2\chi}\right)^{1/2}\times$$
$$\exp\left(i\Psi(\bar{x})-\Gamma(\bar{x})-\frac{k_y^2 H}{\bar{k}_t}+i\left(\frac{\pi}{4}-\frac{\arg\bar{v}^x_N}{2}\right)\right), \tag{44}$$
$$\tilde{B}_{yN}(0) = \frac{k_y}{\bar{k}_{xN}}\tilde{B}_{xN}(0),$$

and when $|x_H - x_{TN}| \ll \Delta x_N$

$$\begin{aligned} \tilde{B}_{xN}(0) &= 2\frac{\lambda_{PN}}{\lambda_{TN}}\bar{B}\; g'(\tilde{\xi}_{TN})\exp(i\bar{\Psi}_N-\bar{\Gamma}_N), \\ \tilde{B}_{yN}(0) &= 2ik_y\lambda_{PN}\cos\chi\bar{B}\; g(\tilde{\xi}_{TN})\exp(i\bar{\Psi}_N-\bar{\Gamma}_N). \end{aligned} \tag{45}$$

Here

$$\begin{aligned} \bar{B} &= \frac{B_A}{\Sigma_P}\left(\Sigma_H - i\frac{k_y}{|k_y|}\Sigma_P\sin\chi\right), \\ x_H &= x+(H-z_A)\tan\chi, \quad \bar{\bar{x}} \;=\; x_H + i\frac{\bar{k}_{xn}H}{\bar{k}_t}, \\ \tilde{\xi}_{PN} &= \frac{\bar{\bar{x}}-x_{PN}}{\lambda_{PN}}+i\epsilon_{PN}, \\ \tilde{\xi}_{TN} &= \frac{\bar{\bar{x}}-x_{TN}}{\lambda_{TN}}+i\left(\epsilon_{TN}+\frac{H}{\lambda_{TN}}\right), \\ \bar{v}^x_N &= v^x_N(\bar{\bar{x}},\omega)+i\frac{\omega k_y^2 H}{2\bar{k}_t^3 l_N}. \end{aligned}$$

Qualitatively, the scheme for the penetration of the electromagnetic field of Alfven oscillations is presented in Fig. 2. In the upper ionosphere, the Alfven wave propagates along field lines to the lower ionosphere where currents generating a magnetosonic waves are induced under the action of its variable electromagnetic field. These currents in magnitude turn out to be much larger than external currents which have generated the Alfven wave (amplitude ”accumulation” effect). The field of these oscillations (largely magnetosonic) is permitted into the atmosphere and reaches the terrestrial surface.

7 Discussion of the results

Let us consider some results that follow from general formulas (40-43). By comparing them with formulas (32-34) governing the Alfven wave field on the upper boundary of the ionosphere, we see that in the case of the penetration to the ground the relationship between the components of the electromagnetic field changes. This is reflected in hodographs of the oscillations constructed at different points inside the transparency region (see Fig. 2). In the magnetosphere, near the poloidal resonance surface (where the Alfven wave is generated) the amplitude of the B_{nN} -component is much larger than the amplitude of B_{yN}. In this case, as follows from (31), the hodograph in the plane (B_n, B_y) rotates clockwise. Near the toroidal resonance surface (where the Alfven oscillations are totally absorbed), on the contrary, the amplitude of B_{yN} is much larger than that of B_{nN} , and the hodograph rotates anticlockwise. On the terrestrial surface, the situation is the opposite. Near a point related along a field line to x_{PN} (the poloidal resonance surface) the amplitude of B_{yN} is much larger than that of B_{xN} . Near a point related to the toroidal resonance surface, x_{TN} , the amplitude of B_{xN} is much larger than the amplitude of B_{yN}. The fact, well known from theory, is thereby confirmed that the polarization ellipse of the electromagnetic field of the Alfven wave rotates through $\pi/2$ as it penetrates to the ground *Hughes*(1974),*Hughes and Southwood*(1976a). Note that this occurs for the model of a horizontally homogeneous ionosphere only. For a horizontally inhomogeneous ionosphere, the rotation of the polarization ellipse can differ greatly from $\pi/2$ *Glassmeier*(1983),*Glassmeier*(1984).

Nevertheless, the results obtained in this study can also be applied, with some constraints, for the horizontally- inhomogeneous ionosphere. These constraints are associated with the the characteristic horizontal scale of a standing Alfven wave in the ionosphere. If this scale is much smaller than the typical horizontal scale of ionospheric inhomogeneity, then the horizontal structure can be expanded, with a certain accuracy, into a Fourier integral in terms of harmonics with a definite k_t. All the subsequent mathematics is similar to that presented in this paper. The largest horizontal scale of the waves under consideration is represented by the wavelength near the poloidal resonance surface (see Paper 2) $\lambda_{PN} \sim l_N\ N^{1/3}/m^{2/3}$, where l_N is a typical horizontal scale of magnetospheric plasma inhomogeneity on the upper boundary of the ionosphere. Also, $N \sim 1$ and $m \gg 1$ according to the condition of the problem formulated. If it is assumed, for estimating purposes, that $l_N = 10^4$ km, $N = 1$, and $m = 125$, then $\lambda_{PN} \sim 40$ km. Thus, if the horizontal scale of ionospheric plasma inhomogeneity becomes larger than this scale, then the results reported in this paper will be applicable for the waves considered. An additional indication of the validity of the theory presented here should be considered to be the presence of a rotation of the plane of polarization of the oscillations as they penetrate from the ionosphere to the ground by $\pi/2$. In this paper we examine the main modes of standing Alfven waves ($N \sim 1$) because, as follows from findings reported in Paper 2, it is for them that the process of excitation by external currents in the ionosphere is most effective. Besides, for these waves in Paper 2 a sufficiently rigorous theory was constructed to describe their total spatial structure in the magnetosphere.

Yet another consequence of the general formulas is that not only the field of currents associated with Hall conductivity but also the field caused by Pedersen conductivity of the ionosphere penetrate to the terrestrial surface. This phenomenon occurs only for the case of an inclined geomagnetic field when the horizontal wave vector lies outside the magnetic meridional plane ($k_y \neq 0$). All preceding work (except Paper 1) addressed situations that did not permit this phenomenon to be uncovered. Those publications considered the case of either a vertical geomagnetic field $(\chi = 0)$ or (with an inclined geomagnetic field) a meridional distribution of waves $(k_y = 0)$, with the result that only oscillations associated with Hall currents in the ionosphere penetrate to the terrestrial surface. It follows from formulas (40-45) that Pedersen conductivity is particularly important in the neighborhood of the poloidal resonance shell (where $k_y \gg k_{xN}$) and becomes unimportant near the toroidal shell. Thus, Pedersen conductivity is vital to the penetration to the ground of the poloidal part of the Alfven oscillations of the ionosphere. Let us consider the way in which the structure of the electromagnetic oscillation field changes in coordinate x as the oscillations penetrate to the ground. We confine ourselves to the case when the scale of localization of the Alfven wave in coordinate x is much larger than the atmospheric thickness

$$\Delta x_{N} \gg H. \qquad (46)$$

As follows from the expressions (43-45), only in this case will the oscillation amplitude on the ground be comparable with that in the ionosphere. By making recourse, for $k_{xN}(x,\omega)$, to the model (36) for $\bar{\bar{x}}$ in the limit of (46), we find

$$\bar{\bar{x}} \approx x_z + iz\sqrt{\frac{x_z - x_{PN}}{\Delta x_N}}.$$

In this case

$$\bar{k}_{xN} \equiv k_{xN}(\bar{\bar{x}},\omega) \approx k_{xN}(x_z,\omega) + i\frac{k_y z\sqrt{\Delta x_N}}{(x_{TN} - x_z)^{3/2}}.$$

It follows from the expressions (43) that near the poloidal resonance surface $\bar{\bar{x}} \approx x_z$, and hence the structure of the electromagnetic oscillation field on the ground is the same as in the magnetosphere (except the rotation of the polarization ellipse through $\pi/2$). The oscillation amplitude on the ground then varies by a factor of $(|B|/B_a)\exp(-k_y H)$.

In the same limit of (46), the functions $\Psi(\bar{\bar{x}},\omega), \Gamma(\bar{\bar{x}},\omega)$ and $k_t(\bar{\bar{x}},\omega)$ in formulas (44) have the form

$$\begin{aligned}
\Psi(\bar{\bar{x}},\omega) &\approx \Psi(x_z,\omega) + i\frac{z}{(\Delta x_N)^{3/2}}\frac{x_z - x_{PN}}{(x_{TN} - x_z)^{1/2}},\\
\Gamma(\bar{\bar{x}},\omega) &\approx \Gamma(x_z,\omega)\left(1 + i\frac{k_y}{k_{xN}(\bar{x}_z,\omega)}\sqrt{\frac{\Delta x_N}{x_{TN} - x_z}}\right),\\
\bar{k}_t(\bar{\bar{x}},\omega) &\approx \left(k_t^2 + ik_y^2\,\frac{z\sqrt{\Delta x_N(x_z - x_{PN})}}{(x_{TN} - x_z)^2}\right)^{1/2}, \qquad (47)
\end{aligned}$$

where

$$\Psi(x_z,\omega) = k_y \Delta x_N \times$$
$$\left[\frac{\pi}{2} - \frac{\sqrt{(x_z - x_{PN})(x_{TN} - x_z)}}{\Delta x_N} - \arcsin\sqrt{\frac{x_{TN} - x_z}{\Delta x_N}}\right]$$
$$\Gamma(x_z,\omega) = 2\frac{\gamma_N}{\omega}k_{xN}(x_z,\omega)l_N,$$
$$k_t = (k_{xN}^2(x_z,\omega) + k_y^2)^{1/2}.$$

Hence it is evident that in the region between the resonance surfaces with the constraint (46) the oscillations receive only a small addition to the phase. A change in their amplitude in this case is determined by the combined contribution of the imaginary part of the phase $\Psi(x,\omega)$ and the real parts of the decrement $\Gamma(x,\omega)$ and $k_y^2 H/k_t$.

Strong distortion of the spatial structure occurs only as one approaches the toroidal resonance surface $(x_z \to x_{TN})$. Note that the expansions (47)become inapplicable. The field structure near the toroidal resonance surface depends considerably from the value of H/λ_{TN} . If $H \ll \lambda_{TN}$, then (when $\epsilon_{TN} \ll 1$ as is assumed here) the field structure is virtually the same as in the magnetosphere (as is viasualized in Fig.3a). In the inverse limit ($H \gg \lambda_{TN}$), we can use for the function $g(\xi)$ its asymptotic representation with larger arguments ($|\xi| \gg 1$):

$$g(\tilde{\xi}_{TN}) \approx \frac{\sqrt{\pi}}{2|\xi_{TN}|^{1/4}} \times \\ \exp\left(-2i\sqrt{\xi_{TN}} - i\left(\frac{\pi}{4} - \frac{3\,\arg\,\xi_{TN}^{-3/2}}{2}\right)\right) \\ \approx \frac{\sqrt{\pi}}{2}\left(\frac{\lambda_{TN}}{H}\right)^{1/4} \times \\ \exp\left(\frac{x_{TN} - x_z}{\sqrt{\lambda_{TN} H}} \mathrm{e}^{-i\pi/4} - \\ \sqrt{\frac{\lambda_{TN}}{H}}\left(\epsilon_{TN} + \frac{H}{\lambda_{TN}}\right)\mathrm{e}^{i\pi/4} - i\frac{\pi}{8}\right).$$

Hence it is evident that the wavelength in coordinate x increases from λ_{TN} in the magnetosphere to $\sqrt{\lambda_{TN} H}$ on the ground (see Fig. 3b). The oscillation amplitude in this case decreases by a factor of $\exp(-\sqrt{H/2\lambda_{TN}}) \ll 1$.

8 Conclusions

The main results of this study may be summarized as follows.

1. We have obtained boundary conditions on the ionosphere for standing Alfven waves in the ionosphere, the sources for which are provided by external currents in the ionospheric E-layer (formulas (27-28)).

2. We have obtained analytic expressions to describe the field of electromagnetic oscillations induced on the terrestrial surface by standing Alfven waves in the magnetosphere. Waves with large wave numbers ($m \gg 1$) have been considered.

3. It has been shown that a substantial role in the process of penetration to the ground of the poloidal part of the electromagnetic field of magnetospheric Alfven oscillations is played by Pedersen conductivity of the ionosphere.

4. Near the toroidal resonance surface, the oscillations can undergo ”blurring” in coordinate x if the wavelength in this coordinate λ_{TN} in the magnetosphere is less than the atmospheric thickness H.

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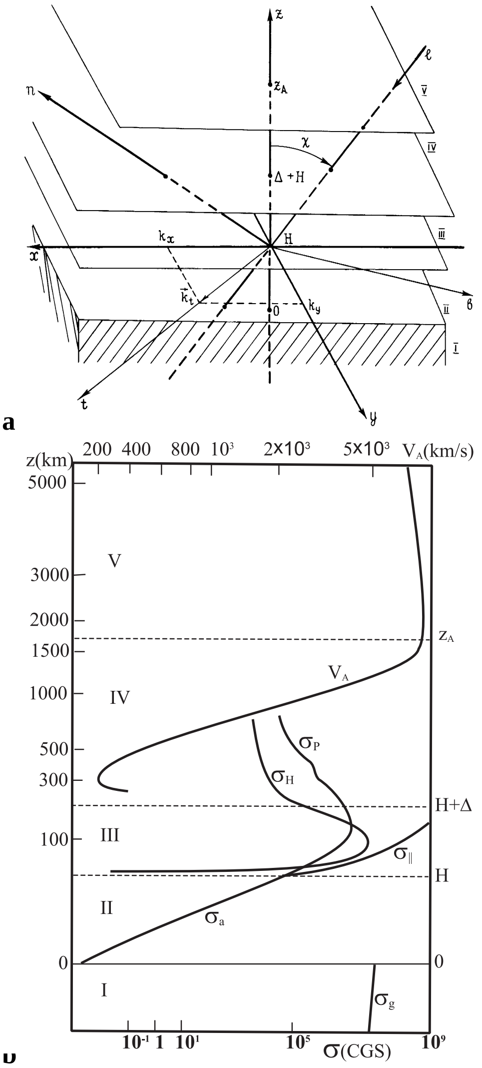
Figure Captions

Fig. 1. (a) - The mutual position of the three coordinate systems used in this paper: (x, y, z), (t, b, z), (n, y, l). (b) - Typical height profiles of the components of the conductivity tensor $\hat{\sigma}$ and Alfven velocity A. Roman numerals refer to the following layers: I - Earth with isotropic conductivity σ_g , II - atmosphere with conductivity σ_a , III - lower ionosphere with transverse Pedersen σ_P and Hall σ_H conductivities, and field-aligned conductivity $\sigma_\|$ (the dash-dotted line slows the model value of $\sigma_\| = \infty$ as used in this paper), IV - upper ionosphere where $\sigma_P, \sigma_H \to 0$, and V-magnetosphere.

Fig. 2. Scheme for propagation from the magnetosphere to the ground of the field of a standing Alfven wave excited by external currents in the ionosphere. The penetration of the wave's field from the magnetosphere to the boundary with the atmosphere $(z = H)$ proceeds along geomagnetic field lines. Both in the magnetosphere and on the ground, the wave is a running wave across magnetic shells from the poloidal resonance surface (where it is generated) to the toroidal resonance surface (where it is totally absorbed). The propagation from the magnetosphere to the ground is accompanied by the rotation of the polarization ellipse of the oscillations through $\pi/2$ and by the rotation reversal of their hodograph.

Fig. 3. Scheme for penetration of the Alfven oscillation field from the magnetosphere to the ground in two limiting cases. Case (a) corresponds to the condition $\lambda_{TN} \gg H$. Here the spatial structure of the oscillation field on the ground is similar to the transverse structure of the Alfven wave's field on the upper boundary of the ionosphere (but for the rotation of the polarization ellipse through $\pi/2$). Case (b) corresponds to $\lambda_{TN} \ll H$. Here the wavelength near the toroidal resonance surface increases from λ_{TN} in the magnetosphere and to $\sqrt{\lambda_{TN} H}$ on the terrestrial surface.

Figures



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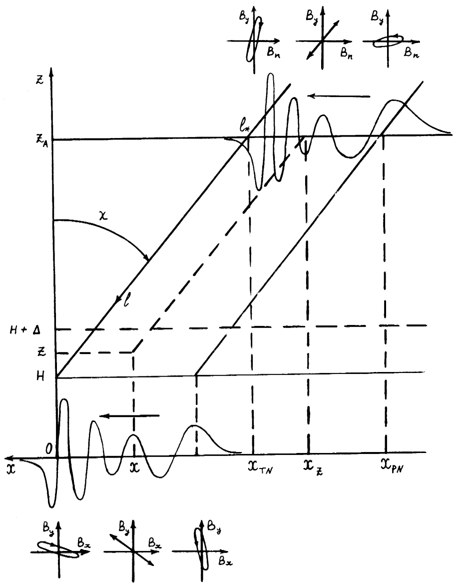

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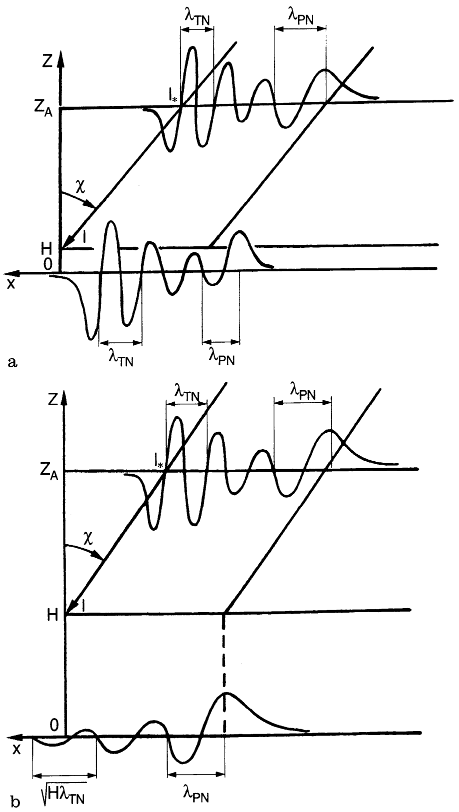

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